FEDERAL RESERVE BANK *of* ATLANTA WORKING PAPER SERIES

# Search Complementarities, Aggregate Fluctuations, and Fiscal Policy

Jesús Fernández-Villaverde, Federico S. Mandelman,
Yang Yu, and Francesco Zanetti

Working Paper 2019-9
April 2019

**Abstract:** We develop a quantitative business cycle model with search complementarities in the inter-firm matching process that entails a multiplicity of equilibria. An active equilibrium with strong joint venture formation, large output, and low unemployment coexists with a passive equilibrium with low joint venture formation, low output, and high unemployment.

Changes in fundamentals move the system between the two equilibria, generating large and persistent business cycle fluctuations. The volatility of shocks is important for the selection and duration of each equilibrium. Sufficiently adverse shocks in periods of low macroeconomic volatility trigger severe and protracted downturns. The magnitude of government intervention is critical to foster economic recovery in the passive equilibrium, while it plays a limited role in the active equilibrium.

JEL classification: C63, C68, E32, E37, E44, G12

Key words: aggregate fluctuations, strategic complementarities, macroeconomic volatility, government spending

https://doi.org/10.29338/wp2019-09

---

The authors thank Davide Debortoli, Edouard Schaal, Shouyong Shi, Martin Uribe, and participants at multiple seminars for valuable comments and suggestions. The views expressed here are those of the authors and not necessarily those of the Federal Reserve Bank of Atlanta or the Federal Reserve System. Any remaining errors are the authors' responsibility. Zanetti gratefully acknowledges financial support from the British Academy. The usual disclaimer applies.

Please address questions regarding content to Jesús Fernández-Villaverde, University of Pennsylvania, Department of Economics, The Ronald O. Perelman Center for Political Science and Economics, 133 South 36th Street, Philadelphia, PA 19104, jesusfv@econ.upenn.edu; Federico S. Mandelman, Federal Reserve Bank of Atlanta, Research Department, 1000 Peachtree Street NE, Atlanta, Georgia, 30309-4470, federico.mandelman@atl.frb.org; Yang Yu, Shanghai University of Finance and Economics, School of Economics. 318 Wuchuan Road, WuJiaoChang, Yangpu Qu, Shanghai Shi, China, 200433, yu.yang@mail.shufe.edu.cn; or Francesco Zanetti, University of Oxford, Manor Road, Oxford, OX1 3UQ, U.K., francesco.zanetti@economics.ox.ac.uk.

# 1 Introduction

Search often involves two parties. Workers search for firms and firms search for workers. Customers search for shops and shops search for customers. Entrepreneurs search for venture capitalists and venture capitalists search for entrepreneurs.

Two-sided searches generate a strategic complementarity. If the probability of a match depends on the search intensity exerted by the parties, an increase in the search effort by one party might lead to a rise in the search effort by the other party. Conversely, a decrease in the search effort by one party might lower the search effort by the other party. Under certain conditions, this strategic complementarity begets multiple Nash equilibria: either both agents search with high effort or both agents search with low effort—even when fundamentals such as technology and preferences are the same.

Based on this intuition, we build a quantitative business cycle model, calibrating it to U.S. data. Firms post job vacancies and fill them with workers from households in an otherwise standard Diamond-Mortensen-Pissarides (DMP) frictional labor market. Once vacancies have been filled, firms must match among themselves to produce output. This mechanism is a simple way to capture the inter-firm linkages embedded in the network structure of a modern economy. For example, a general contractor needs to find an electrician to finish a new house, and an electrician must find a general contractor to transform her skills into output. When general contractors search with high intensity for electricians and electricians search with high intensity for general contractors, output is high and unemployment low. Otherwise, output is low and unemployment high.

In our model, search and matching have strategic complementarities. Returns from establishing a joint venture between firms depend on fundamentals *and* on the search intensity of potential partners. The latter dependence generates, for plausible parameter values, a *passive* equilibrium (where firms search for partners with zero intensity) and an *active* equilibrium (where firms search for partners with positive intensity). In this active equilibrium, firms post more vacancies, output is higher, and unemployment lower than in the passive equilibrium.

In addition, households are subject to discount factor shocks, and firms experience productivity shocks. Since households own the firms in the economy, the discount factor shocks also affect how firms discount the future. When the discount factor is high, households and firms search, *ceteris paribus*, more intensely because the continuation value from search is discounted less (a similar reasoning holds for productivity shocks, which we ignore here for concision). Thus, the passive equilibrium might disappear and only the active equilibrium survives. Conversely, when households and firms discount the future sufficiently, just the passive equilibrium exists. For intermediate values of the discount factor, both equilibria are possible. In this case, we will assume that the economy stays in the same equilibrium as in the previous period: if yesterday firms did not search, today firms still do not search; if yesterday firms searched with positive intensity, today firms still search.

This history-dependent equilibrium selection amplifies and lengthens the effects of shocks. A small drop in the discount factor when the economy is at an active equilibrium, but on the cusp of the disappearance of such an equilibrium, pushes the economy to the passive equilibrium, leading to a large decline in output and a big increase in unemployment. By comparison, when the economy is farther away from the region where the active equilibrium disappears, the same shock has only minor effects on output and unemployment. Thus, our economy features a strong non-linearity and bimodal ergodic distributions of aggregate variables, where the mass around each mode represents the economy living in one equilibrium or another.

Furthermore, once the economy is at the passive equilibrium, it remains there until a sufficiently large discount factor shock terminates the equilibrium. In the meantime, even if the active equilibrium reappears as a possibility, the economy is stuck in the passive equilibrium. Thus, search complementarities transform transitory shocks into long-lived slumps. This phenomenon might explain the aftermath of the Great Recession in the United States, where output has remained below trend for ten years after the outset of the crisis and employment-to-population ratios are still depressed. The economy moved in 2008 to an equilibrium with less search, and it has not abandoned it even after the original negative shocks evaporated.

Quantitatively, if the model starts from the active equilibrium deterministic steady state, a one-period adverse shock to the discount factor of 12% moves the system to the passive equilibrium with low search intensity, increasing the unemployment rate by 3.2% and reducing output by approximately 15%. This reduction in output is in the ballpark of the one observed for the U.S. in the Great Recession if we measure it as a deviation with respect to trend (which we ignore in our model for simplicity).[1] Since Justiniano and Primiceri (2008) estimate a standard

[1]Between 2007.Q4 and 2014.Q4, output per capita fell 12.4% in the U.S. with respect to its post-war trend.

deviation of the discount factor equal to 5% in the U.S. post-war period, a reduction of 12% in the discount factor is approximately a two-and-a-half standard deviation fall in the discount factor, a low probability but not a rare event. Smaller shocks to the discount factor fail to move the system away from the original equilibrium and the properties of the system are similar to those of standard business cycle models. By comparison, productivity shocks have much more limited effects on equilibrium switches. Given the observed U.S. data, the empirically plausible standard deviation of productivity shocks is too small to generate productivity realizations that move the economy from one equilibrium to the other.

The model matches U.S. business cycle statistics, in particular along two moments that have proven to be difficult to replicate in the past. First, the economy generates a strong internal propagation of shocks, and the autocorrelation of the variables is larger and closer to the observed data than in standard models without the need to assume highly persistent exogenous shocks. In our model, instead, the persistence of variables comes from the switches between the two equilibria. Second, our economy generates endogenous movements in labor productivity and a more realistic volatility of unemployment than in standard business cycle models.

The data support the central mechanism in our model. Changes in the discount factor—proxied by a broad range of indexes—are strongly correlated with the volume of intermediate input and co-move tightly with output and unemployment. Observed movements in intermediate inputs are strongly linked to changes in output at the industry level, and fluctuations in intermediate input explain more than 70% of fluctuations in total industry gross output.

We also show how the volatility of shocks plays a crucial role in shaping aggregate fluctuations in the presence of search complementarities. A reduction in macroeconomic volatility, such as the Great Moderation, leads to increased persistence in labor market downturns.[2] Since large shocks are unlikely in the Great Moderation, once the economy is pushed into the passive equilibrium due to one of these rare negative shocks, it takes a long time before a new large rare positive shock arrives, allowing the economy to abandon the passive equilibrium. Under the Great Moderation, recessions are rarer but their consequences more severe. Far from being an anomaly, the last decade of disappointing macroeconomic performance is a direct consequence of the Great Moderation, albeit an unwelcome one.

In comparison, unemployment increased, at its peak, from 4.4% to 10.0%, around 50% more than in our model.

[2]The reports of the death of the Great Moderation have been greatly exaggerated. See Liu et al. (2018) for updated evidence.

Finally, we investigate the role of fiscal policy in our model. In our example above, an electrician can work for a general contractor or wire a new public school. If the government increases its expenses (modeled as an increase in government-owned firms such as a new public school), the search incentives for private firms increase, and the economy can switch from a passive equilibrium to an active one. Thus, the fiscal multipliers can be large (as high as 3.5 when the fiscal stimulus is of just the right size to move the economy from the passive to the active equilibrium) and highly-nonlinear. On the other hand, if search intensity is already high (or the fiscal expansion too small in a passive equilibrium), the fiscal multiplier will be small (as low as 0.15 when the economy is already in the active equilibrium).

There is a long tradition in macroeconomics of linking strategic complementarities to aggregate fluctuations, going back to Diamond (1982), Weitzman (1982), and Diamond and Fudenberg (1989) and explored by Cooper and John (1988) and Chatterjee et al. (1993). Recent examples of that tradition include Huo and Ríos-Rull (2013) and Kaplan and Menzio (2016). Also, similar ideas regarding the large potential effects of fiscal policy appear in the study of a "big push" à la Murphy et al. (1989).

How does our paper add to the literature of strategic complementarities and aggregate fluctuations? First, we embed strategic complementarities into an otherwise standard quantitative general equilibrium business cycle model that matches the data with a conventional calibration and improves upon the empirical performance of other business cycle models.[3] Thus, our experiments regarding the effects of shocks and fiscal policy provide useful quantitative guidance for policymakers. Second, we do not rely on increasing returns to scale on production, trading, or others. This approach is important, as increasing returns are difficult to identify in the data and to distinguish from varying capacity utilization. Third, we provide evidence that supports our particular choice of strategic complementarities in intra-firm matching and an empirically plausible mechanism for equilibrium switches through variations in the discount factor of the household. Fourth, we show the effects of volatility (and changes to it) on our economy, with consequences for the length of equilibria spells and their changes over time.

[3] Other recent quantitative models are related to ours, but with a different mechanism, such as Taschereau-Dumouchel and Schaal (2015) (who employ strategic complementarities in models with varying production capacity utilization and monopolistic competition), Sterk (2016) (who deals with strategic complementarities created by skill losses of unemployed workers), and Eeckhout and Lindenlaub (2018) (who highlight the strategic complementarities between on-the-job search and vacancy posting by firms).

# 2 A simple model with search complementarities

To build intuition, we present a simple model with search complementarities. This environment embodies the mechanisms at work in our fully fledged model with greater transparency, but at the cost of quantitative implications that are not designed to account for the data.

## 2.1 Environment

We start with a deterministic version of the model. The economy is composed of a continuum of islands of unit measure where time is discrete and infinite. Two risk-neutral firms populate each island. Both firms are owned by a representative household, whose only task is to consume the aggregate net profits of all firms in the economy. At the start of the period, firms are in two separate locations within the island, and they must meet to engage in production. If they do not meet, each firm produces zero output. If they do meet, they jointly produce 2 units of output that they split into equal parts. At the end of the period, the match is dissolved, and each firm moves to a new, separate location to search in the next period *ex novo*. Since we will analyze symmetric equilibria where all firms follow the same search intensity, we drop the island index. Although realizations of the search will differ among islands, a law of large numbers will hold in the aggregate economy and individual matching probabilities will equal the aggregate share of islands where matches occur. Similarly, since there are no state variables carrying information across periods, it is unnecessary to specify a discount factor, and, for the moment, we can drop the time index of each variable.

The probability of meeting is given by a matching function that depends on the search intensity of each firm within the island. Specifically, for a search intensity $\sigma_1 \in [0, 1]$ of firm 1 and a search intensity $\sigma_2 \in [0, 1]$ of firm 2, the matching probability function is:

$$\pi(\sigma_1, \sigma_2) = \frac{1 + \sigma_1 + \sigma_2 + \sigma_1\sigma_2}{4}. \tag{1}$$

This function yields a matching probability of 1/4 when $\sigma_1 = \sigma_2 = 0$, a probability of 1 when $\sigma_1 = \sigma_2 = 1$, and probabilities between 0 and 1 in the intermediate cases of search intensity.

For an $\alpha \in [0, 1)$, the cost of search intensity for firm $i \in \{1, 2\}$ is:

$$c(\sigma_i) = \frac{1+\alpha}{4}\sigma_i + \frac{\sigma_i^3}{3}. \tag{2}$$

## 2.2 Nash equilibria

To find the set of Nash equilibria in our model, we look at the problem of firm 1 when it takes the search intensity of firm 2, $\overline{\sigma}_2$, as given. The expected profit function of firm 1 is:

$$J(\sigma_1, \overline{\sigma}_2) = \frac{1 + \sigma_1 + \overline{\sigma}_2 + \sigma_1\overline{\sigma}_2}{4} - \frac{1+\alpha}{4}\sigma_1 - \frac{\sigma_i^3}{3}.$$

Maximizing $J(\sigma_1, \overline{\sigma}_2)$ with respect to $\sigma_1$ and noticing that the optimal solution is, for some values of $\overline{\sigma}_2$, at a corner of zero optimal search intensity, we get the best response function $\Pi(\sigma_2)$ for firm 1:

$$\sigma_1^* = \begin{cases} 0 & \text{if}\, \sigma_2 \leq \alpha \\ \frac{1}{2}\sqrt{\sigma_2 - \alpha} & \text{if}\, \sigma_2 > \alpha. \end{cases} \tag{3}$$

Analogously, the best response function $\Pi(\sigma_1)$ for firm 2 is:

$$\sigma_2^* = \begin{cases} 0 & \text{if}\, \sigma_1 \leq \alpha \\ \frac{1}{2}\sqrt{\sigma_1 - \alpha} & \text{if}\, \sigma_1 > \alpha. \end{cases} \tag{4}$$

These best response functions explain why we imposed the condition that $\alpha \in [0, 1)$. Values of $\alpha < 0$ imply that there is a unique Nash equilibrium and that such an equilibrium has positive search intensity. Values of $\alpha \geq 1$ also yield a unique Nash equilibrium, but now with zero search intensity. Only for $\alpha \in [0, 1)$ can we have multiple Nash equilibria caused by search complementarities.

A (within period) pure Nash equilibrium is a tuple $\{\sigma_1^*, \sigma_2^*\}$ that is a fixed point of the product of both best response functions (3) and (4) (we ignore mixed strategies equilibria; see Footnote 7). Clearly, for all $\alpha \in [0, 1)$, $\{\sigma_1^*, \sigma_2^*\} = \{0, 0\}$ is a Nash equilibrium. We call this case a *passive* equilibrium, where the matching probability is 1/4, aggregate output $y$ is 1/2, and consumption by the representative household $c$ is 1/2.

Depending on the value of $\alpha$, we might have one or two more equilibria in pure strategies with a positive search intensity of $\sigma^* = \sigma_1^* = \sigma_2^* > 0$. The matching probability is now given by

$$\frac{1 + 2\sigma^* + (\sigma^*)^2}{4},$$

aggregate output $y$ by

$$\frac{1 + 2\sigma^* + (\sigma^*)^2}{2},$$

and consumption $c$ by

$$\frac{1 + 2\sigma^* + (\sigma^*)^2}{2} - \frac{1+\alpha}{2}\sigma^* - \frac{2}{3}(\sigma^*)^3.$$

To derive $c$, we subtracted the search costs of both firms from output. We call equilibria with positive search intensity *active*.

**Figure 1:** Three cases of cost parameter $\alpha$

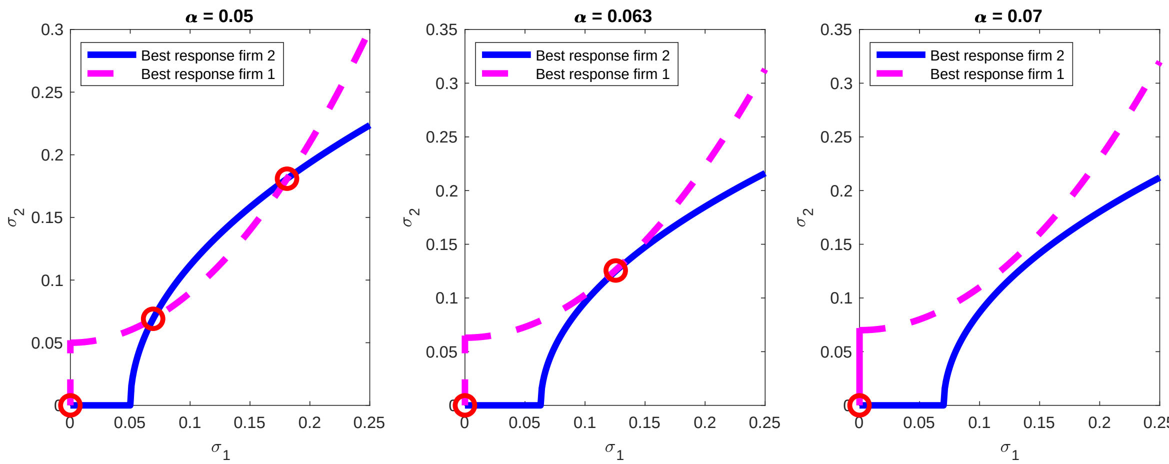



Figure 1 draws three cases: $\alpha = 0.05$ (panel on the left), $\alpha = 0.063$ (central panel), and $\alpha = 0.07$ (panel on the right). The dashed line plots the best response function of firm 1, the solid line the best response function of firm 2, and the red circles each Nash equilibrium. When $\alpha = 0.05$, there are three Nash equilibria in pure strategies: $\sigma^* = \sigma_1^* = \sigma_2^* = 0$, $\sigma^* = \sigma_1^* = \sigma_2^* = 0.069$, and $\sigma^* = \sigma_1^* = \sigma_2^* = 0.181$. These equilibria are Pareto-ranked: consumption (a welfare measure in our environment) is 0.5 in the first equilibrium, 0.535 in the second equilibrium, and 0.598 in the third equilibrium. When $\alpha = 0.063$, there are two Nash equilibria in pure strategies: $\sigma^* = \sigma_1^* = \sigma_2^* = 0$, and $\sigma^* = \sigma_1^* = \sigma_2^* = 0.126$. Again, the equilibria are Pareto-ranked, with consumption in the active equilibrium equal to 0.565. When $\alpha = 0.07$, the only Nash equilibrium in pure strategies is passive, $\sigma^* = \sigma_1^* = \sigma_2^* = 0$.

## 2.3 Stochastic shocks

To generate additional results beyond the multiplicity of equilibria, we introduce stochastic shocks in the production function of matched firms. Instead of jointly producing 2 units of output, as in the baseline case, we now assume that firms produce $2z_t$, where $z_t$ is a productivity shock in period $t$ (we now index variables by $t$, but because of symmetry, there is no need to index them by the island). Productivity shocks will induce movements in the economy along one Nash equilibrium and, sometimes, changes among the Nash equilibria firms play.

The new expected profit function of firm 1 is:

$$J\left(\sigma_{1,t}, \overline{\sigma}_{2,t}, z_t\right) = z_t \frac{1 + \sigma_{1,t} + \overline{\sigma}_{2,t} + \sigma_{1,t}\overline{\sigma}_{2,t}}{4} - \frac{1+\alpha}{4}\sigma_{1,t} - \frac{\sigma_{1,t}^3}{3}.$$

Following the same reasoning as in the deterministic case, the best response function $\Pi\left(\sigma_{2,t}, z_t\right)$ for firm 1 is:

$$\sigma_{1,t}^* = \begin{cases} 0 & \text{if } z_t\left(1+\overline{\sigma}_{2,t}\right) \leq (1+\alpha) \\ \frac{1}{2}\sqrt{z_t\left(1+\sigma_{2,t}\right) - (1+\alpha)} & \text{if } z_t\left(1+\overline{\sigma}_{2,t}\right) > (1+\alpha), \end{cases} \tag{5}$$

and the best response function $\Pi\left(\sigma_{1,t}, z_t\right)$ for firm 2 is:

$$\sigma_{2,t}^* = \begin{cases} 0 & \text{if } z_t\left(1+\overline{\sigma}_{1,t}\right) \leq (1+\alpha) \\ \frac{1}{2}\sqrt{z_t\left(1+\sigma_{1,t}\right) - (1+\alpha)} & \text{if } z_t\left(1+\overline{\sigma}_{2,t}\right) > (1+\alpha). \end{cases} \tag{6}$$

When $z_t = 1$, equations (5) and (6) collapse to equations (3) and (4).

A (with-in period) pure Nash equilibrium is a tuple $\left\{\sigma_{1,t}^*, \sigma_{2,t}^*\right\}$ that is a fixed point of the product of both of the best response functions (5) and (6). As before, we can have one, two, or three Nash equilibria with matching probability given by

$$\frac{1 + 2\sigma_t^* + \left(\sigma_t^*\right)^2}{4},$$

aggregate output $y_t$ by

$$z_t \frac{1 + 2\sigma_t^* + \left(\sigma_t^*\right)^2}{2},$$

and consumption $c_t$ by

$$z_t \frac{1+2\sigma_t^* + (\sigma_t^*)^2}{2} - \frac{1+\alpha}{2}\sigma_t^* - \frac{2}{3}(\sigma_t^*)^3 .$$

To illustrate the behavior of our economy, we fix $\alpha = 0.063$ and assume that $z_t$ follows a Markov chain with support $\{0.93, 1, 1.07\}$. Since the values of the transition matrix for this chain will not matter for the next few paragraphs, we momentarily differ its specification. We pick the average value of $z_t$ to be 1 to make the stochastic model coincide, for that realization, with the deterministic environment. The value of $\alpha = 0.063$ ensures that, when $z_t = 1$, there is only one active Nash equilibrium. We pick the high realization of $z_t$ to be 1.07 to get $z_t > 1 + \alpha$. When this condition holds, zero search intensity is not a Nash equilibrium. We pick the low realization 0.93 for symmetry.

**Figure 2:** Changing productivity $z_t$

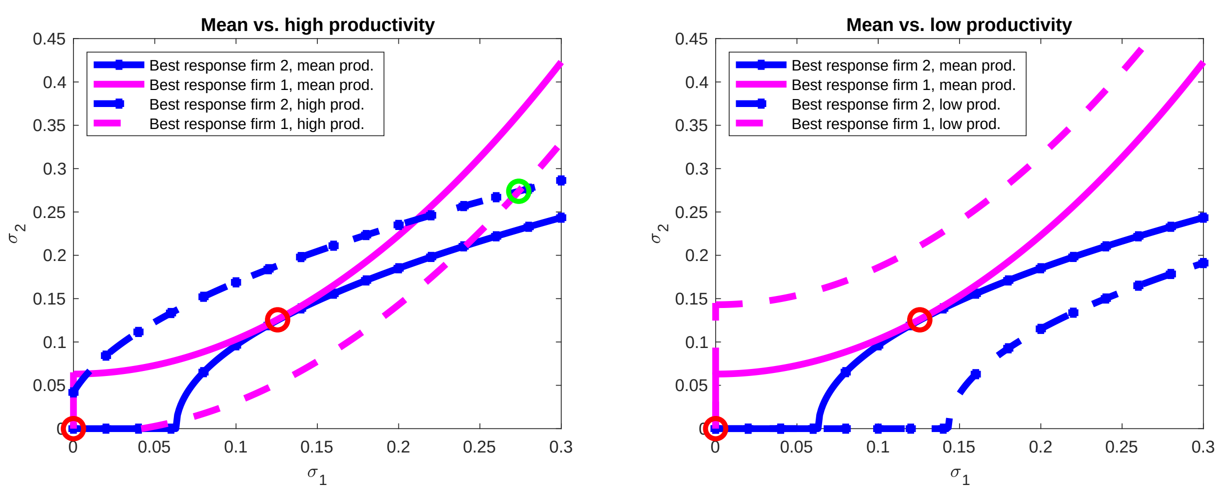



Figure 2 plots the best response functions under each realization of productivity. The left panel shows in solid lines the best responses for $z_t = 1$ (with crosses for the best response of firm 2). These are the same as those drawn in the central panel of Figure 1 and show two fixed points, one with $\sigma_t^* = \sigma_{1,t}^* = \sigma_{2,t}^* = 0$, and one with $\sigma_t^* = \sigma_{1,t}^* = \sigma_{2,t}^* = 0.126$. Consumption in the first equilibrium is 0.5 and 0.565 in the second equilibrium, even if productivity remains the same. The dashed lines in the same panel are the best responses when $z_t = 1.07$ (with crosses for the best response of firm 2). Now we have a unique Nash equilibrium at $\sigma_t^* = \sigma_{1,t}^* = \sigma_{2,t}^* = 0.274$ (the green circle), with consumption at 0.709. The right panel plots in solid lines the best responses for $z_t = 1$, with the same explanation as above. The dashed lines now draw the best responses for $z_t = 0.93$, with a unique Nash equilibrium at $\sigma_t^* = \sigma_{1,t}^* = \sigma_{2,t}^* = 0$ and consumption at 0.465.

Figure 2 illustrates how consumption usually moves more than productivity. For example, consumption increases 27% when the economy starts at the passive equilibrium and $z_t$ moves from 1.0 to 1.07. This amplification mechanism comes from search complementarities: when firm 1 searches more because productivity is higher, firm 2 increases its search intensity in response to the higher search intensity of firm 1 (and vice versa).

Indeed, in our model, the multiplier $\frac{\Delta c_t/c_t}{\Delta z_t/z_t}$ of consumption to a productivity shock is state-dependent: the same productivity shock leads to different changes in consumption depending on the state of the economy. Table 1 documents this point by reporting the multiplier in six relevant cases (and where subindexes denote the productivity level and type of equilibria). The multiplier ranges from as low as 1 –when the economy moves from low productivity to mean productivity, as search intensity is zero in both cases– to nearly 6 –when the economy moves from mean productivity and zero search intensity to high productivity.

**Table 1:** Multiplier

| Productivity shock | $\frac{\Delta c_t/c_t}{\Delta z_t/z_t}$ |
|---|---|
| $z_{low} \to z_{mean,passive}$ | 1 |
| $z_{low} \to z_{high}$ | 3.485 |
| $z_{mean,passive} \to z_{high}$ | 5.969 |
| $z_{mean,active} \to z_{high}$ | 3.627 |
| $z_{high} \to z_{low}$ | 4.009 |
| $z_{high} \to z_{mean,active}$ | 3.095 |

Our last task is to specify a transition matrix $\Pi$ for productivity shocks. We select a standard business cycle parameterization with symmetry and medium persistence:

$$\Pi = \begin{pmatrix} 0.90 & 0.08 & 0.02 \\ 0.05 & 0.90 & 0.05 \\ 0.02 & 0.08 & 0.90 \end{pmatrix}.$$

When $z_t$ is high or low, the Nash equilibrium is unique. When $z_t = 1$, there are two Nash equilibria, and we select between them through history dependence following Cooper (1994). More concretely, if the economy was in a passive equilibrium in the previous period, we stay in such an equilibrium today. Conversely, if the economy was in an active equilibrium in the previous period, firms continue searching with positive intensity today (Taschereau-Dumouchel and Schaal 2015 show that a global game produces, on average, the same equilibrium selection).

This history-dependent equilibrium selection has two implications. First, the effects of a productivity shock persist longer than the shock. In particular, the economy cannot move directly from $z_{low}$ to $z_{mean,active}$ or from $z_{high}$ to $z_{mean,passive}$ (this explains why Table 1 does not report these cases). Instead, to switch equilibria, the economy must transition through an intermediate stage of high productivity (when we start from $z_t = 0.93$) or low productivity (when we start from $z_t = 1.07$). Second, we do not generate fluctuations through sunspots. Changes among Nash equilibria in our economy always derive from the movement in fundamentals.

**Figure 3:** Simulation of aggregate consumption

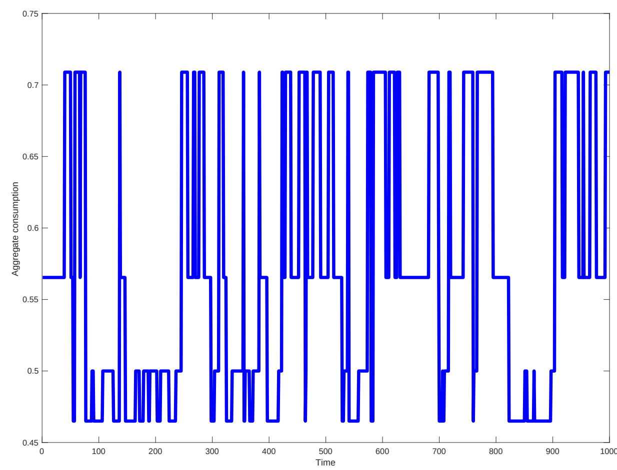



Figure 3 shows a typical realization of consumption for 1,000 periods. Consumption takes four different values: 0.465 ($z_t = 0.93$), 0.5 ($z_t = 1.0$, passive equilibrium), 0.565 ($z_t = 1.0$, active equilibrium), and 0.709 ($z_t = 1.07$). Given $\Pi$, the stationary distribution of productivity is $(0.278, 0.444, 0.278)$. Since our simulations start from $z_t = 1.0$ (and an active equilibrium), we have a slightly higher level of mean realizations of productivity, with a count of $(233, 490, 277)$. Consumption is 0.465 in 233 periods and 0.552 in 277 periods. More interesting is the breakdown of the 490 periods when $z_t = 1.0$: 180 happen in a passive equilibrium and 310 in an active equilibrium. Asymptotically, due to the symmetry of $\Pi$, the realizations of $z_{mean}$ will split evenly between both levels of consumption.

The simple model has illustrated four points. First, search complementarities create multiple Nash equilibria. Second, the interaction of search complementarities with stochastic shocks amplifies the impact of the latter. Third, the effects of shocks are history-dependent: the multiplier of consumption to a productivity shock is a highly non-linear function of the state of the economy. Fourth, history dependence enhances the persistence of aggregate variables to shocks. We move now to show how these four key points appear as well in a fully fledged quantitative business cycle model with search complementaries.

# 3 A search and matching model

We work with a search and matching model where time is discrete and infinite. The economy is composed of households, firms in the intermediate-goods production sector ($I$), and firms in the final-goods production sector ($F$).

## 3.1 Households

There is a continuum of households of size 1. Households are risk neutral and discount the future by $\beta\xi_t$ per period. This term is the product of a constant $\beta < 1$ and a preference shock $\xi_t$. Innovations to $\xi_t$ encapsulate movements in the stochastic discount factor, which Cochrane (2011) and Hall (2016, 2017) highlight as a central source of aggregate fluctuations. Since households own the firms in the economy, firms also employ $\beta\xi_t$ to discount future profits.

Households can either work one unit of time per period for a wage $w$ or be unemployed and receive $h$ utils of home production and leisure. Households do not have preferences for working –or searching for a job– in either sector $i \in \{I, F\}$ of the economy. Households also receive the aggregate profits of all firms, but since those are zero in equilibrium because of free entry, we ignore them.

## 3.2 Labor matching

At the beginning of each period $t$, any willing new firm can post a vacancy in either sector at the cost of $\chi$ per period to hire job-seeking households. Each firm posts a vacancy for one worker. Vacancies and job seekers meet in a DMP frictional labor market.

More precisely, given $u_{i,t}$ unemployed households and $v_{i,t}$ posted vacancies in sector $i$, a constant-return-to-scale matching technology $m(u_{i,t}, v_{i,t})$ determines the number of hires and vacancies filled in period $t$. The new hires start working in period $t+1$. The job finding rate for unemployed households, $\mu_{i,t} = m(u_{i,t}, v_{i,t})/u_{i,t} = \mu(\theta_{i,t})$, and the probability of filling a vacancy, $q_{i,t} = m(u_{i,t}, v_{i,t})/v_{i,t} = q(\theta_{i,t})$, are functions of each sector's labor market tightness ratio $\theta_{i,t} = v_{i,t}/u_{i,t}$. Then, $\mu'(\theta_{i,t}) > 0$ and $q'(\theta_{i,t}) < 0$: in a tighter labor market, unemployed households are more likely to find a job, and firms are less likely to fill vacancies.

At the end of each period $t$, already existing jobs terminate at a rate $\delta$ and unfilled vacancies expire. Newly unemployed workers are equally split across sectors. To simplify the model, households search for a job in one of the two sectors without being allowed to change sector (given the symmetry of our model across sectors and our calibration below, workers do not mind this restriction). Appealing to an appropriate law of large numbers, unemployment evolves as:

$$u_{t+1} = u_t - \underbrace{\left[\mu_I\left(\theta_{I,t}\right) \cdot u_{I,t} + \mu_F\left(\theta_{F,t}\right) \cdot u_{F,t}\right]}_{\text{Job creation}} + \underbrace{\delta \cdot (1 - u_t)}_{\text{Job destruction}} \tag{7}$$

where $u_t = u_{I,t} + u_{F,t}$. Equation (7) shows how unemployment is determined by changes in job creation that depend on sectoral labor market tightness, $\theta_{i,t}$. A slack (tight) labor market in sector $i$ decreases (increases) job creation and increases sectoral unemployment.

## 3.3 Inter-firm matching

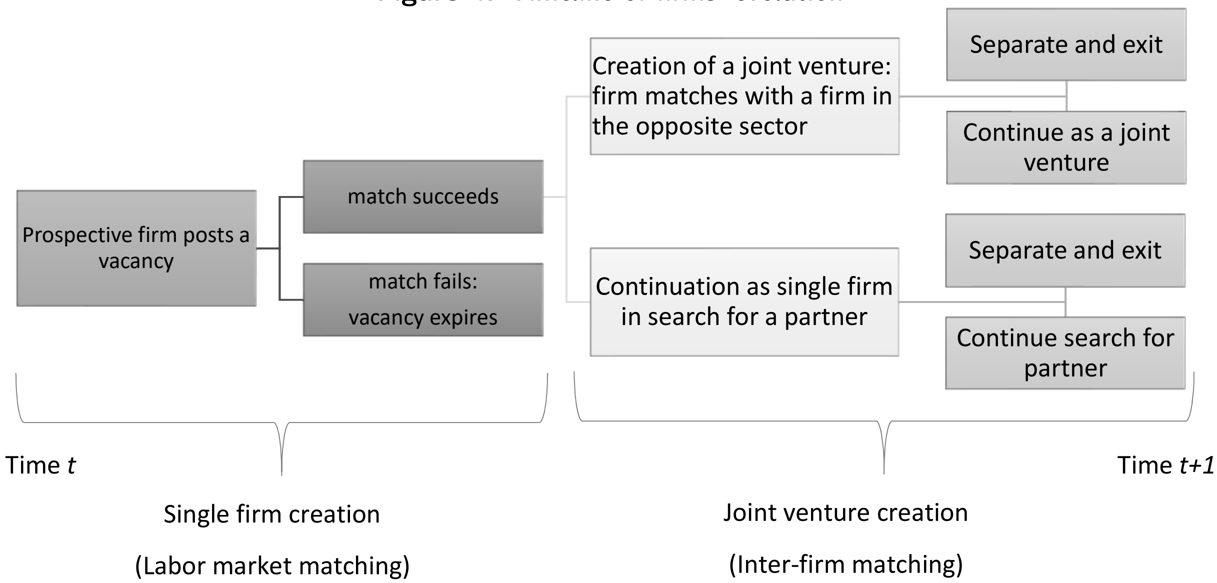
Once job vacancies are filled, a final-goods firm must form a joint venture with an intermediate-goods firm to manufacture together, starting in $t + 1$, the final goods sold to households. This final good is also the numeraire in the economy. If a firm fails to form a joint venture in period $t$, it produces no output and continues searching for a partner in $t + 1$. This simple matching problem summarizes more sophisticated inter-firm network structures such as those in Jones (2013) and Acemoglu et al. (2012).

A technology with variable search intensity governs inter-firm matching. Search intensity is costly, but it reduces the expected duration of remaining a single firm unable to produce. At the end of each period, a constant fraction of already existing joint ventures are destroyed because either the job matches in the component firms terminate or the joint venture fails at a rate $\widetilde{\delta}$.[4] In the former case, the firms dissolve. In the latter case, the firms revert to their status as single

[4] To simplify the algebra, we assume that, in a joint venture, the jobs in the intermediate-goods firm and the final-goods firm terminate simultaneously with probability $\delta$ or survive simultaneously with probability $1 - \delta$. In single firms, the job destruction rate is also $\delta$. Also, we assume that $\delta + \widetilde{\delta} < 1$.

firms, but the jobs survive.

**Figure 4:** Timeline of firms' evolution




The actions of these firms, summarized in Figure 4, require more explanation. In a joint venture, the intermediate-goods firm uses its worker to produce $y_{I,t} = z_t$, where $z_t$ is the stochastic productivity in the intermediate-goods sector. The final-goods firm takes this $y_{I,t}$ and, employing its worker, transforms it one-to-one into the final good, $y_{F,t} = y_{I,t} = z_t$.

Extending the search intensity model in Burdett and Mortensen (1980), we assume that the number of inter-firm matches is $M\left(\widetilde{n}_{F,t}, \widetilde{n}_{I,t}, \eta_{F,t}, \eta_{I,t}\right) = \left(\phi + \eta_{F,t}\eta_{I,t}\right) H\left(\widetilde{n}_{F,t}, \widetilde{n}_{I,t}\right)$, where $\widetilde{n}_{F,t}$ is the number of single firms in sector $F$ with search intensity effort, $\eta_{F,t}$; $\widetilde{n}_{I,t}$ and $\eta_{I,t}$ are the analogous variables for the $I$ sector. The parameter $\phi > 0$ represents the efficiency in matching unrelated with search efforts and it will help us to replicate the inter-firm matching probability in the data. The function $H(\cdot)$ has constant returns to scale and it is strictly increasing in both search intensities. We set up its units by choosing $H(1, 1) = 1$. Variable search intensity generates *strategic complementarities* in the sense of Bulow et al. (1985) since the degree of optimal search intensity by one firm will be (weakly) increasing in the number of firms searching in the opposite sector and their search intensity.

Given the inter-firm market tightness ratio $\widetilde{n}_F/\widetilde{n}_I$, the probability that a sector $I$ firm will form a joint venture with a sector $F$ firm is:

$$\pi_I = \frac{M\left(\widetilde{n}_F, \widetilde{n}_I, \eta_F, \eta_I\right)}{\widetilde{n}_I} = \left(\phi + \eta_F \eta_I\right) H\left(\frac{\widetilde{n}_{F,t}}{\widetilde{n}_{I,t}}, 1\right), \tag{8}$$

and the probability that a sector $F$ firm will form a joint venture with a sector $I$ firm is:

$$\pi_F = \frac{M\left(\widetilde{n}_F, \widetilde{n}_I, \eta_F, \eta_I\right)}{\widetilde{n}_F} = \left(\phi + \eta_F \eta_I\right) H\left(1, \frac{\widetilde{n}_{I,t}}{\widetilde{n}_{F,t}}\right). \tag{9}$$

Search intensity in each sector is given by a fixed component, $\psi > 0$, and a variable, effort-related component, $\sigma_{i,t} \geq 0$:

$$\eta_{i,t} = \psi + \sigma_{i,t}. \tag{10}$$

The fixed component $\psi$ guarantees that the marginal return to searching does not become zero when prospective partners search with zero intensity. In comparison, each firm optimally chooses $\sigma_{i,t} \geq 0$ (we will focus on symmetric equilibria where all firms within one sector make the same choice) to trade off search cost and the profits from matching success.

The cost of searching at intensity $\sigma_{i,t}$ is given by:

$$c\left(\sigma_{i,t}\right) = c_0 \sigma_{i,t} + c_1 \frac{\sigma_{i,t}^{1+\nu}}{1+\nu}, \tag{11}$$

where $c_0 > 0$ creates a linear cost tranche and $\{c_1, \nu\} > 0$ a convex cost tranche.[5] The linear cost implies that the net gain from searching can be negative, in which case the firm chooses $\sigma_i = 0$. This assumption is critical. If $c_0 = 0$, the benefit from an additional unit of search intensity is always positive, and the firm chooses $\sigma_i > 0$ in all states of the economy. Instead, $c_0 > 0$ generates the non-convexity that triggers, as we will see, multiple equilibria.

In a symmetric equilibrium, the two sectors have the same number of single firms ($\widetilde{n}_{F,t} = \widetilde{n}_{I,t}$) and search intensity ($\sigma_{F,t} = \sigma_{I,t}$). Thus, the inter-firm matching probability is:

$$\pi_{F,t} = \pi_{I,t} = \phi + \eta_{F,t}\eta_{I,t} = \phi + \left(\psi + \sigma_{F,t}\right)\left(\psi + \sigma_{I,t}\right). \tag{12}$$

Note that $\psi$ determines the impact of the search intensity in the opposite sector for the total matching probability because of the product $\eta_F \eta_I$ in equation (12), while $\phi > 0$ does not. This will give us identification in our calibration in Section 5.[6]

[5] The cost function (2) in our simple model in Section 2 follows equation (11) when $c_0 = \frac{1+\alpha}{4}$, $c_1 = 1$, and $\nu = 2$.

[6] The matching probability (1) in our model in Section 2 is nearly the same as the matching probability in equation (12) when $\phi = \frac{3}{16}$ and $\psi = \frac{1}{4}$, except for a term $\frac{1}{4}$ missing in front of $\sigma_{I,t}\sigma_{F,t}$, which we introduced in the simple model to ensure that the matching probability was always between $(0, 1)$.

The number of joint ventures in period $t+1$ comprises firms that survive job separation and joint venture destruction plus newly formed joint ventures:

$$n_{t+1} = (1-\delta-\widetilde{\delta})n_t + (\phi + (\psi + \sigma_{F,t})\,(\psi + \sigma_{I,t}))\widetilde{n}_{I,t}. \tag{13}$$

The number of single firms in period $t+1$ includes firms that survive job separation ($(1-\delta)\,\widetilde{n}_{i,t}$), newly created single firms whose vacancies are filled by job-seekers ($\mu_i\,(\theta_{i,t}) \cdot u_{i,t}$), and firms whose joint ventures exogenously terminate ($\widetilde{\delta} n_{i,t}$), net of the number of single firms that form joint ventures ($\pi_{i,t}\widetilde{n}_{i,t}$):

$$\widetilde{n}_{i,t+1} = (1-\delta)\,\widetilde{n}_{i,t} + \mu_i\,(\theta_{i,t}) \cdot u_{i,t} + \widetilde{\delta} n_{i,t} - \pi_{i,t}\widetilde{n}_{i,t}. \tag{14}$$

We will prove below that search complementarities beget multiple equilibria. As in Section 2, one of these equilibria is *passive*, with zero search intensity ($\sigma_{I,t} = \sigma_{F,t} = 0$), low production, and high unemployment. The other equilibria are *active*, with positive search intensity ($(\sigma_{I,t}, \sigma_{F,t}) > 0$), high production, and low unemployment. Also, as we assumed in Section 2, the selection of equilibria is history dependent. Sufficiently large shocks to productivity or the discount factor induce firms to adjust search intensity, and the economy shifts from one equilibrium to the other. Otherwise, the economy stays in the same equilibrium as in the previous period.

Since we require notation to keep track of those equilibria, we specify an indicator function, $\iota_t$, with value 0 if the equilibrium is passive and 1 if active. This indicator function is an endogenous state variable taken as given by all agents.[7]

## 3.4 Values of households and firms

We can now define the Bellman equations that determine the value, for each sector $i$, of an unemployed household ($U_{i,t}$), of an employed household in a single firm ($\widetilde{W}_{i,t}$) and in a joint venture ($W_{i,t}$), of a filled job in a single firm ($\widetilde{J}_{i,t}$) and in a joint venture ($J_{i,t}$), and of a vacant job ($V_{i,t}$). We index all of these value functions by $\iota_t$ since they depend on the type of equilibrium

[7]There might exist a mixed-strategy Nash equilibrium in which firms search with positive intensity with a certain probability. However, Appendix E shows the mixed-strategy is not robust: when one sector changes the probability slightly due to a trembling hand perturbation, the opposite sector would immediately set the probability to either zero or one. Therefore, we forget about these mixed-strategy Nash equilibria for the rest of the paper.

at $t$, which affects the future path of the equilibrium and the match value.

The value of an unemployed household in sector $i$ and equilibrium $\iota$ is:

$$U_{i,t|\iota_t} = h + \beta\xi_t\mathbb{E}_t\left[\mu_{i,t}\widetilde{W}_{i,t+1} + (1-\mu_{i,t})\,U_{i,t+1} \mid \iota_t\right]. \tag{15}$$

In the current period, the unemployed household receives a payment $h$. The household finds a job with probability $\mu_{i,t}$ and circulates into employment during the next period, or it fails to find employment with probability $1-\mu_{i,t}$ and remains unemployed.

The value of a household with a job in a single firm in sector $i$ is:

$$\widetilde{W}_{i,t|\iota_t} = \widetilde{w}_{i,t} + \beta\xi_t\mathbb{E}_t\left\{(1-\delta)\left[\pi_{i,t}W_{i,t+1} + (1-\pi_{i,t})\,\widetilde{W}_{i,t+1}\right] + \delta U_{i,t+1} \mid \iota_t\right\}. \tag{16}$$

The first term on the right-hand side (RHS) is the period wage $\widetilde{w}_{i,t}$ (to be determined below by Nash bargaining). In period $t+1$, the match that survives job destruction may either form a joint venture with a firm in the opposite sector with probability $\pi_{i,t}$, gaining the value $W_{i,t+1}$, or otherwise remain a single firm with probability $1-\pi_{i,t}$, with value $\widetilde{W}_{i,t+1}$. With probability $\delta$, the job is destroyed, and the household transitions into unemployment.

The value of a household with a job in a joint venture in each sector $i$ is:

$$W_{i,t|\iota_t} = w_{i,t} + \beta\xi_t\mathbb{E}_t\left[(1-\delta-\widetilde{\delta})W_{i,t+1} + \widetilde{\delta}\widetilde{W}_{i,t+1} + \delta U_{i,t+1} \mid \iota_t\right]. \tag{17}$$

A worker in a joint venture receives the wage $w_{i,t}$. In period $t+1$, the worker becomes unemployed with probability $\delta$, gaining the value $U_{i,t+1}$. With probability $\widetilde{\delta}$, the joint venture is terminated, and the value becomes $\widetilde{W}_{i,t+1}$. Otherwise, the match continues, gaining the value $W_{i,t+1}$.

The value of a single firm in sector $i$ is:

$$\widetilde{J}_{i,t|\iota_t} = \max_{\sigma_{i,t}\geq 0}\left\{-\widetilde{w}_{i,t} - c\left(\sigma_{i,t}\right) + \beta\left(1-\delta\right)\xi_t\mathbb{E}_t\left[\pi_{i,t}J_{i,t+1} + (1-\pi_{i,t})\,\widetilde{J}_{i,t+1} \mid \iota_t\right]\right\}. \tag{18}$$

Equation (18) tells us that single firms have zero revenues until they form a joint venture with a firm in the opposite sector. Despite zero production, the firm pays the wage ($\widetilde{w}_{i,t}$) and incurs search costs $c\left(\sigma_{i,t}\right)$, as described in equation (11). In period $t+1$, conditional on surviving job destruction with probability $1-\delta$, the firm forms a joint venture with probability $\pi_{i,t}$ given by equation (12), gaining the flow value $J_{i,t+1}$. Otherwise, the firm remains single with flow value $\widetilde{J}_{i,t+1}$. If the job is destroyed, the firm exits the market with zero value.

The value of a joint venture for a sector $I$ firm is:

$$J_{I,t|\iota_t} = z_t p_t - w_{I,t} + \beta \xi_t \mathbb{E}_t \left[ (1 - \delta - \widetilde{\delta}) J_{I,t+1} + \widetilde{\delta} \widetilde{J}_{I,t+1} \mid \iota_t \right]. \tag{19}$$

This profit comprises revenues $z_t p_t$ from selling intermediate goods to the final-goods firm, net of the wage $w_{I,t}$. Both $p_t$ and $w_{I,t}$ are determined by Nash bargaining. In period $t+1$, with probability $\widetilde{\delta}$, the firm is separated from its partner and becomes a single firm, gaining a value of $\widetilde{J}_{I,t+1}$; with probability $\delta$, the job match is destroyed, and the firm exits the market with zero value. Otherwise the joint venture continues with flow value $J_{i,t+1}$.

The value of a joint venture for a sector $F$ firm is:

$$J_{F,t|\iota_t} = z_t(1 - p_t) - w_{F,t} + \beta \xi_t \mathbb{E}_t \left[ (1 - \delta - \widetilde{\delta}) J_{F,t+1} + \widetilde{\delta} \widetilde{J}_{F,t+1} \mid \iota_t \right]. \tag{20}$$

The profit for the joint venture in the final-goods sector comprises revenues from selling $z_t$ units of final goods at a unitary price, net of the costs of purchasing intermediate goods ($z_t p_t$) and paying the wage ($w_{F,t}$). The rest of the equation follows the same interpretation as equation (19).

The value of a vacant job in sector $i$ is:

$$V_{i,t|\iota_t} = -\chi + \beta \xi_t \mathbb{E}_t \left[ q\left(\theta_{i,t}\right) \widetilde{J}_{i,t+1} + \left(1 - q\left(\theta_{i,t}\right)\right) \max\left(0, V_{I,t+1}, V_{F,t+1}\right) \mid \iota_t \right]. \tag{21}$$

Equation (21) shows that the value of a vacant job comprises the fixed cost of posting a vacancy $\chi$ in period $t$. With probability $q\left(\theta_{i,t|\iota_t}\right)$, the vacancy is filled, and a single firm with flow value $\widetilde{J}_{i,t+1}$ is created. Otherwise, the vacancy remains open, generating the flow value of $V_{i,t+1}$. The last term in the equation shows that firms that fail to recruit a worker may choose to be inactive or post a vacancy in either sector in the next period $t+1$.

Due to the free-entry condition by firms, we have $V_{i,t} = 0$ and then:

$$\chi = \beta \xi_t \mathbb{E}_t \left[ q\left(\theta_{i,t}\right) \widetilde{J}_{i,t+1} \mid \iota_t \right], \tag{22}$$

a condition that pins down labor market tightness.

## 3.5 Wages and prices

We are ready now to define the Nash bargaining rules that determine wages and prices. During each period $t$, wages are pinned down by Nash bargaining between firms in joint ventures and workers:

$$\max_{w_{i,t}} (W_{i,t} - U_{i,t})^{1-\tau} J_{i,t}^{\tau} \tag{23}$$

and between single firms and workers:

$$\max_{\widetilde{w}_{i,t}} (\widetilde{W}_{i,t} - U_{i,t})^{1-\tau} \widetilde{J}_{i,t}^{\tau}, \tag{24}$$

where the parameter $\tau \in [0, 1]$ is the firm's bargaining power.

The price for goods manufactured in the intermediate-goods sector is determined by Nash bargaining between the final-goods producer and the intermediate-goods producer within the joint venture:

$$\max_{p_t} (J_{F,t} - \widetilde{J}_{F,t})^{1-\widetilde{\tau}} (J_{I,t} - \widetilde{J}_{I,t})^{\widetilde{\tau}}, \tag{25}$$

where the parameter $\widetilde{\tau} \in [0, 1]$ is the intermediate-goods producer's bargaining power.

## 3.6 Stochastic processes and aggregate resource constraint

The preference shock, $\xi_t$, has a log-normal *i.i.d.* distribution, $\log(\xi_t) \sim \mathcal{N}\left(0, \sigma_\xi^2\right)$. This preference shock is not persistent over time. In this way, we can show that the propagation mechanism created by discount factor shocks in our model is wholly endogenous. Productivity follows an AR(1) process in logs: $\log(z_{t+1}) = \rho_z \log(z_t) + \sigma_z \epsilon_{z,t+1}$ where $\rho_z \leq 1$.

We close the presentation of the model by pointing out that the total resources of the economy, equal to $z_t n_t$ (i.e., production per joint venture times the number of existing joint ventures; $h$ is in util terms and, thus, fails to appear here), are used for aggregate consumption by households, $c_t$, and to pay for vacancies and intra-firm search:

$$c_t + \sum_{i=I,F} \chi v_{i,t} + \sum_{i=I,F} \widetilde{n}_{i,t} \left( c_0 \sigma_{i,t} + c_1 \frac{\sigma_{i,t}^{1+\nu}}{1+\nu} \right) = z_t n_t. \tag{26}$$

# 4 Equilibrium

A recursive equilibrium of type $\iota_t$ for our economy is a collection of Bellman equations $U_{i,t}$, $\widetilde{W}_{i,t}$, $W_{i,t}$, $\widetilde{J}_{i,t}$, $J_{i,t}$, and $V_{i,t}$, a search intensity $\sigma_{i,t}$, and sequences for unemployment $u_t$, single firms $\widetilde{n}_{i,t}$, joint ventures $n_t$, the price of the intermediate good $p_t$, and wages $\widetilde{w}_{i,t}$ and $w_{i,t}$, all for $i \in \{I, F\}$, such that:

1. $U_{i,t}$, $\widetilde{W}_{i,t}$, $W_{i,t}$, $\widetilde{J}_{i,t}$, $J_{i,t}$, and $V_{i,t}$ satisfy equations (15)-(21).

2. The free entry condition $V_{i,t} = 0$ holds.

3. $\sigma_{i,t}$ maximizes the asset value of the single firm $\widetilde{J}_{i,t}$.

4. The sequences of unemployment $u_t$, single firms $\widetilde{n}_{i,t}$, and joint ventures $n_t$ follow the laws of motion in equations (7), (14), and (13), respectively.

5. The intermediate-goods price $p_t$ and wage for single and joint ventures, $\widetilde{w}_{i,t}$ and $w_{i,t}$, respectively, are determined by the Nash bargaining equations (23)-(25).

6. The type of equilibrium $\iota_t$ is consistent with the value of search intensity $\sigma_{i,t}$.

7. $\xi_t$ and $z_t$ follow their stochastic processes.

8. The aggregate resource constraint (26) is satisfied.

We can use this definition to characterize the optimal search strategy of firms and show the existence of multiple equilibria.

## 4.1 Optimal search intensity

Following condition 3 above, the optimal search intensity $\sigma_{i,t}$ maximizes the value of the single firm, $\widetilde{J}_{i,t}$. We can express this value function as a response function to $\sigma_{j,t}$ given an equilibrium $\iota_t$:

$$\Pi_i\left(\sigma_{i,t} \mid \sigma_{j,t}, \iota_t\right) = -\widetilde{w}_{i,t} - c\left(\sigma_{i,t}\right) + \beta \xi_t \left(1 - \delta\right) \mathbb{E}_t \left[\pi_{i,t}(J_{i,t+1} - \widetilde{J}_{i,t+1}) + \widetilde{J}_{i,t+1} \mid \iota_t\right]. \tag{27}$$

A single firm $i$ chooses its search intensity $\sigma_{i,t}$ to maximize $\Pi_i(\sigma_{i,t} \mid \sigma_{j,t}, \iota_t)$. The interior solution $\sigma_{i,t} > 0$ satisfies:

$$c_0 + c_1 \sigma_{i,t}^{\nu} = \widetilde{\beta} \underbrace{(\psi + \sigma_{j,t})}_{\text{Search intensity in sector } j} \underbrace{\xi_t}_{\text{Preference shock}} \underbrace{\mathbb{E}_t\left(J_{i,t+1} - \widetilde{J}_{i,t+1} \mid \iota_t\right)}_{\text{Expected capital gain}} \tag{28}$$

where $\widetilde{\beta} = \beta(1-\delta)/\tau$ (the wage Nash bargaining implies that the firm bears $\tau$ fraction of the search cost). The left-hand side (LHS) of equation (28) is the marginal cost of exerting search intensity to build a joint venture in sector $i$, while the RHS is the expected benefit of searching for a partner, which increases with $\sigma_{j,t}$, and the expected capital gain from entering into a joint venture, $\mathbb{E}_t(J_{i,t+1} - \widetilde{J}_{i,t+1} \mid \iota_t)$ times the preference shock $\xi_t$. Because the optimization problem is non-convex, we also have a corner solution $\sigma_{i,t} = 0$ when the RHS of equation (28) is less than $c_0$, either because the firms in the other sector do not search actively or because the discounted expected gains from matching are small. The next proposition summarizes this argument.

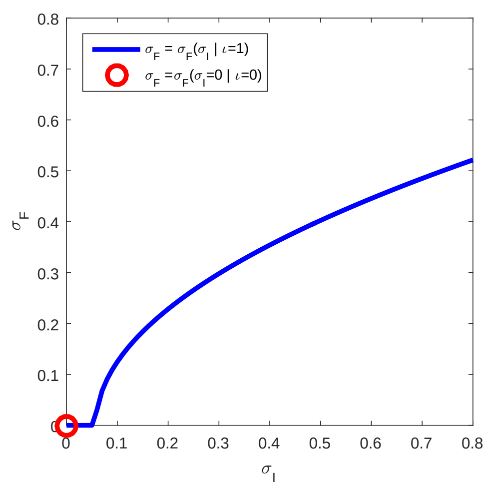
**Proposition 1.** *The optimal search intensity $\sigma_{i,t}$ is equal to:*

$$\sigma_{i,t} = \begin{cases} \left[\frac{\widetilde{\beta}(\psi+\sigma_{j,t})\xi_t \mathbb{E}_t\left(J_{i,t+1} - \widetilde{J}_{i,t+1} \mid \iota_t\right) - c_0}{c_1}\right]^{\frac{1}{\nu}} & \text{if } \widetilde{\beta}(\psi + \sigma_{j,t})\xi_t \mathbb{E}_t\left(J_{i,t+1} - \widetilde{J}_{i,t+1} \mid \iota_t\right) > c_0 \\ 0 & \text{otherwise.} \end{cases} \tag{29}$$

Proposition 1 establishes why search complementarities beget a multiplicity of equilibria (this proposition follows directly from equation (28); the proofs of the other propositions and lemmas in this subsection appear in Appendix C). The firm's optimal search intensity in sector $i$ depends on the expected capital gain from forming a joint venture, which in turn depends on the search intensity of the firms in sector $j$. Positive search intensity in one sector stimulates search intensity in the other sector. Similarly, a termination of search in one sector lowers search intensity and ultimately terminates search in the other sector. The parameter $c_0$ determines whether the firm searches with positive intensity while $c_1$ controls search intensity. A high $c_1$ increases the marginal cost of search and flattens the best response line. Shocks to $\xi_t$ change the incentive for searching. Sufficiently large shocks move the system between equilibria and alternate business cycle phases with robust search intensity, a large number of joint ventures, and low unemployment with phases marked by no search intensity, few joint ventures, and high unemployment.

Figure 5 illustrates Proposition 1 by plotting the optimal search intensity $\sigma_F$ of a firm in the final-goods sector as the best response to the search intensity of firms in the intermediate-goods sector $\sigma_I$. The red circle shows the best response in the passive equilibrium when search intensity in the intermediate-goods sector is zero. The solid line shows the best response in the active equilibrium with positive search intensity. Here and in the rest of this section, we calibrate the model using the parameter values described in Section 5.[8]

**Figure 5:** Best response function for firm in the final-goods sector




*Note*: The figure shows the best response function in the final-goods sector conditional on the *active* equilibrium ($\iota = 1$, solid line) and on the *passive* equilibrium ($\iota = 0$, circle marker).

Two lessons emerge from Figure 5. First, the active equilibrium with positive search intensity involves complementarities in search intensity. The upward sloping optimal response curve shows that the final-goods producing firm (weakly) increases its search intensity when the firm in the intermediate-goods sector increases its own search intensity. Second, the optimal search intensity for the final-goods firm remains equal to zero for values of $\sigma_I$ below 0.05. In such a region, the marginal cost of forming a joint venture is larger than the benefit of the joint venture and, thus, final-goods producing firms choose $\sigma_F = 0$.

[8] Also, we use the expected capital gain in the stable and active deterministic steady state (DSS) when computing the best response curve in the active equilibrium. Analogously, we use the expected capital gain in the passive DSS when computing the best response in the passive equilibrium. We will follow the same assumptions regarding the expected capital gain and parameter values in Figures 8 and 9 below.

## 4.2 The deterministic steady states of the model

We study now the existence and stability properties of the deterministic steady states (DSSs) of the model that appear when we shut down the shocks to the discount factor and productivity. The model encompasses two types of DSSs: a passive DSS with zero search intensity ($\sigma_I = \sigma_F = 0$) and active DSSs with positive search intensity ($\sigma_I > 0$, $\sigma_F > 0$). The level of economic activity is different across DSSs.

**Proposition 2.** *The level of output is strictly lower and the unemployment rate is strictly higher in a passive DSS than in an active DSS.*

Proposition 2 shows that the passive DSS is associated with weak economic activity compared to an active DSS. Intuitively, zero search intensity in the passive DSS implies few joint ventures and low production. A small probability of forming a joint venture reduces the value of a single firm and generates a fall in posted vacancies and an increase in unemployment.

The next two propositions establish conditions for the existence of the different DSSs.

**Proposition 3.** *The passive DSS exists if and only if*

$$\frac{\tilde{\beta}\psi}{2 - 2\beta\left[\left(1-\delta-\tilde{\delta}\right) - (1-\delta)\left(\phi+\psi^2\right)\right]} < c_0. \tag{30}$$

Proposition 3 states that the passive DSS exists for any sufficiently large value of $c_0$—that is, when the benefit from an additional unit of search intensity is lower than the cost associated with it. In such a case, optimal search intensity is zero (i.e., $\sigma_I = \sigma_F = 0$). The critical cost for the existence of the passive DSS is $c_0$. In comparison, $c_1$ does not appear in Proposition 3.

**Proposition 4.** *The active DSS exists if and only if there exists $\sigma \in \left(0, \sqrt{1-\phi} - \psi\right)$ that solves*

$$\tilde{\beta}\left(\psi+\sigma\right)\frac{1+\left(c_0\sigma + c_1\frac{\sigma^{1+\nu}}{1+\nu}\right)}{2-2\beta\left[\left(1-\delta-\tilde{\delta}\right) - (1-\delta)\left(\phi+(\sigma+\psi)^2\right)\right]} = c_0 + c_1\sigma^{\nu}. \tag{31}$$

The LHS of equation (31) captures the marginal gain of searching with positive intensity in the active equilibrium. The RHS reflects the marginal cost of searching. In the active DSS, both quantities must be equal. Proposition 4 defines the parameter values that guarantee the existence of the active DSS. The restriction $\sigma \in \left(0, \sqrt{1-\phi}-\psi\right)$ ensures that the matching probability $\phi + (\psi + \sigma_I)(\psi + \sigma_F)$ is within (0,1).

**Proposition 5.** *The active and passive DSSs coexist if and only if equations (30) and (31) hold simultaneously.*

Equations (30) and (31) can hold simultaneously, since they depend on different parameter combinations. Intuitively, the passive DSS characterized by equation (30) is uniquely pinned down when search intensity is zero and the stochastic shocks are fixed at their mean value. In comparison, the system allows for multiple active DSSs, since equation (31) can hold for different symmetric values of search intensity across the two sectors.

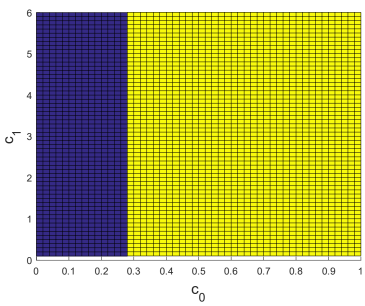
Using Figure 5, we determine the active DSS by the crossing between the best response function and the 45-degree line that represents the intersection of the best response function in the two sectors. When the best response function is strictly concave (i.e., $\nu > 1$), the system admits, at most, two DSSs (if $\nu < 1$, we would only have one active and unstable equilibrium). The argument is formalized in the lemma below.

**Lemma 1.** *The system has a unique passive DSS and at most two active DSSs.*

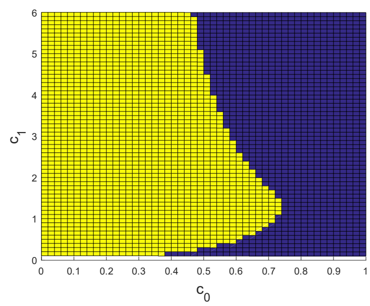
Figure 6 numerically illustrates, for a range of values of $c_0$ (x-axes) and $c_1$ (y-axes), the conditions for the existence of a passive DSS, an active DSS, and the coexistence of DSSs (the computation of the DSS is described in Appendix B). The yellow-shaded area shows the combination of $c_0$ and $c_1$ values that guarantee the existence of such a DSS, while the blue area shows the nonexistence region. Panel (a) shows that the passive DSS exists for values of $c_0$ larger than 0.28, irrespective of $c_1$. As stated in Proposition 3, a sufficiently large value of $c_0$ leads firms to search with zero intensity. Panel (b) demonstrates that the active DSS exists for sufficiently low values of $c_0$. An increase in the value of $c_1$ has two opposing effects on the incentive to form a joint venture. On the one hand, it increases the cost of search intensity and, on the other hand, it decreases the value of remaining a single firm, which raises the relative value of forming a joint venture. On balance, the second effect dominates, and a large $c_1$ expands the range of values of $c_0$ that satisfy Proposition 4. Panel (c) shows that two active DSSs exist when $c_1$ is sufficiently large. Panel (d) combines panel (a) and panel (b) to draw the values for $c_0$ and $c_1$ that support the coexistence of passive and active DSSs. Lastly, panel (e) plots the values of $c_0$ and $c_1$ that allow for the coexistence of a passive and two active DSSs.

**Figure 6:** Existence of DSSs

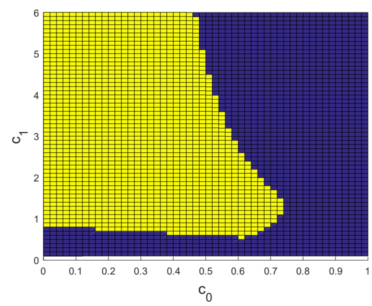
**(a)** Passive DSS



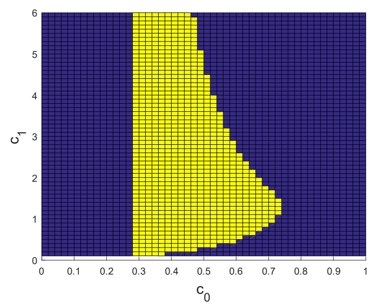

**(b)** At least one active DSS



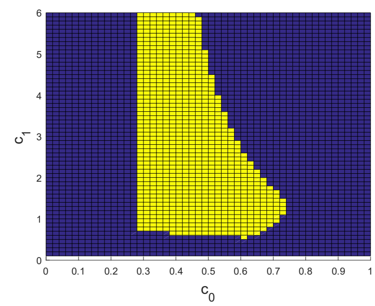

**(c)** Two active DSS




**(d)** Coexistence of passive and at least one active DSS




**(e)** Coexistence of passive and two active DSSs



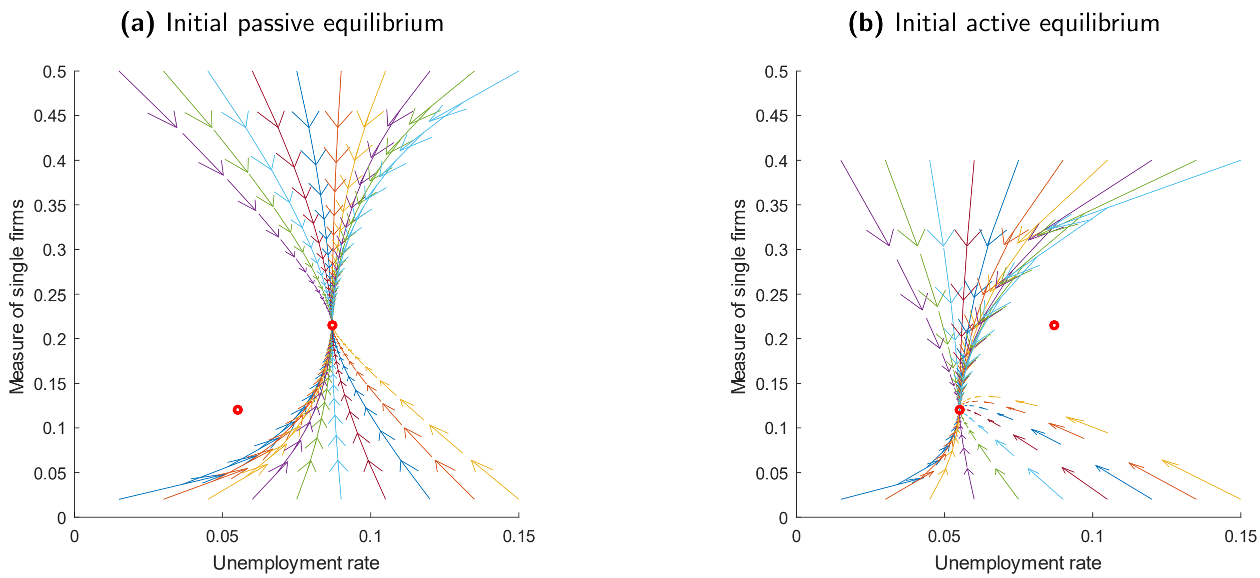

The next proposition establishes the stability of the DSSs. This stability guarantees that a slight deviation of a subset of firms from their best response will fail to cause the system to deviate from the initial DSS permanently.

**Proposition 6.** *Suppose the active and passive DSSs coexist. The passive DSS is stable. When two active DSSs coexist, one DSS is stable and the other DSS is unstable. When only one active DSS exists, it is unstable.*

For the remainder of the analysis, we mainly focus on stable DSSs. Also, we can study the transition path from an arbitrary point in the state space of the system to the DSS. The endogenous state variables of the system are the unemployment rates $(u_{I,t}, u_{F,t})$, the measure of single firms $(\tilde{n}_{I,t}, \tilde{n}_{F,t})$, the measure of firms in joint ventures $(n_{I,t}, n_{F,t})$, and the current equilibrium $(\iota_t)$. Knowledge of $\tilde{n}_{i,t}$ and $u_{i,t}$ gives us $n_{i,t} = 1 - \tilde{n}_{i,t} - u_{i,t}$.

Figure 7 shows the transition path of the system to the DSS for different initial values of the unemployment rate (x-axes) and the measure of single firms (y-axes). Since we consider the case of a symmetric economy, the analysis is representative of the equilibrium in each sector. Panel (a) shows the transition path to the DSS when the system starts from a passive equilibrium (with each red dot representing a DSS of the system). Given the history dependence of the equilibrium selection, the system remains in the passive equilibrium and converges to the passive DSS indicated by the higher red circle, where the unemployment rate is 8.7% and the measure of single firms is 22%. Analogously, panel (b) shows the system converges to the active and stable DSS, when it starts from an active equilibrium. In the active DSS (the lower red dot), the unemployment rate is 5.5%, and the measure of single firms is 12%.

**Figure 7:** Transition path to the DSS




## 4.3 Existence of two (stochastic) equilibria

Once we have characterized the DSSs of the model, we can reintroduce the shocks to the discount factor and productivity. The following propositions characterize the conditions for the existence of (stochastic) passive and active equilibria and their coexistence.

**Proposition 7.** *The passive equilibrium exists if and only if*

$$\frac{\partial \Pi_i \left(0|0, \iota_t = 0\right)}{\partial \sigma_{i,t}} \leq 0 \quad \text{for } i = I, F \tag{32}$$

*or equivalently*

$$c_0 > \widetilde{\beta} \psi \xi_t \mathbb{E}_t (J_{i,t+1} - \widetilde{J}_{i,t+1} \mid \iota_t = 0). \tag{33}$$

Proposition 7 states that the passive equilibrium exists when the marginal benefit from increasing search intensity is negative. Equation (33) highlights that the existence of the passive equilibrium requires either a low $\xi_t$ or a small $z_{t+1}$ (and, hence, a low $\mathbb{E}_t(J_{i,t+1} - \widetilde{J}_{i,t+1} \mid \iota_t = 0)$).

**Proposition 8.** *The active equilibrium exists if and only if there exists a pair of positive search efforts ($\{\sigma_{I,t}\sigma_{F,t}\} > 0$) that satisfies:*

$$\frac{\partial \Pi_i \left(\sigma_{i,t} \mid \sigma_{j,t}, \iota_t = 1\right)}{\partial \sigma_{I,t}} = 0 \quad \text{for } i = \{I, F\} \tag{34}$$

*or, equivalently,*

$$c_0 + c_1 \sigma_{i,t}^{\nu} = \widetilde{\beta} \left(\psi + \sigma_{j,t}\right) \xi_t \mathbb{E}_t(J_{i,t+1} - \widetilde{J}_{i,t+1} \mid \iota_t = 1), \tag{35}$$

*with $(\sigma_{I,t}, \sigma_{F,t}) > 0$ and the second derivatives of $\Pi_i$ are negative.*

Proposition 8 states that an active equilibrium exists when the optimal response of the firm is to choose a positive search intensity that satisfies equation (35).

**Proposition 9.** *The active and passive equilibria coexist if and only if Propositions 7 and 8 hold simultaneously.*

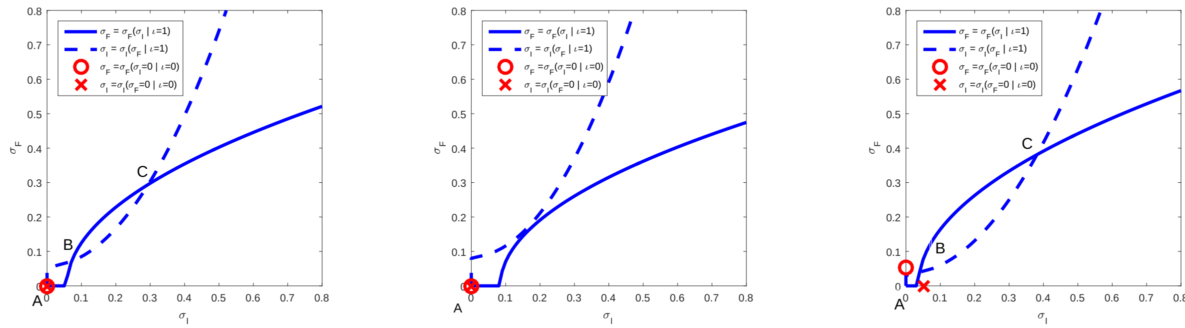
Proposition 9 states the condition for the coexistence of the two equilibria. History dependence selects between them.

## 4.4 Multiple equilibria, dynamics, and $\xi_t$

We can also investigate the role of $\xi_t$ in creating a multiplicity of equilibria. Panel (a) in Figure 8 plots the optimal search intensity in each sector for $\xi_t = 1$. The solid and dashed lines show the optimal response for the final-goods firm and the intermediate-goods firm, respectively, for the active equilibrium (i.e., $\iota = 1$). The circle and cross markers show the optimal response for the final-goods firm and the intermediate-goods firm, respectively, in the passive equilibrium (i.e., $\iota = 0$). Panel (a) shows three crossings of the best response functions. Point A has zero search intensity (i.e., $\sigma_I = \sigma_F = 0$). Point C shows a positive optimal level of search intensity (i.e., $\sigma_I = \sigma_F = 0.30$). Furthermore, this optimal level of search intensity is stable. Point B also entails positive search intensity (i.e., $\sigma_I = \sigma_F = 0.06$), but this choice is unstable. If one firm increases its search intensity, the firm in the opposite sector also increases its search intensity until the system reaches point C. Similarly, if a firm decreases its search intensity, the firm in the other sector also decreases its search intensity until we arrive at point A.

**Figure 8:** Coexistence of equilibria

**(a)** Two equilibria ($\xi_t = 1$) **(b)** Unique passive eq. ($\xi_t = 0.85$) **(c)** Unique active eq. ($\xi_t = 1.10$)



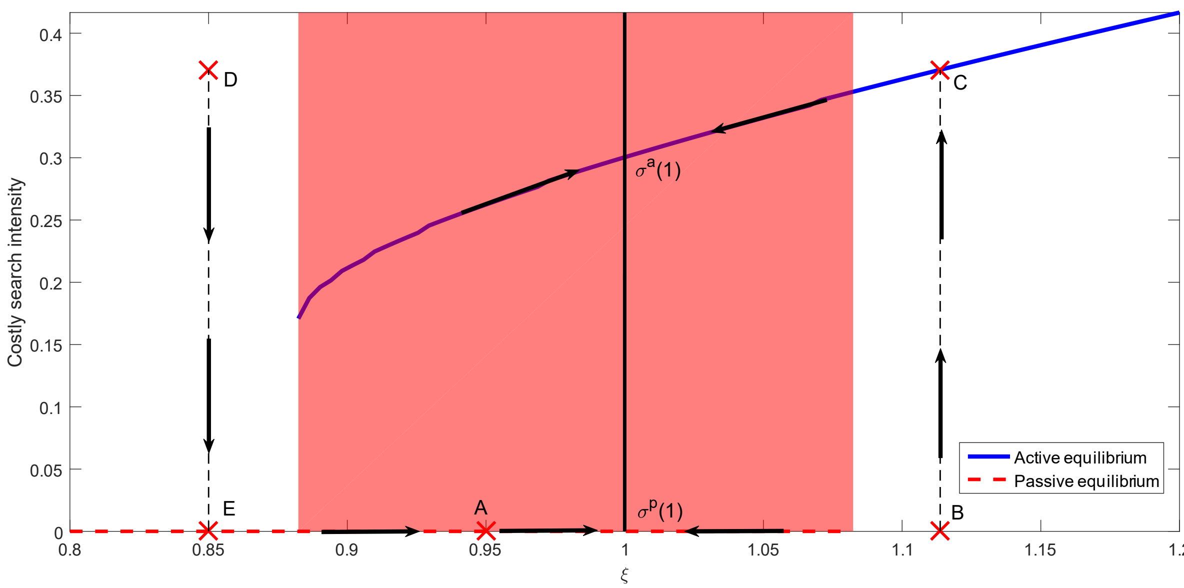

*Note*: Each panel shows the best response function of the final-goods sector (solid line) and the intermediate-goods sector (dashed line) for the active equilibrium and the best response for the passive equilibrium for the final-goods sector (circle marker) and the intermediate-goods sector (cross marker).

We consider now the case when $\xi_t = 0.85$. Panel (b) in Figure 8 shows that point A ($\sigma_I = \sigma_F = 0$) continues to exist. However, there is no other crossing of the best response functions: a low $\xi_t$ reduces the gain from forming a joint venture, and firms exert no search intensity. Finally, when $\xi_t = 1.10$, panel (c) of Figure 8 shows that the system retains the crossings at points B and C (with the same stability properties as when $\xi_t = 1$). Point A disappears, since firms in both sectors optimally choose positive search intensity even when the other firms do not search, as shown by the circle and cross markers around 0.05.

To illustrate the properties of the stochastic system and the transition dynamics between equilibria, Figure 9 draws the phase diagram summarizing movements in search intensity as a function of $\xi_t$ (a similar figure could be drawn for changes in $z_t$). The dashed line plots the passive equilibrium path with low search intensity and the solid line the active equilibrium path with high search intensity. The arrows indicate the direction of the transition dynamics for the endogenous variable to reach the basins of attraction of the system, represented by point $\sigma^p(1)$ for the passive DSS and $\sigma^a(1)$ for the active DSS. The shaded area indicates the range of values of $\xi_t$ that support multiple equilibria. The passive equilibrium fails to exist for sufficiently large values of $\xi_t$ and, conversely, the active equilibrium fails to exist for sufficiently small values of $\xi_t$. In the absence of innovations to $\xi_t$, the system converges and remains on the original basins of attraction in the passive equilibrium, $\sigma^p(1)$, and the active equilibrium, $\sigma^a(1)$, depending on the starting equilibrium.

Temporary shifts to $\xi_t$, which are sufficiently strong to change search intensity, move the

**Figure 9:** Phase diagram for search intensity




system to a new equilibrium. For example, if the system starts in the passive equilibrium at point A and a large and positive innovation to $\xi_t$ moves the system to point B, the passive equilibrium disappears, and the equilibrium of the system becomes active. The economy moves to the new active equilibrium at point C, converging to the stationary basin of attraction $\sigma^a(1)$ in the long run. The system remains in the active equilibrium until a sufficiently large and negative innovation to $\xi_t$ returns the system to the passive equilibrium. For instance, a large negative innovation to $\xi_t$, which moves the system from point C to point D, triggers the new passive equilibrium at point E, converging to the stationary basin of attraction $\sigma^p(1)$. In comparison, innovations to $\xi_t$ that move the equilibrium of the system within the shaded area, where both equilibria coexist, fail to shift the equilibrium because of history dependence.

# 5 Calibration

We calibrate the model at a monthly frequency for U.S. data over the post-WWII period. Table 2 summarizes the value, and the source or target for each parameter.

The constant $\beta$ in the discount factor is set to 0.996 (equivalent to 0.99 at a quarterly frequency) to replicate an average annual interest rate of 5% over the sample period. We assume a Cobb-Douglas matching function $m(u, v) = u^{1-\alpha}v^{\alpha}$ in the labor market and calibrate the elasticity of vacancies in the matching function $\alpha = 0.4$, which is the average value estimated in

**Table 2:** Parameter calibration

| Parameter | Value | Source or Target |
|---|---|---|
| $\beta$ | 0.996 | 5% annual risk-free rate |
| $\alpha$ | 0.4 | Shimer (2005) |
| $\tau$ | 0.4 | Hosios condition |
| $\chi$ | 0.28 | 0.45 monthly job-finding rate |
| $\kappa$ | 1.25 | den Haan et al. (2000) |
| $h$ | 0.3 | Thomas and Zanetti (2009) |
| $\widetilde{\tau}$ | 0.5 | Sectoral symmetry |
| $\delta$ | 0.027 | 5.5% unemployment rate in active DSS |
| $\widetilde{\delta}$ | 0.017 | 5 years duration of joint venture |
| $\phi$ | 0.135 | 22% rate of idleness in recessions |
| $\psi$ | 0.114 | Condition of Propositions 3 and 4 and 15% recession periods |
| $c_0$ | 0.33 | Condition of Propositions 3 and 4 and 15% recession periods |
| $c_1$ | 5 | 12% rate of idleness in booms |
| $\nu$ | 2 | Ensure concavity of best response function |
| $\sigma_\xi$ | 0.05 | Justiniano and Primiceri (2008) |
| $\rho_z$ | $0.95^{1/3}$ | BLS |
| $\sigma_z$ | 0.008 | BLS |

the literature (see Petrongolo and Pissarides, 2001). We set the wage bargaining power equal to $\tau = \alpha = 0.4$, which satisfies the Hosios (1990) condition for efficiency. The inter-firm matching function is

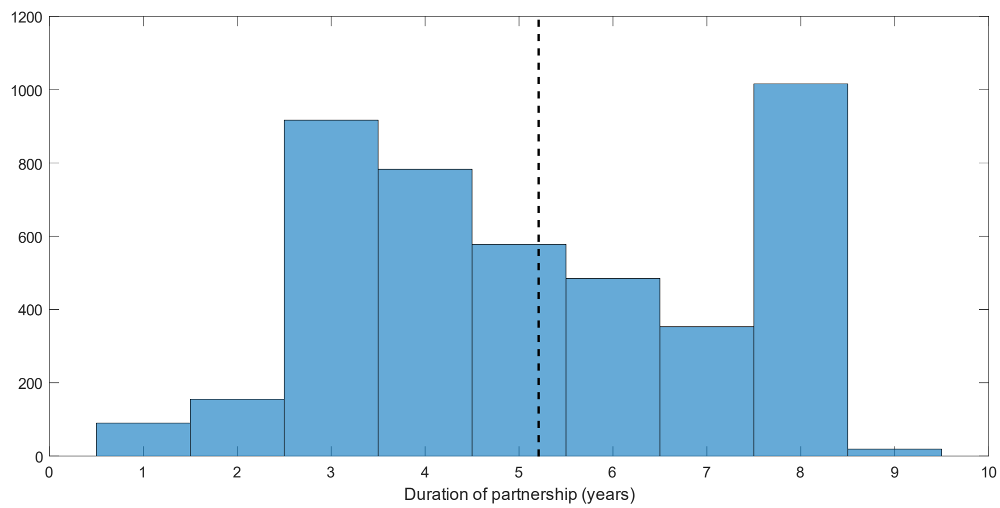
$$H\left(\widetilde{n}_F, \widetilde{n}_I\right) = \frac{\widetilde{n}_F \cdot \widetilde{n}_I}{\left(\widetilde{n}_F^\kappa/2 + \widetilde{n}_I^\kappa/2\right)^{1/\kappa}}. \tag{36}$$

We follow den Haan et al. (2000) and set $\kappa = 1.25$.

We pick the cost of posting a vacancy $\chi = 0.28$ to match the monthly job-finding rate in the active DSS, $\mu(\theta) = 0.45$, as in Shimer (2005). Conditional on $\chi = 0.28$, we select a job-separation rate $\delta = 0.027$ to match an unemployment rate of 5.5% in the active DSS. The flow value of unemployment $h$ is set at 0.3, which consists of the value of leisure and home production and the unemployment benefit, as in Thomas and Zanetti (2009). In this calibration, the flow value of unemployment is about 61% of the average wage in the active DSS, which is in the range of replacement rates documented by Hall and Milgrom (2008).

Compared to a standard DMP economy, our model includes seven new parameters: $\widetilde{\tau}$, $\widetilde{\delta}$, $\phi$, $\psi$, $c_0$, $c_1$, and $\nu$. The bargaining share of the intermediate-goods firm $\widetilde{\tau}$ is set to 0.5, to evenly split between firms the total surplus from matching and make the workers indifferent between working in either sector. The rate of termination of inter-firm matches $\widetilde{\delta}$ is 0.017 to target it to the 5 years' average duration of joint ventures. As shown in Figure 10, the median and the mean of the duration of inter-firm matches is around 5 years in the Compustat segment data, which report the major trading partners for a subset of listed companies in the United States on a yearly basis.

**Figure 10:** The distribution of the inter-firm trading relationship duration




Once we set values for $\widetilde{\delta}$ and the labor market parameters, the convex component of the search cost $c_1$ and the constant component of inter-firm matching efficiency $\phi$ pin down the measure of single firms in the active DSS and passive DSS, respectively. The ratio of the measure of single firms to employment corresponds to the rate of idleness, indicating the share of time when employed workers are idle due to a lack of activity (see Michaillat and Saez, 2015). The Institute for Supply Management constructs the operating rates (one minus the rate of idleness) in the United States. According to its measurements, the rate of idleness is about 30% for the non-manufacturing sector and 20% for the manufacturing sector during the Great Recession, and 12% for both sectors before this event. Thus, we set $\phi = 0.135$ and $c_1 = 5$ to yield a rate of idleness equal to 0.22 and 0.12 in the passive DSS and the active DSS, respectively. Finally, $\nu = 2$ ensures the concavity of the best response function of search intensity.

There is no direct empirical guidance for the calibration of $c_0$ and $\psi$. We calibrate them as 0.33 and 0.114, respectively, to satisfy the conditions for the coexistence of passive and active DSSs in Proposition 5. Our calibration of $c_0$ and $c_1$ generates similar costs of hiring workers and costs of searching for intermediate-goods firms. This finding is consistent with Michaillat and Saez (2015), who establish that the number of workers whose occupation is buying, purchasing, and procurement is about the same as the number of workers whose job relates to recruitment.

Formally, our calibration implies that hiring costs and search costs are the same, i.e., the values for $c_0$ and $c_1$ are such that $\chi \cdot v_i \approx n_i^* \cdot \left(c_0\sigma + c_1\frac{\sigma^{1+\nu}}{1+\nu}\right)$ in the DSSs.

We set $\sigma_\xi$ to 0.05.[9] Such a value, given the rest of the calibration, generates a passive equilibrium with 15% probability, consistent with the frequency of recessions in the post-WWII United States. The persistence of the productivity shock, $\rho_z$, is set to $0.88^{1/13}$ to match the observed quarterly autocorrelation of 0.88, and the standard deviation, $\sigma_z$, is set to 0.0027 to match the quarterly standard deviation of 0.02, as in Shimer (2005).

Once the model is calibrated, we compute the different value functions using value function iteration and exploit the equilibrium conditions of the model to find all other endogenous variables of interest. See Appendices A and D for technical details.

# 6 Quantitative analysis

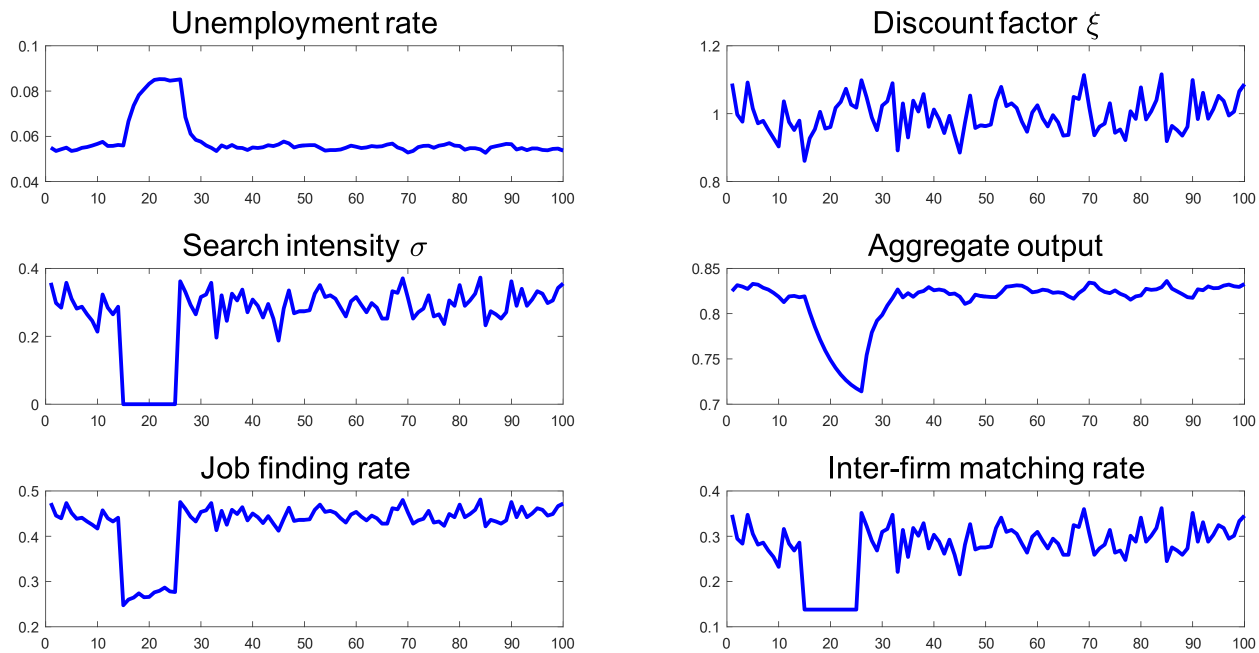
In this section, we study the dynamic properties of the model by simulating it for 3,000,000 months and time-averaging the resulting variables to generate quarterly data. We start the simulation from the active DSS, focusing on the case when only discount factor shocks are present. Appendix F provides a quantitative analysis of properties of the model with productivity shocks. We relegate that case to the appendix because we find that productivity shocks of plausible magnitude are unable to move the system between different equilibria, unless those shocks to technology are permanent.

Figure 11 reports the responses of key variables to shocks to $\xi_t$ for the first 100 periods. The economy begins at a positive search intensity with high output, low unemployment, and a high job-finding rate. Then, in period 15, a sufficiently large shock to the discount factor pushes the economy to the low search equilibrium until period 25, with a prolonged drop in output (as joint ventures terminate faster than they are replaced), high unemployment, and a low job-finding rate. In that period, a large positive discount factor shock shifts the economy back to the active equilibrium with positive search intensity.

Figure 12 plots the ergodic distribution of selected variables implied by the entire simulation.

[9] Justiniano and Primiceri (2008, Table 1) find that the quarterly $\sigma_{\epsilon_\xi} = 3.13\%$, with a persistence of 0.84. This implies that $\sigma_\xi = 0.0313/\sqrt{1-0.84^2} = 0.0577$. If we extrapolate the quarterly AR(1) process to a monthly AR(1) process, the implied standard deviation is about 0.056. Since we ignore persistence, we lower $\sigma_\xi$ to 0.05, to be on the conservative side.

**Figure 11:** Simulated variables for the first 100 periods with shocks to $\xi_t$

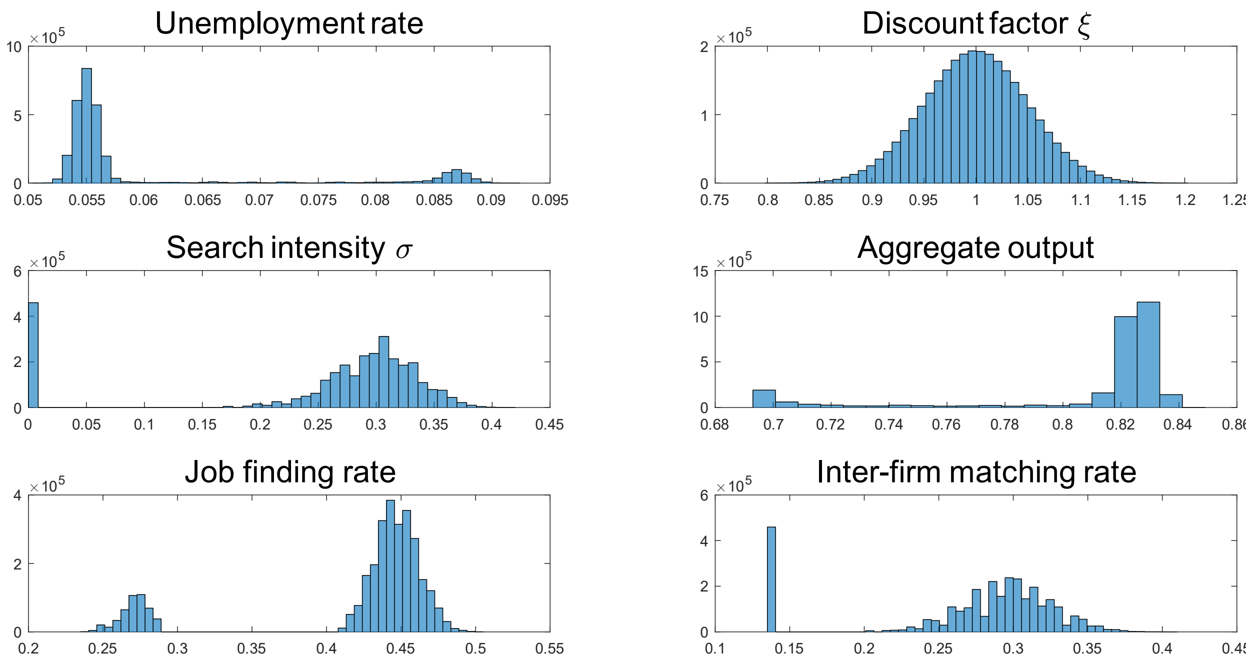



Endogenous switches between the two equilibria generate a distinctive bimodal distribution of aggregate variables that bring significant differences between the two equilibria. As required by our calibration, the figure implies that the economy spends about 85% of the time in the active equilibrium and 15% in the passive equilibrium. In the active equilibrium, the unemployment rate fluctuates around 5.5%. In the passive equilibrium with zero search intensity, unemployment fluctuates around 8.7%. Similarly, the job-finding rate moves around 45% in the active equilibrium and 27% in the passive equilibrium.

Panel (a) of Table 3 reports various second moments of observed business cycle statistics following the same structure as in Shimer (2005, Table 1). Panel (b) reports second moments of the benchmark model with two DSSs. Finally, Panel (c) reports second moments of a version of the model without search complementarities and calibrated on the active equilibrium. Each entry presents the autocorrelation coefficient, the standard deviation, and the correlation matrix for the variables listed across the first row of the table.

Several lessons come from Table 3. First, our benchmark model generates a robust internal propagation: the autocorrelation coefficients of the aggregate variables are significantly larger than in the model without complementarities and much closer to the observed ones. Complementarities in search intensity amplify and prolong the effect of shocks.

**Figure 12:** Ergodic distribution with *i.i.d.* shocks to $\xi_t$




Second, our benchmark model generates large and empirically plausible standard deviations for the selected variables that are substantially larger than those in the model without complementarities. This property of the model comes from the amplification of shocks created by the shift between equilibria.

Third, the benchmark model produces endogenous movements in labor productivity ("lp" in the table) that would be otherwise absent. Our benchmark model assumes that firms manufacture goods after matching with a partner. Hence, measured labor productivity depends on the fraction of the joint ventures over the total number of firms, $n_{i,t}/\left(\widetilde{n}_{i,t}+n_{i,t}\right)$, which is endogenously determined. In comparison, in a version of the model without joint ventures, labor productivity is exogenous. Table 3 shows that business cycle statistics for labor productivity generated by our benchmark model are close to those in the data.

Fourth, the benchmark model generates a correlation between unemployment and vacancies (i.e., the Beveridge curve) equal to -0.71, which is close to the value of -0.92 in the data and much larger than the correlation of -0.27 in the model without complementarities. The large negative correlation between vacancies and unemployment is a direct consequence of strategic complementarities in search intensity. For instance, in the active equilibrium with high search efforts, there is robust vacancy posting and low unemployment, while the relationship is reversed

**Table 3:** Second moments

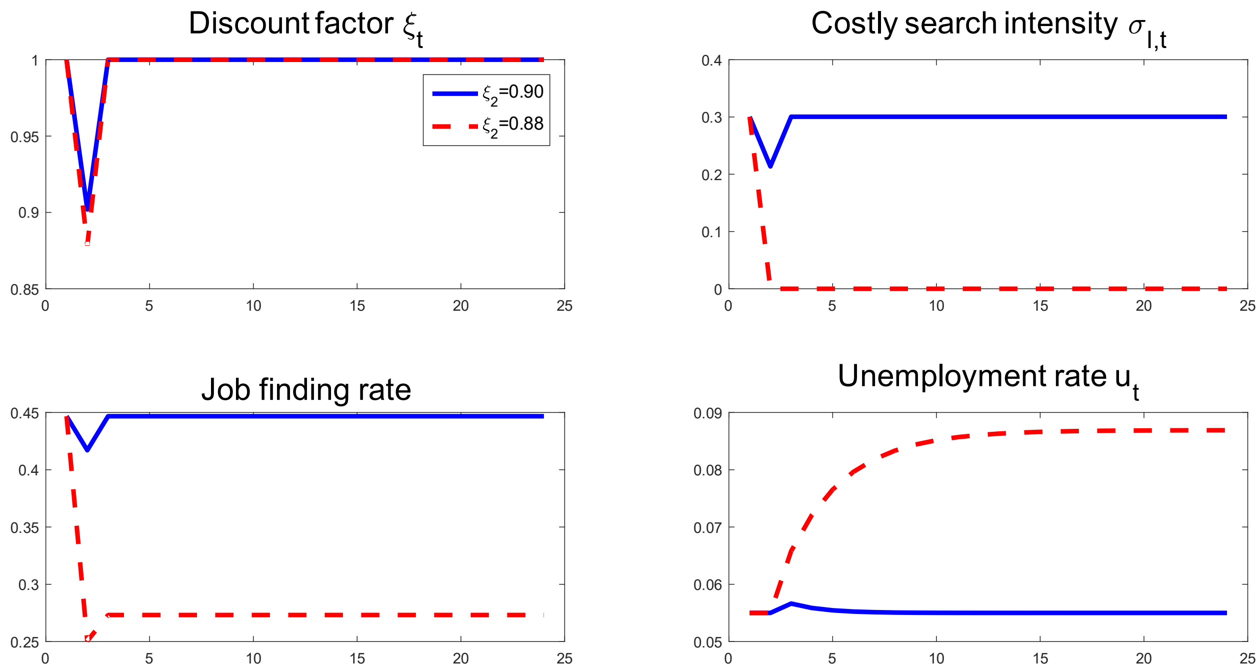
| | | $u$ | $v$ | $v/u$ | lp | $\xi$ |
|---|---|---|---|---|---|---|
| (a) Quarterly U.S. data, 1951-2016 | | | | | | |
| Autocorrelation coefficient | | 0.95 | 0.95 | 0.95 | 0.90 | – |
| Standard deviation | | 0.20 | 0.21 | 0.40 | 0.02 | – |
| | $u$ | 1 | -0.92 | -0.98 | -0.25 | – |
| Correlation matrix | $v$ | | 1 | 0.98 | 0.29 | – |
| | $v/u$ | | | 1 | 0.27 | – |
| | $lp$ | | | | 1 | – |
| (b) Benchmark model | | | | | | |
| Autocorrelation coefficient | | 0.82 | 0.55 | 0.71 | 0.88 | 0 |
| Standard deviation | | 0.10 | 0.21 | 0.28 | 0.02 | 0.03 |
| | $u$ | 1 | -0.71 | -0.85 | -0.94 | -0.06 |
| Correlation matrix | $v$ | | 1 | 0.97 | 0.54 | 0.39 |
| | $v/u$ | | | 1 | 0.72 | 0.30 |
| | $lp$ | | | | 1 | 0.00 |
| | $\xi$ | | | | | 1 |
| (c) Model without search complementarities | | | | | | |
| Autocorrelation coefficient | | 0.06 | -0.27 | -0.08 | 1 | 0 |
| Standard deviation | | 0.02 | 0.04 | 0.05 | 0 | 0.03 |
| | $u$ | 1 | -0.27 | -0.56 | 0 | -0.56 |
| Correlation matrix | $v$ | | 1 | 0.95 | 0 | 0.95 |
| | $v/u$ | | | 1 | 0 | 1.00 |
| | $lp$ | | | | 1 | 0 |
| | $\xi$ | | | | | 1 |

in the passive equilibrium. The switching between equilibria results in periods with a consistently negative relationship between vacancies and unemployment that generates the downward sloping Beveridge curve that is absent in the standard search and matching model.

In summary, introducing complementarities in the inter-firm search intensity magnifies the effect of exogenous shocks on the system and enables the model to replicate important business cycle statistics.

Finally, Figure 13 shows generalized impulse response functions (GIRF) of selected variables to a 12% (solid line) and 10% (dashed line) shock to $\xi_t$, respectively (we are not dealing with a linear model, thus the description of "generalized" is used). In period $t = 1$, the economy starts from the active DSS. In period $t = 2$, an exogenous and one-period disturbance to the discount factor hits the economy. When the contractionary shock to $\xi_t$ is 10%, the firm's search intensity temporarily declines in response to the fall in the stream of benefits in forming a joint venture,

**Figure 13:** GIRFs to a negative discount factor shock




*Note*: Each panel shows the response of a variable to a negative discount factor shock ($\xi_t$) with magnitudes of 0.10 (solid line) and 0.12 (dashed line).

generating a temporary fall in labor market tightness and a rise in the unemployment rate. This shock is too small to move the system to the passive equilibria and the variables return to the original DSS. However, when the fall in $\xi_t$ is sufficiently large, search intensity across firms ends, and the system moves to the equilibrium with zero search intensity, low output, and high unemployment. Notice how the shock is only 2% larger (12% instead of 10%), and yet the effects are quite different: search complementarities induce large non-linearities in the model.

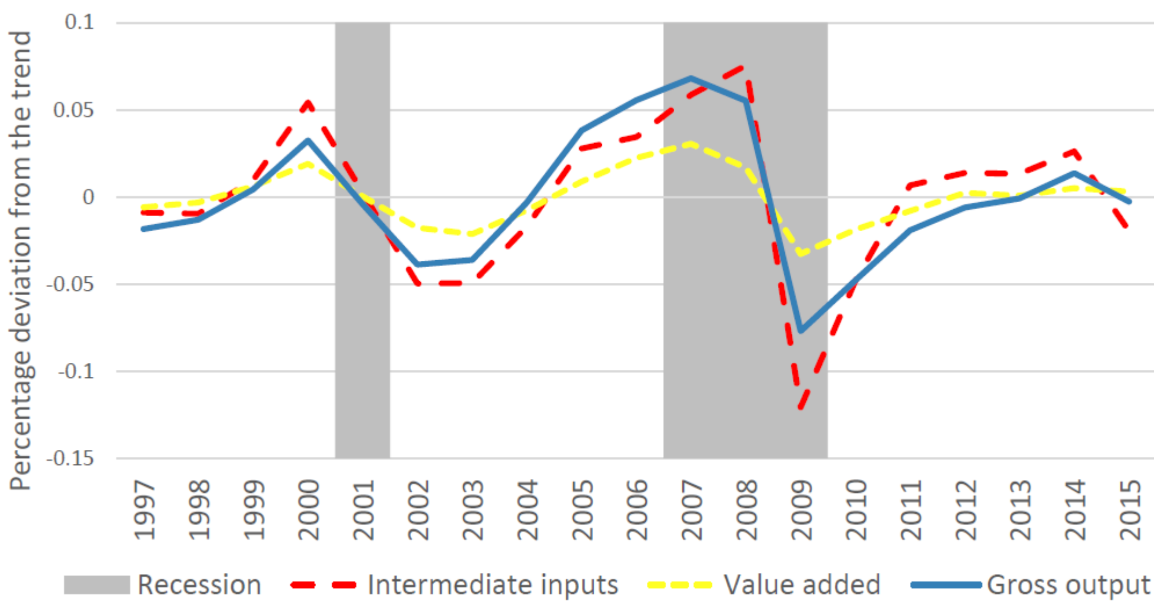
# 7 Evidence on the theoretical mechanism

The central mechanism in our model builds on two legs: first, the cyclical role of intermediate goods for changes in production and, second, the relevance of the discount factor for movements in intermediate goods and other measures of real activity. We investigate, in turn, the empirical foundation of each of these legs.

The Bureau of Economic Analysis (BEA) compiles a measure of gross output ($O$) equal to the sum of an industry's value added ($VA$) and intermediate inputs ($II$), i.e., $O = VA + II$. BEA data are annual and comprise 402 industries for 69 commodities over the period 1997-2015.

Figure 14 plots the cyclical component of gross output (blue line), intermediate inputs (red line), and industry value added (yellow line) together with NBER-dated recession periods (grey bands). We extract the cyclical component of the variable using an HP filter. The figure reveals that fluctuations in intermediate inputs are more procyclical than those in output. The period of the Great Recession is characterized by a sharp fall in intermediate input and gross production across industries, while the value added remained more stable.

**Figure 14:** Intermediate inputs, value added, and gross output




To establish the relative contribution of value added and industry input to the overall volatility of gross output, we decompose the variance of the gross industrial output in terms of the covariance terms: $\text{Var}(O) = \text{Cov}(VA, O) + \text{Cov}(II, O)$. Using this identity, together with the definition $O = VA + II$, and plugging in observed data, we find that the contribution of industry inputs to movements in industrial gross output is:

$$\frac{\text{Cov}(II, VA + II)}{\text{Var}(VA + II)} = 0.71. \tag{37}$$

Thus, fluctuations in intermediate input account for 71% of overall movements in gross industry output. This average contribution increases during recessions. For instance, in 2008, industry intermediate input decreased by 1.9 trillion, making up 84% of the decline in gross industrial output (2.3 trillion).

The second critical theoretical mechanism embedded in the model is the relevance of changes in the discount factor for fluctuations in intermediate inputs and aggregate fluctuations. We use the standard definition of the discount factor as the ratio of the current market price of a future cash receipt to the expected value of the payment (our households are risk neutral and, hence, we do not need to adjust for risk).

To investigate this link, we relate three popular measures of the discount factor to changes in aggregate output and unemployment. In measure 1, we follow Hall (2017) and construct the series for the market discount rate for dividends payable from one year (12 months) to two years (24 months) as: $\xi_t = p_t/(\mathbb{E}_t \sum_{\tau=13}^{24} d_{t+\tau})$, where $p_t$ is the market price in month $t$ of the claim of future dividends inferred from option prices and the stock price, and $d_t$ is the dividend paid in month $t$. The data on $p_t$ are from Binsbergen et al. (2012). In measure 2, we proxy the discount factor using the price-dividend ratio (p/d) of the stock market, as described in Cochrane (2011). Finally, in measure 3, we proxy the discount rate $r_t$ using the measure of expected returns from the S&P stock price index. We obtain the median 12-months-ahead forecast of the stock market index (mnemonics: SPIF, Forecast12month) from the Livingston Survey. Then, we divide by the index of the base period to calculate the expected gross return $1 + r_t$ and compute the discount factor as $\xi_t = 1/\left(1 + r_t\right)$.

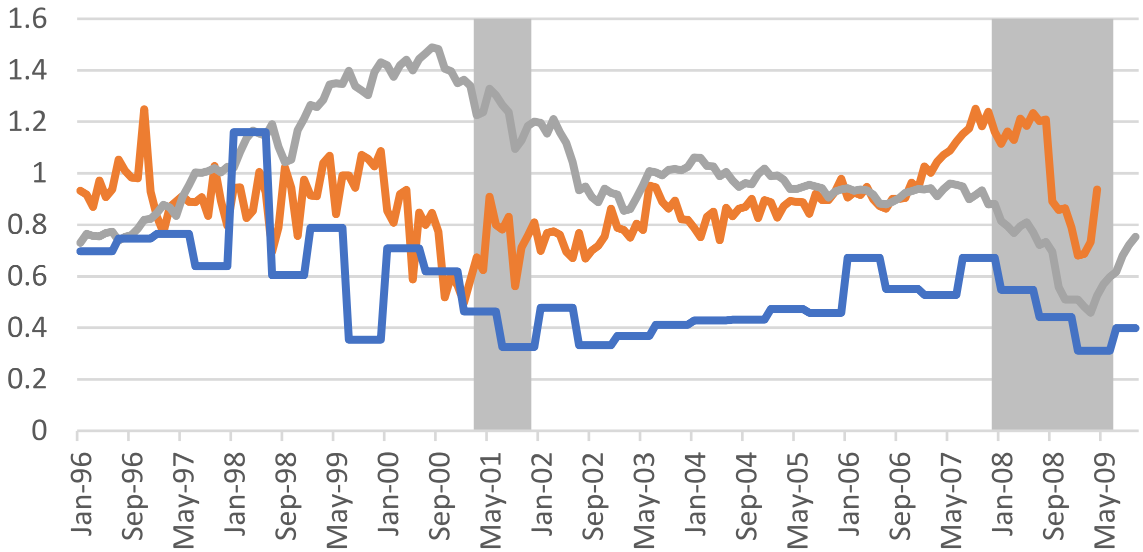
**Table 4:** Discount factor: standard deviation and correlation matrix

| | (a) Livingston Survey | (b) S&P dividend strip | (c) P/d ratio |
|---|---|---|---|
| Standard Deviation | 0.04 | 0.12 | 0.10 |
| Correlation matrix | | | |
| Livingston Survey | 1 | 0.19 | 0.46 |
| S&P dividend strip p/d ratio | | 1 | 0.34 |
| P/d ratio | | | 1 |

Figure 15 plots the three alternative measures of the discount factor for the period between January 1996 and May 2009. Importantly, all three measures agree that i) there was a sizable decline in the discount factor during the Great Depression (as our theory requires) and ii) the series display high variance (reflecting the large sensitivity of the discount factor over the business cycle, also required by our theory). The low correlation across the three measures (see Table 4 and, for similar results, Hall, 2017) is not surprising, since each of these series reflects discounting from different financial players and assets.

Finally, Table 5 shows that the three measures of the discount factor are positively correlated

**Figure 15:** Alternative measures of the discount factor




*Note*: Alternative measures of the discount factor from dividend strip (red line), the price-to-dividend ratio (green line), and the Livingston Survey (blue line) between January 1996 and May 2017.

with GDP and input of intermediate goods and negatively correlated with unemployment. This pattern corroborates the important relation between shocks to the discount factor and movements in production and unemployment highlighted by our model.

**Table 5:** Correlation between discount rates and aggregate variables

| Correlation coefficient | Unemployment rate | GDP | Intermediate input |
|---|---|---|---|
| Livingston Survey | -0.55 | 0.53 | 0.42 |
| S&P dividend strip p/d ratio | -0.33 | 0.50 | 0.21 |
| P/d ratio | -0.75 | 0.80 | 0.53 |

*Note*: Discount rates and unemployment: monthly data from January 1996 to May 2009. GDP: quarterly data from 1996Q1 to 2009Q1. Intermediate input: annual data from 1997 to 2009. Series are HP filtered.

# 8 The volatility of shocks

In this section, we study how the volatility of the shocks to the economy is critical for the dynamic properties of the model and the likelihood and duration of each equilibrium.

## 8.1 Analytical illustration with a simplified model

To gain intuition, and before we report the quantitative results from the full model, we derive an analytical characterization of the effect of volatility for the likelihood and duration of each equilibrium by simplifying the model in Section 3. First, we assume firms produce their output without the need of workers. Thus, we can drop the whole DMP module of the model and set a constant measure of size 1 of firms in each sector (instead of their number being determined by the free-entry condition in job posting). Second, we assume that $\widetilde{\delta} = 1$, i.e., all joint ventures terminate after one period. Also, joint ventures start producing in the same period where firms match. Hence, the firm's problem is equivalent to a sequence of static maximization problems and we do not need to specify a discount factor. To ease the algebra, we also set $\rho_z = 0$, and as we did in the calibration in Section 5, $\widetilde{\tau} = 0.5$ and $\nu = 2$.[10]

Under these simplifications, each firm optimally chooses the level of its search intensity, $\sigma_{i,t}$, given the search intensity of the firms in the opposite sector, $\sigma_{-i,t}$, and productivity, $z_t$, by maximizing its static profits:

$$J_{i,t}(\sigma_{i,t}, \sigma_{-i,t}, z_t) = (\phi + (\psi + \sigma_{i,t})(\psi + \sigma_{-i,t}))\frac{z_t}{2} - c_0\sigma_{i,t} - c_1\frac{\sigma_{i,t}^3}{3}.$$

The first term of the RHS is the inter-firm matching probability defined in equation (12) multiplied by half the expected production, $\pi_{i,t}z_t$ (recall the equal split of output between the firms given $\widetilde{\tau} = 0.5$) minus the cost of searching.

The interior solution $\sigma_{i,t} > 0$ satisfies:

$$c_0 + c_1\sigma_{i,t}^2 = (\psi + \sigma_{-i,t})\frac{z_t}{2}. \tag{38}$$

Otherwise, $\sigma_{i,t} = 0$. Hence, as in the benchmark model, the simplified model entails passive and active equilibria. The passive equilibrium with zero search intensity exists if and only if

$$c_0 > \psi\frac{z_t}{2}. \tag{39}$$

From equation (39), we can define a threshold of productivity $\bar{z} = \frac{2c_0}{\psi}$ that determines whether

[10] This simplified model is nearly identical to the model in Section 2. The only differences are a slightly different matching function and that now we have an AR(1) process for $z_t$.

the passive equilibrium exists.

**Lemma 2.** *The passive equilibrium exists if and only if* $z_t < \bar{z}$.

Recall that we assumed that $\psi > 0$. If $\psi = 0$, a passive equilibrium always exists regardless of the value of $z_t$.

In an active equilibrium, firms in each sector optimally choose a positive search intensity that comes from finding the fixed point of the product of equation (38) for each sector:

$$\sigma_{F,t} = \sigma_{I,t} = \frac{z_t + \sqrt{z_t^2 + 8\psi z_t - 16c_0c_1}}{4c_1}. \tag{40}$$

This optimal search intensity is increasing in $z_t$.[11]

From equation (40), we define the threshold for the active equilibrium $\underline{z} = 4\left(\sqrt{\psi^2 c_1^2 + c_1c_0} - \psi c_1\right)$, and we get the following lemma.[12]

**Lemma 3.** *An active equilibrium exists if and only if* $z_t \geq \underline{z}$.

Proposition 10 merges lemmas 2 and 3 to characterize the range of values $z_t$ compatible with multiple equilibria.

**Proposition 10.** *The economy retains multiple equilibria if* $z_t \in (\underline{z}, \bar{z})$. *The passive equilibrium is the unique equilibrium if* $z_t \leq \underline{z}$. *The active equilibrium is the unique equilibrium if* $z_t \geq \bar{z}$.

Proposition 10 establishes that if economic fundamentals are sufficiently weak or strong, the equilibrium is unique, either passive or active; otherwise, we have two equilibria. Sufficiently large shocks to $z_t$ move the system between the two alternative equilibria. Proposition 10 is empirically relevant because we can calibrate $\psi$ to a small number so that $\bar{z}$ is low and $c_1$ to a large number so that $\underline{z}$ is high. In that way, the model will allow multiple equilibria for a wide range of productivity $\underline{z} < 1 < \bar{z}$.

Since we have set $\rho_z = 0$, we have that $\log(z_t) \sim \mathcal{N}(0, \sigma_z^2)$. Using the distribution for $z_t$ and the thresholds $\underline{z}$ and $\bar{z}$ that determine changes between equilibria, we derive the transition matrix between equilibria:

[11] There is a second fixed point, $\sigma_{i,t} = \frac{z_t - \sqrt{z_t^2 + 8\psi z_t - 16c_0c_1}}{4c_1}$. However, this solution is locally unstable.

[12] To prevent the marginal search cost from converging to zero when $\sigma_{i,t}$ is zero, the term $c_0$ must be positive. If $c_0 = 0$, it yields $\underline{z} = 0$. In such an instance, the active equilibrium exists for any positive value of $z_t$.

| | Active | Passive |
|---|---|---|
| Active | $1-\Phi\left[\log\left(\underline{z}\right)/\sigma_z\right]$ | $\Phi\left[\log\left(\underline{z}\right)/\sigma_z\right]$ |
| Passive | $1-\Phi\left[\log\left(\bar{z}\right)/\sigma_z\right]$ | $\Phi\left[\log\left(\bar{z}\right)/\sigma_z\right]$ |

where $\Phi(\cdot)$ is the cdf of the standard normal distribution. The next proposition characterizes the role that the volatility of exogenous disturbances plays in the duration of each equilibrium.

**Proposition 11.** *The expected duration of a passive equilibrium spell is* $\frac{1}{\Phi[\log(\underline{z})/\sigma_z]}$, *and the expected duration of an active equilibrium spell is* $\frac{1}{1-\Phi[\log(\bar{z})/\sigma_z]}$. *The duration of each equilibrium is inversely related to the volatility of* $z_t$.

Proposition 11 establishes that aggregate volatility plays a critical role in the selection and duration of each equilibrium and how it interacts with search complementarities. A reduction in volatility induces the system to remain for a prolonged spell in one equilibrium, with a decreased probability for the system to move to the alternative equilibrium. However, if a sufficiently large change in fundamentals triggers a change in the equilibrium, the economy would move to the alternative equilibrium and stay there for a long time.

The dynamics in the simple model are consistent with the large and persistent increase in the unemployment rate in the aftermath of the financial crisis of 2007-2009 and the prolonged period before the series returned to its initial level in 2017. The financial crisis was preceded by a long spell of stable economic conditions during the Great Moderation that started in the mid-1980s, which the model identifies as a prerequisite for the unprecedented persistence in unemployment.

## 8.2 Simulation with the benchmark model

With the intuition from the simplified model, we return to our benchmark model to assess the quantitative effect of changes in the volatility of shocks. Table 6 reports business cycle statistics for a low (column (a)) and a high (column (b)) variance of shocks to the discount factor ($\sigma_\xi$). As before, we simulate the model for 3,000,000 months and time average to obtain quarterly data. The first and second rows report the number of periods and the average duration of the passive equilibrium, respectively, and the third row reports the transition matrix between equilibria. We calibrate high and low volatility by following Justiniano and Primiceri (2008), who estimate that the volatility of preference shocks is equal to 0.07 before 1984 and 0.04 after that date.

**Table 6:** Variance of shocks and duration of equilibria

| | (a) $\sigma_\xi = 0.04$ | | (b) $\sigma_\xi = 0.07$ | |
|---|---|---|---|---|
| Fraction of periods in passive equilibrium | 0.11 | | 0.27 | |
| Average number of quarters at a passive equilibrium | 11 | | 3.4 | |
| Transition matrix | | | | |
| | Active | Passive | Active | Passive |
| Active | 0.98 | 0.02 | 0.89 | 0.11 |
| Passive | 0.09 | 0.91 | 0.29 | 0.71 |

The passive equilibrium materializes with a probability of around 11% in the low-volatility economy, in contrast with 27% probability in the high-volatility economy. Despite the lower chance of moving to a passive equilibrium, the low-volatility economy stays longer on average in a passive equilibrium, 11 quarters, than the high-volatility economy, 3.4 quarters. Low volatility induces less frequent but long-lasting periods of low output and high unemployment.

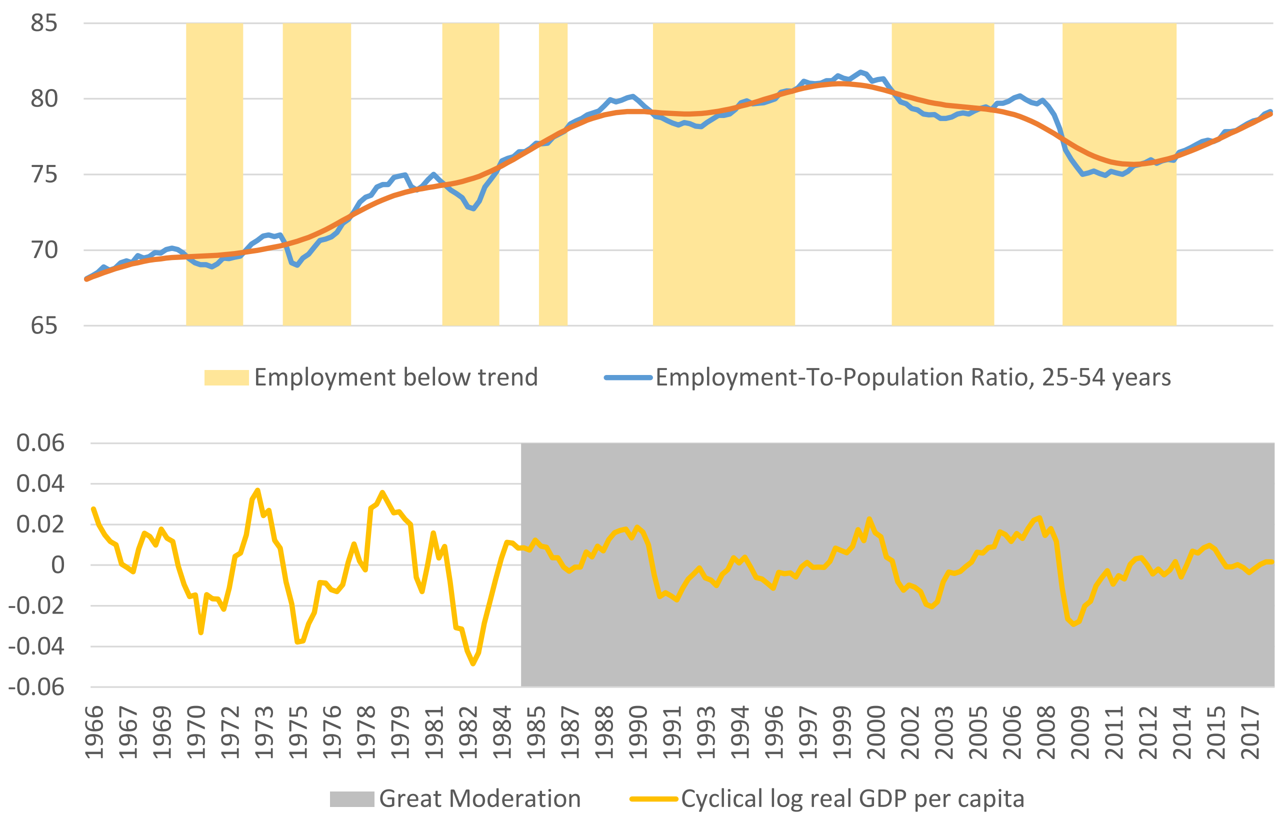
The last two rows in Table 6 report the transition matrix between equilibria. The entries reveal that the low-volatility economy transitions between equilibria infrequently. The probability of moving from active equilibrium to passive equilibrium is equal to 2%, and the probability of a reverse move from passive equilibrium to active equilibrium is equal to 9%. The rotation among equilibria gets much higher in the high-volatility economy, as the probability of moving from an active to a passive equilibrium is 11%, and the probability of a reverse move is 29%.

Appendix G includes the histograms of endogenous variables of interest in the model with high and low probability. The most important lesson from those figures is the long left tail of output when the volatility of $\xi_t$ is high.

### 8.3 The Great Moderation and the persistence of business cycles

Our model predicts that a lower volatility of fundamentals is associated with more prolonged equilibrium spells. This prediction is consistent with the empirical pattern in the U.S. data. In Figure 16, the upper panel plots the U.S. employment rate (blue curve) and its trend (orange curve) estimated from an HP filter with $\lambda = 1600$ from 1996 to 2017. The light-orange bars indicate labor market downturns. Inspired by the NBER's methodology in defining recessions, we define that a labor market downturn starts when the employment rate falls below the trend for two quarters and ends when the employment rate rises above the trend for two quarters. As noted by many researchers (see Jaimovich and Siu 2012 and references therein), the figure shows how the three labor market downturns that occurred after 1984 were longer than the previous ones. Precisely after 1984, the U.S. economy experienced a substantial reduction in the volatility of business cycle fluctuations, which Justiniano and Primiceri (2008) and Fernández-Villaverde et al. (2015) attribute, in part, to a lower volatility of shocks to fundamentals. To illustrate this point, the bottom panel in Figure 16 plots the cyclical component of real GDP per capita, with a grey area to indicate the Great Moderation that started in the mid-1980s.

**Figure 16:** The Great Moderation and labor market downturns




Our model suggests an intrinsic linkage between the Great Moderation and the increasing persistence in labor market downturns. While the Great Moderation improves macroeconomic stability and reduces the occurrences of recessions, it makes these recessions and the associated labor market downturns more durable.

# 9 The role of fiscal policy

In our model, government spending that stimulates joint venture formation may permanently move the system from a passive to an active equilibrium, inducing a large fiscal multiplier. To study this hypothesis, we embed government spending in the economy and derive the analytical conditions for fiscal policy to move the system from a passive to an active equilibrium. We then investigate the effect of public spending on the DSSs in the model. Finally, we study the size and state dependence of the impact of government spending.

## 9.1 Government spending as a set of final-goods producers

We focus our investigation on government spending defined as government consumption expenditures and gross investment. We ignore transfers because our model abstracts from aggregate demand considerations. We model government spending as an exogenous increase in the number of single firms in the final-goods sector, where these additional firms can be interpreted as new public projects such as building a new school. Thus, we have government-owned single final-goods firms, $\widetilde{n}^G_{F,t}$, that operate together with private single firms in both sectors. The formation of private firms remains endogenous, as described by equation (14). We assume that government spending is financed by lump-sum taxes.

The law of motion for government single final-goods firms, $\widetilde{n}^G_F$, is:

$$\widetilde{n}^G_{F,t+1} = (1 - \delta - \pi_F)\,\widetilde{n}^G_{F,t} + \epsilon^G_t, \tag{41}$$

where $\epsilon^G_t$ are the new government-owned single firms created in period $t$. Like the private firms in the final-goods sector, government-owned firms must form a joint venture with firms in the intermediate-goods sector to manufacture goods (for example, a public school requires bricks produced by private firms). Joint ventures with government-owned firms, $n^G_F$, follow:

$$n^G_{F,t+1} = \left(1 - \delta - \widetilde{\delta}\right) n^G_{F,t} + \pi_F \widetilde{n}^G_{F,t}. \tag{42}$$

A government firm exits the market when its job match or joint venture is terminated.

The inflow $\epsilon_t^G$ changes the matching probabilities in the inter-firm matching market:

$$\pi_{I,t} = \left[\phi + (\psi + \sigma_I)(\psi + \sigma_F)\right] H\left(1, \widetilde{\theta}_t\right), \tag{43}$$

and

$$\pi_F = \left[\phi + (\psi + \sigma_I)(\psi + \sigma_F)\right] H\left(\frac{1}{\widetilde{\theta}_t}, 1\right), \tag{44}$$

where $\widetilde{\theta}_t = (\widetilde{n}_{F,t} + \widetilde{n}_{F,t}^G)/\widetilde{n}_{I,t}$ is the inter-firm matching market tightness ratio in the presence of government single firms.

Since $H$ is increasing in both arguments, $\epsilon_t^G > 0$ increases the matching probability for intermediate-goods firms (more potential partners) and decreases the matching probability for final-goods firms (stiffer competition for partners). These changes in matching probabilities, in turn, move search intensity and, potentially, the equilibrium of the economy.

Total government spending is equal to the output produced by government-owned firms in joint ventures and the single government-owned firms' search cost:

$$g_t = z_t n_F^G + \widetilde{n}_F^G\left(c_0\sigma_F + c_1\frac{\sigma_F^{1+\nu}}{1+\nu}\right).$$

Gross aggregate output comprises government and private production: $y_t = z_t(n_{F,t}^G + n_{F,t})$, and it is used for private consumption, government spending, and search costs. The aggregate resource constraint is:

$$y_t = c_t + g_t + \sum_{i=I,F}\chi v_i + \sum_{i=I,F}\widetilde{n}_i\left(c_0\sigma_i + c_1\frac{\sigma_i^{1+\nu}}{1+\nu}\right). \tag{45}$$

## 9.2 Shocks to government spending and equilibria switches

We assume that the economy is in the passive equilibrium (i.e., $\sigma_I = \sigma_F = 0$) before the arrival of a positive government spending shock, $\epsilon_t^G$.

Upon the realization of the shock, the passive equilibrium continues to exist if:

$$\widetilde{\beta}\xi_t\psi H\left(1, \widetilde{\theta}_t\right)\mathbb{E}_t\left(J_{I,t+1} - \widetilde{J}_{I,t+1} \mid \iota = 0\right) < c_0, \tag{46}$$

and

$$\widetilde{\beta}\xi_t\psi H\left(\widetilde{\theta}_t^{-1}, 1\right)\mathbb{E}_t\left(J_{F,t+1} - \widetilde{J}_{F,t+1} \mid \iota = 0\right) < c_0. \tag{47}$$

Equation (46) shows that the passive equilibrium disappears if the increase of a government-owned single firm tightens the inter-firm matching market enough and makes the expected capital gain of intermediate-goods firms so high that these firms search with positive intensity even if the final-goods firms search with zero intensity.

**Proposition 12.** *Starting from the passive equilibrium, the size of government spending needed to move the system to the active equilibrium is:*

$$\frac{\widetilde{n}_{F,t}^{G}}{\widetilde{n}_{I,t}} > \Psi\left[\frac{c_0}{\beta\xi\psi\mathbb{E}_t\left(J_{I,t+1}-\widetilde{J}_{I,t+1}\mid\iota=0\right)}\right]-\frac{\widetilde{n}_{F,t}}{\widetilde{n}_{I,t}}, \tag{48}$$

*with* $\Psi' > 0$.[13]

Equation (48) shows that the magnitude of the policy intervention that moves the economy to an active equilibrium is proportional to the cost-to-benefit ratio of forming a joint venture; and it decreases with the measure of the private firms in the final-goods sector relative to intermediate-goods firms. A large quantity of private final-goods firms improves the joint venture prospects for intermediate-goods firms, decreasing the magnitude of government spending needed to move to the active equilibrium.

## 9.3 Quantitative results

We investigate the dynamic response of the economy to expansionary fiscal policy shocks and the size of the fiscal multiplier. See Appendix D.2 for details of the computation of the model in this case. Once we introduce government spending, we have 12 state variables. Due to this large number of state variables, we implement a dimensionality reduction algorithm inspired by Krusell and Smith (1998) that is of interest in itself and potentially applicable to similar problems.

Panel (a) of Figure 17 plots the best response functions for the final-goods firm and the intermediate-goods firm in the active equilibrium (magenta solid and dotted lines, respectively) and the passive equilibrium (black circle and cross markers, respectively) for a 20% exogenous

[13] Denote $h\left(\widetilde{\theta}\right)=H\left(1,\widetilde{\theta}\right)$. $\Psi$ is the inverse function of $h(\cdot)$. As $h(\cdot)$ is strictly increasing in $\widetilde{\theta}$ by assumption, $\Psi$ is also a strictly increasing function. In our calibration: $h(\theta)=2^{\frac{1}{\kappa}}\left(1+\widetilde{\theta}_t^{-\kappa}\right)^{-\frac{1}{\kappa}}$, $\Psi(x)=\left(2x^{-\kappa}-1\right)^{-\frac{1}{\kappa}}$.

increase in the relative size of the final-goods sector. The blue lines and the red circle and cross markers show the best response functions when the fiscal shock is absent. Following Proposition 12, this 20% fiscal shock is sufficiently powerful to move the system to the active equilibrium. Given our calibration in Section 5, the threshold for an equilibrium switch is 19%. The fiscal stimulus increases the matching probability for intermediate-goods firms, raising the payoff to forming joint ventures and leading to an optimal positive search intensity in the intermediate-goods sector ($\sigma_I = 0.03$ as in the cross marker). The passive equilibrium ceases to exist, and point C remains the unique stable equilibrium. In comparison, panel (b) in Figure 17 plots the best response functions associated with a 15% increase in the relative size of the final-goods sector. This fiscal stimulus fails to eliminate the passive equilibrium with zero search intensity in the final-goods sector and the intermediate-goods sector (circle and cross markers, respectively).

**Figure 17:** Increase in government spending

**(a)** 30% increase **(b)** 15% increase

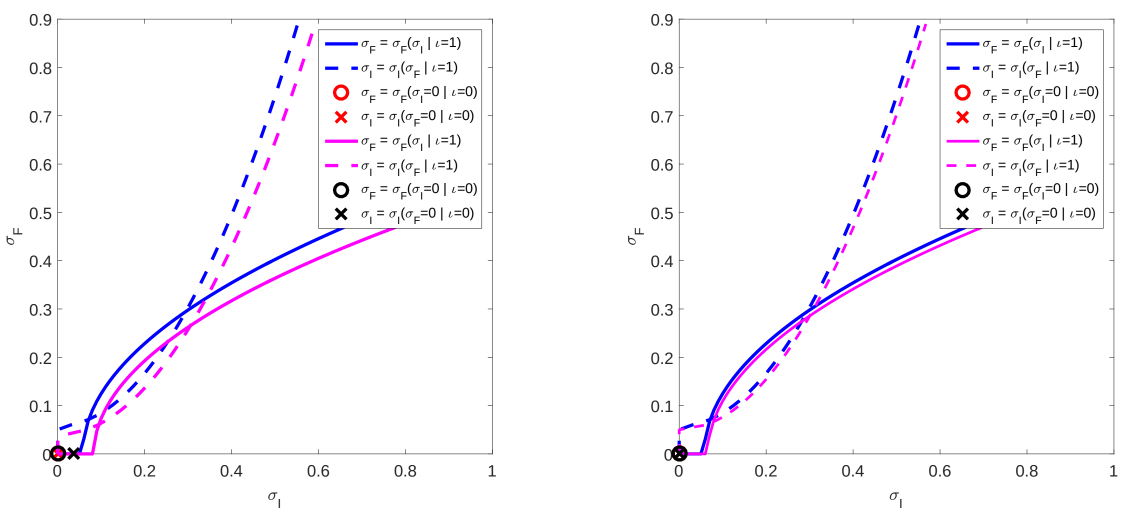
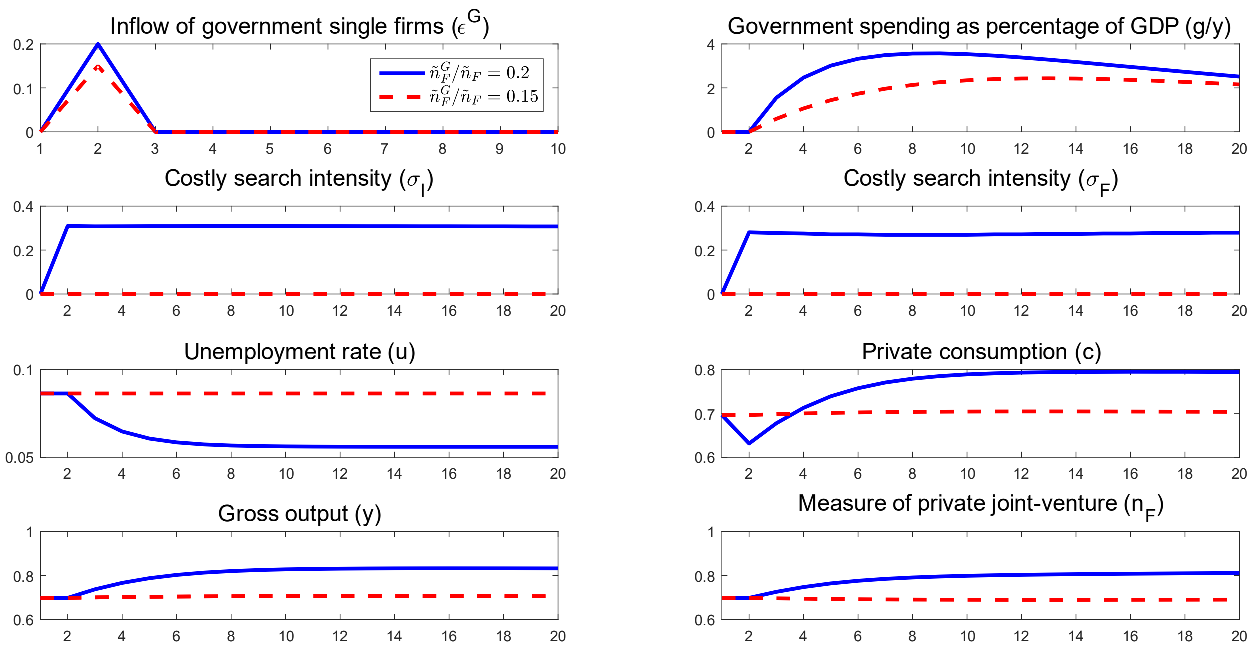



Figure 18 shows the dynamic reaction of selected variables to the same 15% (dotted line) and 20% (solid line) shocks to the relative size of the final-goods sector that we just described when the economy starts at the passive equilibrium DSS (Appendix H shows the responses for the system that starts from the active equilibrium). Since the 20% fiscal expansion satisfies Proposition 12, it produces a significant and persistent increase in output and a fall in unemployment. Nevertheless, this fiscal expansion crowds out private consumption upon impact. This reaction is due to two mechanisms: first, a rise in government-owned firms reduces, in the short run, the formation of joint ventures that produce for private consumption, and second, the shift of equilibrium triggers an increase in the cost associated with vacancy posting and joint venture formation, which further reduces private consumption. The first mechanism still exists in the 15% fiscal expansion, inducing a small drop in private consumption even when the system stays in the passive equilibrium.

**Figure 18:** GIRFs to positive government spending shock

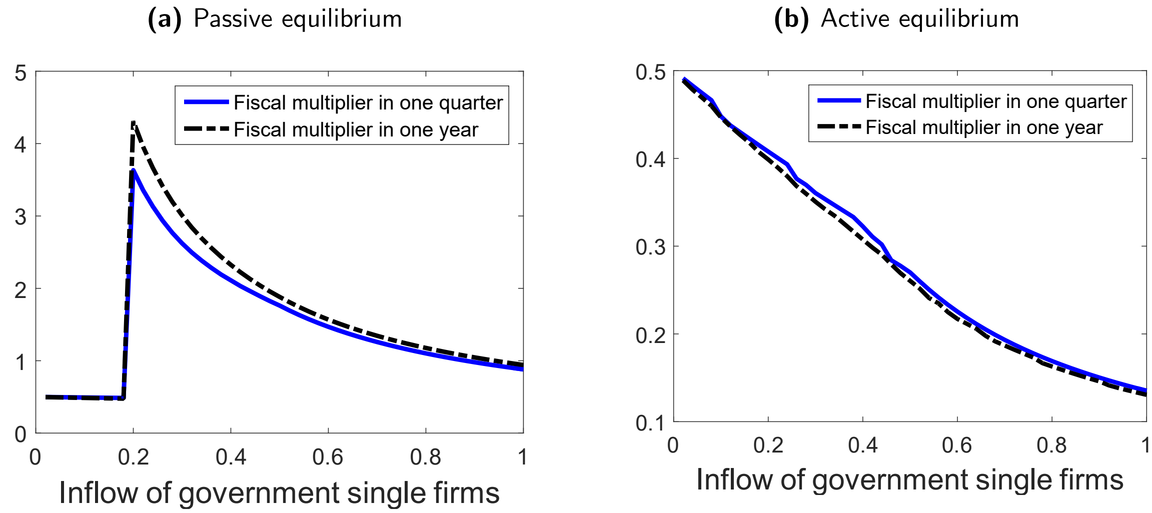



*Note*: Each panel shows the response of a variable to a one-period, 15% (dashed line) and 20% (solid line) increase in government spending.

We also calculate the fiscal multiplier for our economy, defined as the ratio of the cumulative change in output over one quarter and one year, generated by the one-period change in government spending triggered by the inflow of government-owned single firms in the final-goods sector (we could compute the fiscal multiplier at other horizons if desired). Panel (a) in Figure 19 shows the fiscal multiplier as a function of the inflow of government-owned single firms when the economy is in the passive equilibrium at the start of the fiscal expansion. Panel (b) replicates the exercise for the case when the economy is in the active equilibrium.

In the passive equilibrium, a sufficiently large fiscal expansion generates a multiplier larger than 1 since it triggers a rise in search intensity. The fiscal multiplier peaks at the threshold where we shift from the passive to the active equilibrium. In our calibration, the peak quarterly fiscal multiplier, 3.5, is at a 19% increase in the number of government-owned firms, which is

**Figure 19:** Fiscal multiplier




equivalent to a 3.8% increase in government spending relative to output in the first quarter (since the increase in government spending is persistent, the overall size of the fiscal intervention is larger than the impact change of 3.8%). Any stimulus beyond this level reduces the fiscal multiplier because the crowding out of private consumption outweighs the increase in output from the fiscal expansion. A doubling in the number of government-owned final-goods firms generates a fiscal multiplier of around 1 over a quarter. Similarly, a fiscal expansion below the threshold generates a less than unitary fiscal multiplier since it creates a large crowding out effect and no equilibrium switch.

Panel (b) in Figure 19 shows that the fiscal multiplier is substantially lower in the active equilibrium. The increased costs of forming joint ventures for private firms in the final-goods sector reduce private output, and we have a less than unitary fiscal multiplier for any size of the fiscal stimulus. The multiplier declines with the size of government spending for a crowding out effect across a wide range of time horizons.

Our results in Figure 19 agree with the recent empirical literature that has documented the acute state-dependence of fiscal multipliers. See, for example, Auerbach and Gorodnichenko (2012), Owyang et al. (2013), and Ghassibe and Zanetti (2019). Our model proposes a mechanism to account for the state-dependence of fiscal multipliers parsimoniously.

# 10 Conclusion

This paper shows that search complementarities in inter-firm joint venture formation have broad implications for the persistence of business cycle fluctuations and the effect of fiscal policy. The optimal degree of search intensity is either zero or positive, and the system entails two equilibria: an active one with large economic activity and a passive one with high economic activity. Sufficiently large changes in fundamentals that change search intensity move the system between the two equilibria.

The dynamic properties of our economy are unlike those of standard models. Search complementarities generate bimodal ergodic distributions of variables and protracted slumps. Macroeconomic volatility plays an essential role in the selection and duration of each equilibrium. In particular, large negative shocks during spells of low volatility generate a persistent shift to the passive equilibrium, which is consistent with the large and persistent deviation of economic variables from trend after the financial crisis that started in 2007 in the aftermath of the Great Moderation. Fiscal policy operates markedly different than in standard models, and it is powerful in stimulating the economy in the passive equilibrium, with a non-monotonic effect on economic activity, while its effectiveness significantly declines in the active equilibrium.

The analysis opens exciting avenues for additional research. A direct extension would be to embed strategic complementarities in richer models of the business cycle such as those including money, nominal rigidities, and financial frictions. Furthermore, the role of agent heterogeneity deserves further exploration. We will explore some of those avenues in future work.

# References


Acemoglu, D., Carvalho, V. M., Ozdaglar, A., and Tahbaz-Salehi, A. (2012). The network origins of aggregate fluctuations. *Econometrica*, 80(5):1977–2016.

Auerbach, A. J. and Gorodnichenko, Y. (2012). Measuring the output responses to fiscal policy. *American Economic Journal: Economic Policy*, 4(2):1–27.

Binsbergen, J. v., Brandt, M., and Koijen, R. (2012). On the timing and pricing of dividends. *American Economic Review*, 102(4):1596–1618.

Bulow, J. I., Geanakoplos, J. D., and Klemperer, P. D. (1985). Multimarket oligopoly: Strategic substitutes and complements. *Journal of Political Economy*, 93(3):488–511.

Burdett, K. and Mortensen, D. (1980). Search, layoffs, and labor market equilibrium. *Journal of Political Economy*, 88(4):652–72.

Chatterjee, S., Cooper, R., and Ravikumar, B. (1993). Strategic complementarity in business formation: Aggregate fluctuations and sunspot equilibria. *Review of Economic Studies*, 60(4):795–811.

Cochrane, J. H. (2011). Discount rates. *Journal of Finance*, 66(4):1047–1108.

Cooper, R. (1994). Equilibrium selection in imperfectly competitive economies with multiple equilibria. *Economic Journal*, 104(426):1106–1122.

Cooper, R. and John, A. (1988). Coordinating coordination failures in Keynesian models. *Quarterly Journal of Economics*, 103(3):441–463.

den Haan, W. J., Ramey, G., and Watson, J. (2000). Job destruction and propagation of shocks. *American Economic Review*, 90(3):482–498.

Diamond, P. (1982). Aggregate demand management in search equilibrium. *Journal of Political Economy*, 90(5):881–894.

Diamond, P. and Fudenberg, D. (1989). Rational expectations business cycles in search equilibrium. *Journal of Political Economy*, 97(3):606–619.

Eeckhout, J. and Lindenlaub, I. (2018). Unemployment cycles. Mimeo, UCL.

Fernández-Villaverde, J., Guerrón, P., and Rubio-Ramírez, J. F. (2015). Estimating dynamic equilibrium models with stochastic volatility. *Journal of Econometrics*, 185(1):216–229.

Ghassibe, M. and Zanetti, F. (2019). State dependence of fiscal multipliers: the source of fluctuations matters. Mimeo, University of Oxford.

Hall, R. E. (2016). *Macroeconomics of Persistent Slumps*, volume 2 of *Handbook of Macroeconomics*, pages 2131–2181. Elsevier.

Hall, R. E. (2017). High discounts and high unemployment. *American Economic Review*, 107(2):305–330.

Hall, R. E. and Milgrom, P. (2008). The limited influence of unemployment on the wage bargain. *American Economic Review*, 98(4):1653–1674.

Hosios, A. J. (1990). On the efficiency of matching and related models of search and unemployment. *Review of Economic Studies*, 57(2):279–298.

Huo, Z. and Ríos-Rull, J.-V. (2013). Paradox of thrift recessions. Working Paper 19443, National Bureau of Economic Research.

Jaimovich, N. and Siu, H. E. (2012). The trend is the cycle: Job polarization and jobless recoveries. Working Paper 18334, National Bureau of Economic Research.

Jones, C. I. (2013). Misallocation, economic growth, and input-output economics. In Acemoglu, D., Arellano, M., and Dekel, E., editors, *Advances in Economics and Econometrics: Tenth World Congress*, volume 2, pages 419–456. Cambridge University Press.

Justiniano, A. and Primiceri, G. E. (2008). The time-varying volatility of macroeconomic fluctuations. *American Economic Review*, 98(3):604–641.

Kaplan, G. and Menzio, G. (2016). Shopping externalities and self-fulfilling unemployment fluctuations. *Journal of Political Economy*, 124(3):771–825.

Krusell, P. and Smith, A. A. (1998). Income and wealth heterogeneity in the macroeconomy. *Journal of Political Economy*, 106(5):867–896.

Liu, P., Theodoridis, K., Mumtaz, H., and Zanetti, F. (2018). Changing macroeconomic dynamics at the zero lower bound. *Journal of Business & Economic Statistics*, 0(0):1–14.

Michaillat, P. and Saez, E. (2015). Aggregate demand, idle time, and unemployment. *Quarterly Journal of Economics*, 130(2):507–569.

Murphy, K. M., Shleifer, A., and Vishny, R. W. (1989). Industrialization and the big push. *Journal of Political Economy*, 97(5):1003–1026.

Owyang, M. T., Ramey, V. A., and Zubairy, S. (2013). Are government spending multipliers greater during periods of slack? evidence from twentieth-century historical data. *American Economic Review*, 103(3):129–134.

Petrongolo, B. and Pissarides, C. A. (2001). Looking into the black box: A survey of the matching function. *Journal of Economic Literature*, 39(2):390–431.

Shimer, R. (2005). The cyclical behavior of equilibrium unemployment and vacancies. *American Economic Review*, 95(1):25–49.

Sterk, V. (2016). The Dark Corners of the Labor Market. Discussion Papers 1603, Centre for Macroeconomics (CFM).

Taschereau-Dumouchel, M. and Schaal, E. (2015). Coordinating Business Cycles. 2015 Meeting Papers 178, Society for Economic Dynamics.

Thomas, C. and Zanetti, F. (2009). Labor market reform and price stability: An application to the Euro Area. *Journal of Monetary Economics*, 56(6):885–899.

Weitzman, M. (1982). Increasing returns and the foundations of unemployment theory. *Economic Journal*, 92(368):787–804.

# Appendix

We include, for completeness, a series of appendices. First, in Appendix A, we show the derivation of the total surplus of a filled job and the capital gain from forming a joint venture. Second, in Appendix B, we describe how we compute the DSSs of the model. Third, in Appendix C, we present the proofs of several propositions in the main text. Fourth, in Appendix D, we outline how to compute the model. Fifth, in Appendix E, we discuss the role of mixed-strategy Nash equilibria in our model. Sixth, in Appendix F, we complete our discussion of the effects of technology shocks in the model. Seventh, in Appendix G, we look at the ergodic distribution of variables of interest in cases of high and low volatility of the shocks to $\xi_t$. Last, in Appendix H, we report the GIRFs to government spending shocks in the active equilibrium.

# A Total surplus

The total surplus of a labor market match at time $t$ in a joint venture in either sector $i \in \{I, F\}$ of the economy is $TS_{i,t} = W_{i,t} - U_{i,t} + J_{i,t}$. Analogously, the total surplus of a filled job in a single firm is $\widetilde{TS}_{i,t} = \widetilde{W}_{i,t} - U_{i,t} + \widetilde{J}_{i,t}$.

Nash bargaining for wages implies that:

$$
\begin{aligned}
J_{i,t} &= \tau TS_{i,t}, \\
W_{i,t} - U_{i,t} &= (1-\tau)\, TS_{i,t}, \\
\widetilde{J}_{i,t} &= \tau \widetilde{TS}_{i,t}, \\
\widetilde{W}_{i,t} - U_{i,t} &= (1-\tau)\, \widetilde{TS}_{i,t},
\end{aligned}
$$

where recall that the parameter $\tau$ is the firm's bargaining weight, common across sectors. The free-entry condition of the labor market is:

$$
\chi = \beta \xi_t \tau \theta_{i,t}^{\alpha-1} \mathbb{E}_t \left( \widetilde{TS}_{i,t+1} \right). \tag{49}
$$

The total surplus of establishing a joint venture is the sum of the capital gain from matching for the firms in the intermediate-goods sector, $J_{I,t} - \widetilde{J}_{I,t}$, and final-goods sector, $J_{F,t} - \widetilde{J}_{F,t}$:

$$
TSJV_t = J_{I,t} - \widetilde{J}_{I,t} + J_{F,t} - \widetilde{J}_{F,t}.
$$

The price for intermediate goods, $p_t$, is set according to the Nash bargaining rule:

$$
\begin{aligned}
J_{I,t} - \widetilde{J}_{I,t} &= \widetilde{\tau} TSJV_t \\
J_{F,t} - \widetilde{J}_{F,t} &= (1 - \widetilde{\tau}) \, TSJV_t,
\end{aligned}
$$

where the parameter $\widetilde{\tau}$ is the intermediate goods producer's bargaining power.

We derive now value functions for $TS_{i,t}$, $\widetilde{TS}_{i,t}$, and $TSJV_t$. We start with the total surplus of a filled job in a joint venture, $TS_{i,t}$. Using the equations for $W_{I,t}$, $J_{I,t}$, and $U_{I,t}$ in the definition of $TS_{i,t}$, we get:

$$
\begin{aligned}
W_{I,t} + J_{I,t} - U_{I,t} = z_t p_t - h& \\
+ \beta \xi_t \mathbb{E}_t &\left[ \begin{array}{c} \left(1 - \delta - \tilde{\delta}\right) (W_{i,t+1} + J_{i,t+1} - U_{i,t+1}) \\ + \tilde{\delta} \left( \widetilde{W}_{i,t+1} + \widetilde{J}_{i,t+1} - U_{i,t+1} \right) - \mu_{I,t} \left( \widetilde{W}_{I,t+1} - U_{I,t+1} \right) \end{array} \right], \qquad (50)
\end{aligned}
$$

or, equivalently,

$$
TS_{I,t} = z_t p_t - h + \beta \xi_t \mathbb{E}_t \left[ \left(1 - \delta - \tilde{\delta}\right) TS_{I,t} + \left( \tilde{\delta} - \mu_{I,t} (1 - \tau) \right) \widetilde{TS}_{I,t} \right], \qquad (51)
$$

where, in the interest of space, we omit the variable $\iota_t$.

Analogously, the total surplus of a filled job in a joint venture for the firm in the final-goods sector $F$ is:

$$
TS_{F,t} = z_t (1 - p_t) - h + \beta \xi_t \mathbb{E}_t \left[ \left(1 - \delta - \tilde{\delta}\right) TS_{F,t} + \left( \tilde{\delta} - \mu_{F,t} (1 - \tau) \right) \widetilde{TS}_{F,t} \right]. \qquad (52)
$$

Next, we derive the total surplus of a filled job in a single firm, $\widetilde{TS}_{i,t}$. The equations for $\widetilde{W}_{I,t}$, $\widetilde{J}_{I,t}$, and $U_{I,t}$ yield:

$$
\begin{aligned}
\widetilde{J}_{I,t} + \widetilde{W}_{I,t} - U_{I,t} = -h - c\left(\sigma^*_{I,t}\right) +& \\
\beta \xi_t \mathbb{E}_t &\left[ \begin{array}{c} (1 - \delta) \left(1 - \pi^*_{I,t}\right) \left( \widetilde{J}_{I,t+1} + \widetilde{W}_{I,t+1} - U_{I,t+1} \right) + \\ (1 - \delta) \, \pi^*_{I,t} (W_{i,t+1} + J_{i,t+1} - U_{i,t+1}) - \mu_{I,t} \left( \widetilde{W}_{I,t+1} - U_{I,t+1} \right) \end{array} \right], \qquad (53)
\end{aligned}
$$

where $\sigma^*_{I,t}$ is the search intensity that maximizes $\widetilde{J}_{I,t}$ and $\pi^*_{I,t}$ is the matching probability induced by $\sigma^*_{I,t}$. By using the definition of $\widetilde{TS}_{i,t}$ above, we re-arrange the previous equation as:

$$\begin{aligned}\widetilde{TS}_{I,t} &= -h - c\left(\sigma^*_{I,t}\right) \\ &\quad + \beta\xi_t\mathbb{E}_t\left[(1-\delta)\,\pi_{I,t}TS_{I,t+1} + ((1-\delta)\,(1-\pi_{I,t}) - (1-\tau)\,\mu_{I,t})\,\widetilde{TS}_{I,t+1}\right].\end{aligned} \tag{54}$$

Since $\sigma^*_{I,t}$ maximizes $\widetilde{J}_{I,t}$, it also maximizes $\widetilde{TS}_{I,t}$. Thus, equation (54) becomes:

$$\begin{aligned}\widetilde{TS}_{I,t} = \max_{\sigma_{I,t}\geq 0}\Big\{ &-h - c\left(\sigma_{I,t}\right) \\ &+ \beta\xi_t\mathbb{E}_t\left[(1-\delta)\,\pi_{I,t}TS_{I,t+1} + ((1-\delta)\,(1-\pi_{I,t}) - (1-\tau)\,\mu_{I,t})\,\widetilde{TS}_{I,t+1}\right]\Big\},\end{aligned} \tag{55}$$

and where $\pi_{I,t}$ is an increasing function of $\sigma_{I,t}$.

We denote the gain for total surplus from forming a joint venture as $\Delta TS_{i,t} = TS_{i,t} - \widetilde{TS}_{i,t}$, and rewrite equation (55) as:

$$\begin{aligned}\widetilde{TS}_{I,t} = \max_{\sigma_{I,t}\geq 0}\Big\{ &-h - c\left(\sigma_{I,t}\right) \\ &+ \beta\xi_t\mathbb{E}_t\left[(1-\delta)\,\pi_{I,t}\Delta TS_{I,t+1} + ((1-\delta) - (1-\tau)\,\mu_{I,t})\,\widetilde{TS}_{I,t+1}\right]\Big\}.\end{aligned} \tag{56}$$

Similarly, we write the total surplus for single firms in the final-goods sector as:

$$\begin{aligned}\widetilde{TS}_{F,t} = \max_{\sigma_{F,t}\geq 0}\Big\{ &-h - c\left(\sigma_{F,t}\right) \\ &+ \beta\xi_t\mathbb{E}_t\left[(1-\delta)\,\pi_{F,t}TS_{F,t+1} + ((1-\delta)\,(1-\pi_{F,t}) - (1-\tau)\,\mu_{F,t})\,\widetilde{TS}_{F,t+1}\right]\Big\},\end{aligned} \tag{57}$$

or, equivalently,

$$\begin{aligned}\widetilde{TS}_{F,t} = \max_{\sigma_{F,t}\geq 0}\Big\{ &-h - c\left(\sigma_{F,t}\right) \\ &+ \beta\xi_t\mathbb{E}_t\left[(1-\delta)\,\pi_{F,t}\Delta TS_{F,t+1} + ((1-\delta) - (1-\tau)\,\mu_{F,t})\,\widetilde{TS}_{F,t+1}\right]\Big\}.\end{aligned} \tag{58}$$

Finally, we derive the total surplus of a joint venture, $TSJV_{i,t}$. The Nash bargaining for the intermediate goods price and wage yields $\Delta TS_{I,t} = \frac{\tilde{\tau}}{\tau}TSJV_t$ and $\Delta TS_{F,t} = \left(\frac{1-\tilde{\tau}}{\tau}\right)TSJV_t$. Using equations (51) and (55) in the definition of $\Delta TS_{i,t}$ produces:

$$\Delta TS_{I,t} = \min_{\sigma_{I,t}}\left\{ z_t p_t + c\left(\sigma_{I,t}\right) + \beta\left[\left(1-\delta-\tilde{\delta}\right) - (1-\delta)\,\pi_{I,t}\right]\xi_t\mathbb{E}_t\left(\Delta TS_{I,t+1}\right)\right\}, \tag{59}$$

or after using the Nash bargaining condition $\Delta TS_{I,t} = \frac{\tau}{\widetilde{\tau}} TSJV_t$:

$$TSJV_t = \min_{\sigma_{I,t}} \left\{ \frac{\tau}{\widetilde{\tau}} \left[ z_t p_t + c\left(\sigma_{I,t}\right)\right] + \beta \left[ \left(1 - \delta - \tilde{\delta}\right) - \left(1-\delta\right) \pi_{I,t} \right] \xi_t \mathbb{E}_t \left(TSJV_{t+1}\right) \right\}. \tag{60}$$

Analogously, the total surplus of a joint venture from sector $F$'s optimization problem is:

$$\begin{aligned} TSJV_t = \min_{\sigma_{F,t}} \Bigg\{ & \frac{\tau}{1-\widetilde{\tau}} \left[ z_t \left(1-p_t\right) + c\left(\sigma_{F,t}\right)\right] \\ & + \beta \left[ \left(1 - \delta - \tilde{\delta}\right) - \left(1-\delta\right) \pi_{F,t} \right] \xi_t \mathbb{E}_t \left(TSJV_{t+1}\right) \Bigg\}. \end{aligned} \tag{61}$$

Combining Equation (60)$\times\widetilde{\tau}$+Equation (61)$\times\left(1-\widetilde{\tau}\right)$, $p_t$ cancels out and we find:

$$\begin{aligned} TSJV_t = {} & \tau \cdot z_t + \beta \left(1 - \delta - \tilde{\delta}\right) \xi_t \mathbb{E}_t \left(TSJV_{t+1}\right) \\ & + \min_{\sigma_{I,t}} \left\{ \tau \cdot c\left(\sigma_{I,t}\right) - \beta\left(1-\delta\right) \pi_{I,t} \xi_t \mathbb{E}_t \left(\widetilde{\tau} \cdot TSJV_{t+1}\right) \right\} \\ & + \min_{\sigma_{F,t}} \left\{ \tau \cdot c\left(\sigma_{F,t}\right) - \beta\left(1-\delta\right) \pi_{F,t} \xi_t \mathbb{E}_t \left[\left(1-\widetilde{\tau}\right) \cdot TSJV_{t+1}\right] \right\}. \end{aligned} \tag{62}$$

The first-order conditions for $\{\sigma_{I,t}, \sigma_{F,t}\}$ in equation (62) are:

$$\beta\left(1-\delta\right)\left(\psi + \sigma_{F,t}\right) H\left(1, \widetilde{\theta}_t\right) \widetilde{\tau} \xi_t \mathbb{E}_t \left(TSJV_{t+1}\right) = \tau \left[c_0 + c_1 \left(\sigma_{I,t}\right)^{\nu}\right], \tag{63}$$

$$\beta\left(1-\delta\right)\left(\psi + \sigma_{I,t}\right) H\left(1, \widetilde{\theta}_t^{-1}\right) \left(1-\widetilde{\tau}\right) \xi_t \mathbb{E}_t \left(TSJV_{t+1}\right) = \tau \left[c_0 + c_1 \left(\sigma_{F,t}\right)^{\nu}\right]. \tag{64}$$

The active equilibrium exists if and only if there exists a pair $\left(\sigma_{I,t}, \sigma_{F,t}\right) > 0$ that jointly solves equations (63) and (64). In the symmetric equilibrium for which $\widetilde{\tau} = 1/2$ and $\widetilde{\theta}_t = 1$, equations (63) and (64) become:

$$\widetilde{\beta}\left(\psi + \sigma_{F,t}\right) \xi_t \mathbb{E}_t \left(J_{I,t+1} - \widetilde{J}_{I,t+1}\right) = c_0 + c_1 \left(\sigma_{I,t}\right)^{\nu}, \tag{65}$$

$$\widetilde{\beta}\left(\psi + \sigma_{I,t}\right) \xi_t \mathbb{E}_t \left(J_{F,t+1} - \widetilde{J}_{F,t+1}\right) = c_0 + c_1 \left(\sigma_{F,t}\right)^{\nu}, \tag{66}$$

where $\widetilde{\beta} = \beta\left(1-\delta\right)/\tau$. Equivalently, we can express the first-order conditions as:

$$\beta\left(1-\delta\right)\left(\psi + \sigma_{F,t}\right) \xi_t \mathbb{E}_t \left(\Delta TS_{I,t+1}\right) = c_0 + c_1 \left(\sigma_{I,t}\right)^{\nu} \tag{67}$$

$$\beta\left(1-\delta\right)\left(\psi + \sigma_{I,t}\right) \xi_t \mathbb{E}_t \left(\Delta TS_{F,t+1}\right) = c_0 + c_1 \left(\sigma_{I,t}\right)^{\nu}. \tag{68}$$

# B Solving for the DSSs

To solve for the DSSs of our model, we evaluate the set of equilibrium conditions when the variables remain constant over time and the exogenous state variables take their average value. The model entails a passive and an active (stable) DSS. We denote the variables referring to the passive and active DSS with superscript "pas" and "act," respectively.

Using equation (62), the total surplus of a joint venture in the passive DSS is:

$$\begin{aligned} TSJV^{pas} &= \tau \cdot z^{ss} + 2\tau \cdot c\left(0\right) \\ &+ \beta \xi^{ss} \left[ \left(1 - \delta - \tilde{\delta}\right) - \widetilde{\tau}\left(1-\delta\right) \pi_I^{pas} - \left(1 - \widetilde{\tau}\right)\left(1-\delta\right)\pi_F^{pas} \right] TSJV^{pas} \end{aligned} \tag{69}$$

where $c\left(0\right) = 0$, $\pi_I^{pas} = \left(\phi + \psi^2\right) H\left(1, \widetilde{\theta}^{pas}\right)$, and $\pi_F^{pas} = \left(\phi + \psi^2\right) H\left(1, 1/\widetilde{\theta}^{pas}\right)$. As in our baseline calibration, we set $\widetilde{\tau} = 0.5$ and assume a symmetric equilibrium so that $\widetilde{\theta}^{pas} = 1$. Applying these conditions in equation (69) yields:

$$TSJV^{pas} = \frac{\tau \cdot z^{ss}}{1 - \beta \xi^{ss} \left[ \left(1 - \delta - \tilde{\delta}\right) - \left(1-\delta\right)\left(\phi + \psi^2\right) \right]}. \tag{70}$$

The gain of total surplus from forming a joint venture in the passive DSS is determined by $\Delta TS_I^{pas} = \frac{\widetilde{\tau}}{\tau} TSJV^{pas}$ and $\Delta TS_F^{pas} = \left(\frac{1-\widetilde{\tau}}{\tau}\right) TSJV^{pas}$, which are useful in deriving the total surplus of a filled job in the passive DSS.

Analogously, the total surplus of a joint venture in the active DSS is determined by:

$$TSJV^{act} = \frac{\tau \cdot \left[ z^{ss} + \left( c_0 \sigma_I^{act} + c_1 \frac{\left(\sigma_I^{act}\right)^{\nu+1}}{1+\nu} \right) + \left( c_0 \sigma_F^{act} + c_1 \frac{\left(\sigma_F^{act}\right)^{\nu+1}}{1+\nu} \right) \right]}{1 - \beta \xi^{ss} \left[ \left(1 - \delta - \tilde{\delta}\right) - \widetilde{\tau}\left(1-\delta\right)\pi_I^{act} - \left(1-\widetilde{\tau}\right)\left(1-\delta\right)\pi_F^{act} \right]}, \tag{71}$$

where:

$$\pi_I^{act} = \left[\phi + \left(\sigma_F^{act} + \psi\right)\left(\sigma_I^{act} + \psi\right)\right] H\left(1, \widetilde{\theta}^{act}\right),$$

and

$$\pi_F^{act} = \left[\phi + \left(\sigma_F^{act} + \psi\right)\left(\sigma_I^{act} + \psi\right)\right] H\left(1, 1/\widetilde{\theta}^{act}\right).$$

By imposing the symmetry conditions $\widetilde{\tau} = 1/2$, $\widetilde{\theta}^{act} = 1$ and $\sigma_F^{act} = \sigma_I^{act} = \sigma^{act}$, equation (71) becomes:

$$TSJV^{act} = \frac{\tau \cdot \left[z^{ss} + 2\left(c_0\sigma^{act} + c_1\frac{\left(\sigma^{act}\right)^{\nu+1}}{1+\nu}\right)\right]}{1-\beta\xi^{ss}\left[\left(1-\delta-\tilde{\delta}\right)-(1-\delta)\left[\phi+\left(\sigma^{act}+\psi\right)^2\right]\right]}. \tag{72}$$

In the active DSS, the first-order condition for $\{\sigma_{I,t}, \sigma_{F,t}\}$ described by equations (63) and (64) is:

$$\frac{\beta(1-\delta)\left(\psi+\sigma^{act}\right)\xi^{ss}TSJV^{act}}{2} = \tau\left[c_0 + c_1\left(\sigma^{act}\right)^{\nu}\right]. \tag{73}$$

Equations (72) and (73) can be used to solve numerically for $\sigma^{act}$ and $TSJV^{act}$.

The gain of total surplus from forming a joint venture in the active DSS is determined by $\Delta TS_I^{act} = \frac{\tilde{\tau}}{\tau}TSJV^{act}$ and $\Delta TS_F^{act} = \left(\frac{1-\tilde{\tau}}{\tau}\right)TSJV^{act}$. We then derive the total surplus of a filled job in a single firm and the job finding rate in the DSS. Using equation (54), the total surplus of a filled job for a single firm in sector $I$ in the passive DSS is:

$$\widetilde{TS}_I^{pas} = -h + \beta\left\{(1-\delta)\,\pi_I^{pas}\cdot\Delta TS_I^{pas} + \left[(1-\delta)-\mu_I^{pas}(1-\tau)\right]\widetilde{TS}_I^{pas}\right\}, \tag{74}$$

where $\Delta TS_I^{pas}$ and $\pi_I^{pas}$ are solved analytically. Using the matching function and free-entry condition in the labor market, the job finding rate in the passive DSS is:

$$\mu_I^{pas} = \left(\frac{\beta\tau\widetilde{TS}_I^{pas}}{\chi}\right)^{\frac{\alpha}{1-\alpha}}. \tag{75}$$

Equations (74) and (75) are solved numerically for $\widetilde{TS}_I^{pas}$ and $\mu^{pas}$.

Applying the same approach, we solve for $\widetilde{TS}_F^{pas}$ and $\mu_F^{pas}$. Analogously, the total surplus of a filled job in a single firm and the job finding rate in the active DSS solves:

$$\begin{aligned}\widetilde{TS}^{act} = & -h + \left[c_0\sigma^{act} + c_1\frac{\left(\sigma^{act}\right)^{\nu+1}}{1+\nu}\right] \\ & + \beta\left\{(1-\delta)\,\pi^{act}\cdot\Delta TS^{act} + \left[(1-\delta)-\mu^{act}(1-\tau)\right]\widetilde{TS}^{act}\right\},\end{aligned} \tag{76}$$

and

$$\mu^{act} = \left(\frac{\beta\tau\widetilde{TS}^{act}}{\chi}\right)^{\frac{\alpha}{1-\alpha}}. \tag{77}$$

The total surplus of a filled job in a joint venture in the DSS is:

$$TS_i^l = \widetilde{TS}_i^l + \Delta TS_i^l,\ i \in \{I, F\}, l \in \{act, pas\}.$$

The firm's asset value in the DSS is:

$$J_i^l = \tau TS_i^l,\ \widetilde{J}_i^l = \tau \widetilde{TS}_i^l,\ i \in \{I, F\}, l \in \{act, pas\}.$$

Finally, we can derive the DSS value for the remaining variables. Substituting the job finding rate into the matching function of the labor market, we get $\theta^{pas} = (\mu^{pas})^{\frac{1}{\alpha}}$ and $\theta^{act} = (\mu^{act})^{\frac{1}{\alpha}}$.

The value for the unemployment rate, the measure of single firms, and the measure of joint ventures in the passive and active DSS are:

$$\begin{aligned}
u^{pas} &= \frac{\delta}{\delta + \mu^{pas}} \\
u^{act} &= \frac{\delta}{\delta + \mu^{act}} \\
\tilde{n}^{pas} &= \frac{\tilde{\delta} + \left(\mu^{pas} - \tilde{\delta}\right) u^{pas}}{\delta + \pi^{pas} + \tilde{\delta}} \\
\tilde{n}^{act} &= \frac{\tilde{\delta} + \left(\mu^{act} - \tilde{\delta}\right) u^{act}}{\delta + \pi^{act} + \tilde{\delta}} \\
n^{pas} &= 1 - u^{pas} - \tilde{n}^{pas} \\
n^{act} &= 1 - u^{act} - \tilde{n}^{act}.
\end{aligned}$$

The value for total final output in the passive and active DSSs are $y^{pas} = z^{ss} n^{pas}$ and $y^{act} = z^{ss} n^{act}$, respectively.

# C Proof of propositions

### Proof of Proposition 2

*Proof.* We consider the case of symmetric sectors, so we drop the sector subscripts. We first show that the labor market tightness ratio is strictly lower in the passive DSS, i.e., $\theta^{pas} < \theta^{act}$, or, equivalently $\widetilde{TS}^{pas} < \widetilde{TS}^{act}$, as implied by the free-entry condition of the labor market.

We start with

$$\widetilde{TS}^{act} = -h + \left[c_0\sigma^{act} + c_1\frac{\left(\sigma^{act}\right)^{\nu+1}}{1+\nu}\right] \\ + \beta\left\{(1-\delta)\,\pi^{act}\cdot\Delta TS^{act} + \left[(1-\delta) - \mu^{act}\,(1-\tau)\right]\widetilde{TS}^{act}\right\}, \tag{78}$$

and

$$\mu^{act} = \left(\frac{\beta\tau\widetilde{TS}^{act}}{\chi}\right)^{\frac{\alpha}{1-\alpha}}. \tag{79}$$

The values for $\widetilde{TS}^{act}$ and $\theta^{act}$ solve to equations (76) and (77). We rewrite equation (77), for both DSSs, as

$$\theta = \left(\frac{\beta\tau\widetilde{TS}}{\chi}\right)^{\frac{1}{1-\alpha}}. \tag{80}$$

Applying equation (80) to equation (76), delivers:

$$\left(\frac{1-\tau}{\tau}\right)\chi\theta = \Bigg\{-h - \left[c_0\sigma^{act} + c_1\frac{\left(\sigma^{act}\right)^{\nu+1}}{1+\nu}\right] \\ + \beta\,(1-\delta)\,\pi^{act}\cdot\Delta TS^{act} - \left[1-\beta\,(1-\delta)\right]\widetilde{TS}\Bigg\}. \tag{81}$$

In equation (81), labor market tightness, $\theta$, is linear and strictly decreasing in the total surplus for a single firm, $\widetilde{TS}$. In equation (80), $\theta$ is strictly increasing in $\widetilde{TS}$.

Similarly, values for $\widetilde{TS}^{pas}$ and $\theta^{pas}$ solve:

$$\left(\frac{1-\tau}{\tau}\right)\chi\theta = \left[-h + \beta\,(1-\delta)\,\pi^{pas}\cdot\Delta TS^{pas}\right] - \left[1-\beta\,(1-\delta)\right]\widetilde{TS}, \tag{82}$$

$$\theta = \left(\frac{\beta\tau\widetilde{TS}}{\chi}\right)^{\frac{1}{1-\alpha}}. \tag{83}$$

In equation (82), $\theta$ is linear and strictly decreasing in $\widetilde{TS}$. In equation (83), $\theta$ is strictly increasing in $\widetilde{TS}$.

For $\theta^{act} > \theta^{pas}$, it must be that the intercept term in equation (81) is greater than the intercept term in equation (82), which occurs if:

$$-h - \left[c_0\sigma^{act} + c_1\frac{\left(\sigma^{act}\right)^{\nu+1}}{1+\nu}\right] + \beta\,(1-\delta)\,\pi^{act}\cdot\Delta TS^{act} > -h + \beta\,(1-\delta)\,\pi^{pas}\cdot\Delta TS^{pas}. \tag{84}$$

To simplify notation, denote $W(\sigma) = -h + \frac{W_1(\sigma)}{W_2(\sigma)}$, where

$$W_1(\sigma) = \left[\beta\left(1-\delta-\tilde{\delta}\right)-1\right]\left[c_0\sigma + c_1\frac{\sigma^{\nu+1}}{1+\nu}\right] + \beta(1-\delta)\left[\phi + (\psi+\sigma)^2\right],$$
$$W_2(\sigma) = 1-\beta\left\{\left(1-\delta-\tilde{\delta}\right)-(1-\delta)\left[\phi+(\psi+\sigma)^2\right]\right\}.$$

We can rewrite equation (84) as $W(\sigma^{act}) > W(0)$ and verify that, for $\sigma \in \left(0, \sqrt{1-\phi}-\psi\right)$, $\frac{dW_1}{d\sigma}/W_1 > \frac{dW_2}{d\sigma}/W_2$, which implies $dW/d\sigma > 0$. Consequently, equation (84) holds, and $\theta^{act} > \theta^{pas}$.

Since the job finding rate is strictly increasing in labor market tightness, $\mu^{act} > \mu^{pas}$. Since $u = \delta/(\delta+\mu)$ in the DSS, $u^{act} < u^{pas}$ holds.

Finally, we show that $y^{act} > y^{pas}$. Since $y = n$ and $n = 1-\tilde{n}-u$, $y^{act} > y^{pas}$ is equivalent to show that $\tilde{n}^{act} + u^{act} < \tilde{n}^{pas} + u^{pas}$.

In the DSS, it holds that:

$$\tilde{n} + u = \frac{\tilde{\delta} + (\pi+\mu+\delta)\frac{\delta}{\delta+\mu}}{\delta+\pi+\tilde{\delta}}. \tag{85}$$

The RHS of equation (85) is strictly decreasing in both $\mu$ and $\pi$. Given that $\mu^{pas} < \mu^{act}$ and $\pi^{pas} < \pi^{act}$, it holds that $\tilde{n}^{act} + u^{act} < \tilde{n}^{pas} + u^{pas}$, or, equivalently$y^{act} > y^{pas}$. □

**Proof of Proposition 3**

*Proof.* Proposition 3 holds if it is optimal for firms in one sector to search with zero intensity when firms in the opposite sector search with zero intensity. In such a case, the Nash equilibrium with zero search intensity exists in the passive DSS.

The firm's maximization problem in the passive DSS is:

$$\widetilde{TS}^{pas} = \max_{\sigma\geq 0} -h - \left(c_0\sigma + c_1\frac{\sigma^{\nu+1}}{1+\nu}\right) + \beta\begin{Bmatrix}(1-\delta)\left[\phi+\psi(\psi+\sigma)\right]\cdot\Delta TS^{pas} \\ +\left[(1-\delta)-\mu^{pas}(1-\tau)\right]\widetilde{TS}^{pas}\end{Bmatrix}. \tag{86}$$

The total surplus of a single firm $\widetilde{TS}^{pas}$ is strictly concave in $\sigma$, for $\sigma > 0$. Hence, the corner solution $\sigma = 0$ is optimal if and only if the first-order derivative is non-positive at $\sigma = 0$:

$$c_0 + c_1 0^{\nu} > \beta(1-\delta)\psi\Delta TS^{pas}, \tag{87}$$

or, equivalently:

$$c_0 > \frac{\beta\left(1-\delta\right)\psi}{2-2\beta\left[\left(1-\delta-\tilde{\delta}\right)-\left(1-\delta\right)\left(\phi+\psi^2\right)\right]}, \tag{88}$$

where we assume $z^{ss}=1$, $\xi^{ss}=1$, and $\widetilde{\tau}=0.5$. $\square$

### Proof of Proposition 4

*Proof.* Proposition 4 holds if there exist $\sigma \in \left(0, \sqrt{1-\phi}-\psi\right)$ (to guarantees that the matching probability is bounded by one) and $\Delta TS \in \mathbb{R}$ that solve equations (73) and (72).

By substituting equation (73) into equation (72), we get:

$$\frac{1+\left(c_0\sigma+c_1\frac{\sigma^{\nu+1}}{1+\nu}\right)}{2-2\beta\left[\left(1-\delta-\tilde{\delta}\right)-\left(1-\delta\right)\left[\phi+\left(\sigma+\psi\right)^2\right]\right]}=\frac{c_0+c_1\sigma^{\nu}}{\beta\left(1-\delta\right)\left(\psi+\sigma\right)}, \tag{89}$$

where we assume $\widetilde{\tau}=1/2$, $\xi^{ss}=1$, $z^{ss}=1$. $\square$

### Proof of Proposition 6

*Proof.* We first show that the Nash equilibrium in the passive DSS is stable. To do so, we demonstrate that there exists a $\epsilon>0$, such that when a firm in sector $j$ deviates from the passive DSS by searching with a small and positive effort bounded by $\epsilon$, it remains optimal for the firm in the opposite sector $i$ to search with zero effort:

$$c_0+c_1 0^{\nu} > \beta\left(1-\delta\right)\left(\psi+\sigma_j\right)\mathbb{E}\left(\Delta TS_i\right), \tag{90}$$

where $\sigma_j \in (0,\epsilon)$. The RHS of equation (90) is a function of $\sigma_j$, which is continuous at $\sigma_j=0$ (note that $\mathbb{E}\left(\Delta TS_i\right)$ is a continuous function of $\sigma_j$). Given the existence of the passive DSS, we know that $c_0+c_1 0^{\nu}>\beta\left(1-\delta\right)\psi\Delta TS^{pas}$. Because of continuity, there exists $\epsilon>0$, so that equation (90) holds when $\sigma_j<\epsilon$.

Next, we show that one Nash equilibrium in the active DSS is stable when two active DSSs exist. The best response function of sector $i$ implied by equations (67) and (68) in the active DSS is:

$$\sigma_i=\begin{cases}\left[\frac{\beta(1-\delta)(\psi+\sigma_j)\Delta TS^{act}-c_0}{c_1}\right]^{\frac{1}{\nu}} & \text{if } \beta\left(1-\delta\right)\left(\psi+\sigma_j\right)\Delta TS^{act}\geq c_0\\ 0 & \text{if } \beta\left(1-\delta\right)\left(\psi+\sigma_j\right)\Delta TS^{act}<c_0,\end{cases} \tag{91}$$

which is strictly increasing and concave in $\sigma_j$ since $c_1 > 0$ and $\nu > 1$. When two active DSSs exist, the best response curve (91) intersects with the 45-degree line at $\sigma_F = \sigma_I = \sigma^*$ and $\sigma_F = \sigma_I = \sigma^{**}$ with $0 < \sigma^* < \sigma^{**} < \sqrt{1-\phi} - \psi$. Due to strict concavity, we have $\frac{d\sigma_i}{d\sigma_j}\mid_{\sigma_i=\sigma_j=\sigma^*} > 1$ and $\frac{d\sigma_i}{d\sigma_j}\mid_{\sigma_i=\sigma_j=\sigma^{**}} < 1$. Therefore, the active Nash equilibrium at $\sigma_F = \sigma_I = \sigma^*$ is unstable, while the one at $\sigma_F = \sigma_I = \sigma^{**}$ is stable.

Finally, consider the case when the passive DSS and one active DSS exist, where $\sigma_F = \sigma_I = \sigma^*$ and $0 < \sigma^* < \sqrt{1-\phi} - \psi$. Since the passive DSS exists, the inequality $c_0 > \beta(1-\delta)\psi\Delta TS^{pas}$ holds. In addition, we have that $\Delta TS^{act} < \Delta TS^{pas}$, which results from equations (70) and (71). We also have that $c_0 > \beta(1-\delta)\psi\Delta TS^{act}$. So $\sigma_i(\sigma_j) = 0$ in the active DSS for $\sigma_j \in [0, \hat{\sigma}]$ with $\hat{\sigma} = \frac{c_0}{\beta(1-\delta)\Delta TS^{act}} - \psi$. Since $\sigma_F = \sigma_I = \sigma^*$ is the only intersection between $\sigma_i(\sigma_j)$ and the 45-degree line in the range $\sigma_j \in [\hat{\sigma}, \sigma^*]$ with $\sigma_i(\hat{\sigma}) = 0$, we must have $\frac{d\sigma_i}{d\sigma_j}\mid_{\sigma_i=\sigma_j=\sigma^*} > 1$. Thus, the active Nash equilibrium at $\sigma_F = \sigma_I = \sigma^*$ is unstable. $\square$

# D Model solution

In this appendix, we outline the algorithm to solve the model numerically.

## D.1 Solution without government spending

We first discuss the solution to the benchmark case with sectoral symmetry without government spending. The vector of state variables is: $S_t = (z_t, \xi_t, \iota_{t-1}, u_t, n_t, \widetilde{n}_t)$, where we omit the sector subscripts. At the beginning of period $t$, $S_t$ is taken as given. The states $z_t$ and $\xi_t$ are exogenous, and the states $\iota_{t-1}, u_t, n_t$, and $\widetilde{n}_t$ are endogenous and predetermined. To derive the solution of the system, we require the value functions $TSJV(S_t)$, and $\widetilde{TS}(S_t)$; two policy functions $\sigma(S_t)$, and $\theta(S_t)$; and the transition rule of $\iota_t = \iota(\iota_{t-1}, S_t)$. The transition rule for the other endogenous states ($u_t$, $n_t$ and $\widetilde{n}_t$) is directly given by the model once the other functions have been found.

Because of sectoral symmetry, $\widetilde{\theta}_t = \widetilde{n}_{F,t}/\widetilde{n}_{I,t} = 1$. As we will show below, a fixed $\widetilde{\theta}$ implies that the value functions, policy functions, and the transition rule for $\iota_t$ depend on $(z_t, \xi_t, \iota_{t-1})$ only.

**Step 1: Solve for $TSJV$, $\sigma$ and $\iota$.** Equation (62) can be rewritten as:

$$TSJV\left(z_t, \xi_t, \iota_{t-1}\right) = \min_{\sigma_t \geq 0} \tau \cdot \left[z_t + 2c\left(\sigma_t\right)\right] + \beta\left\{\left(1-\delta-\tilde{\delta}\right) - (1-\delta)\left[\phi + \left(\psi+\sigma_t\right)\left(\psi+\overline{\sigma_t}\right)\right]\right\} \\ * \, \xi_t \mathbb{E}_t\left[TSJV\left(z_{t+1}, \xi_{t+1}, \iota_t\right)\right], \tag{92}$$

where $\overline{\sigma_t}$ is the search intensity in the opposite sector, taken as given by the firms. In the symmetric equilibrium, $\sigma_t = \overline{\sigma_t}$.

The equilibrium type $\iota_t$ is determined by the best response functions implied by equation (92) and the history-dependence of equilibrium selection. Specifically, if $\iota_{t-1} = 0$ (passive equilibrium in period $t-1$), we first verify whether the passive equilibrium continues to exist in period $t$ by checking whether:

$$\arg\min_{\sigma_t \geq 0} 2c\left(\sigma_t\right) - \widetilde{\beta}\left[\phi + \left(\psi+\sigma_t\right)\psi\right]\xi_t \mathbb{E}_t\left[TSJV\left(z_{t+1}, \xi_{t+1}, \iota_t = 0\right)\right] = 0 \tag{93}$$

holds. If it does, the passive equilibrium exists and persists (i.e., $\iota_t = \iota_{t-1} = 0$). Otherwise, the passive equilibrium fails to exist and the active equilibrium is selected (i.e., $\iota_t = 1$).

Analogously, if $\iota_{t-1} = 1$ (active equilibrium in period $t-1$), we verify whether the active equilibrium continues to exist in period $t$ by checking whether:

$$\arg\min_{\sigma_t \geq 0} 2c\left(\sigma_t\right) - \widetilde{\beta}\left[\phi + \left(\psi+\sigma_t\right)\left(\psi+\sigma^*\right)\right]\xi_t \mathbb{E}_t\left[TSJV\left(z_{t+1}, \xi_{t+1}, \iota_t = 1\right)\right] > 0 \tag{94}$$

holds. If it does, the active equilibrium exists and persists (i.e., $\iota_t = \iota_{t-1} = 1$). Otherwise, the active equilibrium fails to exist and the passive equilibrium is selected (i.e., $\iota_t = 0$).

We use value function iteration methods to solve for the value function $TSJV$, the policy function $\sigma$ and the transition rule of $\iota$ using equation (92) and conditions (93) and (94).

**Step 2: Solve for $\widetilde{TS}$ and $\theta$.** Equation (54) can be rewritten as:

$$\widetilde{TS}\left(z_t, \xi_t, \iota_{t-1}\right) = -h - c\left(\sigma_t\right) + \beta\xi_t\mathbb{E}_t\left[\begin{array}{c}(1-\delta)\,\pi_t \Delta TS\left(z_{t+1}, \xi_{t+1}, \iota_t\right) + \\ \left((1-\delta) - (1-\tau)\,\theta^\alpha\left(z_t, \xi_t, \iota_{t-1}\right)\right)\widetilde{TS}\left(z_{t+1}, \xi_{t+1}, \iota_t\right)\end{array}\right], \tag{95}$$

where we used $\Delta TS_{t+1} = TS_{t+1} - \widetilde{TS}_{t+1}$ and $\mu_t = \theta_t^\alpha$.

The free-entry condition of the labor market (equation 49) can be rewritten as:

$$\chi = \beta \xi_t \tau \theta^{\alpha-1} \left(z_t, \xi_t, \iota_{t-1}\right) \mathbb{E}_t \left[\widetilde{TS} \left(z_{t+1}, \xi_{t+1}, \iota_t\right)\right]. \tag{96}$$

With $\Delta TS_t = \tilde{\tau} TSJV_t/\tau$, $\sigma_t$, and $\iota_t$ being solved in step 1, we find the value function $\widetilde{TS}$ and the policy function $\theta$ with equations (95) and (96) by using value function iteration.

## D.2 Solution with government spending

We consider now the case with government spending. This case is challenging to solve since, in general, it implies sectoral asymmetry. The model's vector of state variables is: $S_t = \left(z_t, \xi_t, \epsilon_t^G, \iota_{t-1}, u_t^F, u_t^I, n_t^F, n_t^I, n_t^G, \widetilde{n}_t^F, \widetilde{n}_t^I, \widetilde{n}_t^G\right)$. States $z_t, \xi_t$, and $\epsilon_t^G$ are exogenous, and states $\iota_{t-1}, u_t^F, u_t^I, n_t^F, n_t^I, n_t^G, \widetilde{n}_t^F, \widetilde{n}_t^I$, and $\widetilde{n}_t^G$ are endogenous. To derive the solution of the system, we need the solution for the value functions $TSJV\left(S_t\right)$, $\widetilde{TS}_F\left(S_t\right)$, and $\widetilde{TS}_I\left(S_t\right)$ (the other value functions can be derived from these three value functions), the four policy functions $\sigma_I\left(S_t\right)$, $\sigma_I\left(S_t\right)$, $\theta_I\left(S_t\right)$ and $\theta_F\left(S_t\right)$, and the transition rule of $\iota_t = \iota\left(\iota_{t-1}, S_t\right)$. The transition rule of the other endogenous states is directly given by the model once the other functions have been found.

In the asymmetric case, the value functions, the policy functions, and the transition rule of $\iota_t$ depend on the entire vector of states $S_t$ rather than a subset of $S_t$ as in Appendix D.1. The reason is that the measure of single firms $\left(\widetilde{n}_t^F, \widetilde{n}_t^I, \widetilde{n}_t^G\right)$ determines the inter-firm market tightness ratio $\widetilde{\theta}_t$, which affects firms' value and policy. In addition, the transition rule of $\left(\widetilde{n}_t^F, \widetilde{n}_t^I, \widetilde{n}_t^G\right)$ depends on the $\left(u_t^F, u_t^I, n_t^F, n_t^I, n_t^G\right)$.

Given the high dimension of the state space, we simplify the model solution with a forecast rule for $\tilde{\theta}$ that only depends on a small number of state variables. This approach is inspired by similar ideas in Krusell and Smith (1998). Intuitively, firms do not need to know $\left(u_t^F, u_t^I, n_t^F, n_t^I, n_t^G, \widetilde{n}_t^F, \widetilde{n}_t^I, \widetilde{n}_t^G\right)$ to make decisions if the forecast rule is accurate, which greatly reduces the dimension of the state space when solving the value and policy functions.

We choose the forecast rule:

$$\begin{aligned}\log\left(\widetilde{\theta}_{t+1}\right) = &\left(a_{\tilde{\theta}} + a_{\tilde{\theta},\iota}\iota_{t-1}\right)\log\left(\widetilde{\theta}_t\right) + \left(a_z + a_{z,\iota}\iota_{t-1}\right)\log\left(z_t\right) \\ &+ \left(a_\xi + a_{\xi,\iota}\iota_{t-1}\right)\log\left(\xi_t\right) + \left(a_G + a_{G,\iota}\iota_{t-1}\right)\epsilon_t^G,\end{aligned} \tag{97}$$

where $A = \left(a_{\tilde{\theta}}, a_{\tilde{\theta},\iota}, a_z, a_{z,\iota}, a_\xi, a_{\xi,\iota}, a_G, a_{G,\iota}\right)$ is the vector of coefficients to be determined.

To do so, we proceed as follows:

**Step 1: Initialize the algorithm.** We initialize the forecast rule with some initial guess:

$$A^{(0)}=\left(a_{\tilde{\theta}}^{(0)},a_{\tilde{\theta},\iota}^{(0)},a_{z}^{(0)},a_{z,\iota}^{(0)},a_{\xi}^{(0)},a_{\xi,\iota}^{(0)},a_{G}^{(0)},a_{G,\iota}^{(0)}\right). \tag{98}$$

**Step 2: Solve for $TSJV$, $\sigma_F$, $\sigma_I$, and $\iota$.** Equation (62) can be rewritten as:

$$\begin{aligned}TSJV\left(z_t,\xi_t,\epsilon_t^G,\iota_{t-1},\widetilde{\theta}_t\right)&=\tau\cdot z_t+\beta\left(1-\delta-\tilde{\delta}\right)\xi_t\mathbb{E}_t\left[TSJV\left(z_{t+1},\xi_{t+1},\epsilon_{t+1}^G,\iota_t,\widetilde{\theta}_{t+1}\right)\right]\\&+\min_{\sigma_{I,t}}\tau\cdot c\left(\sigma_{I,t}\right)-\beta\left(1-\delta\right)\pi_{I,t}\xi_t\mathbb{E}_t\left[\widetilde{\tau}TSJV\left(z_{t+1},\xi_{t+1},\epsilon_{t+1}^G,\iota_t,\widetilde{\theta}_{t+1}\right)\right]\\&+\min_{\sigma_{F,t}}\tau\cdot c\left(\sigma_{F,t}\right)-\beta\left(1-\delta\right)\pi_{F,t}\xi_t\mathbb{E}_t\left[\left(1-\widetilde{\tau}\right)TSJV\left(z_{t+1},\xi_{t+1},\epsilon_{t+1}^G,\iota_t,\widetilde{\theta}_{t+1}\right)\right],\end{aligned} \tag{99}$$

where $\pi_{I,t}=\left[\phi+\left(\psi+\sigma_{F,t}\right)\left(\psi+\sigma_{I,t}\right)\right]H\left(1,\widetilde{\theta}_t\right)$, $\pi_{F,t}=\left[\phi+\left(\psi+\sigma_{F,t}\right)\left(\psi+\sigma_{I,t}\right)\right]H\left(1,\widetilde{\theta}_t\right)$,

$$\begin{aligned}\log\left(\widetilde{\theta}_{t+1}\right)&=\left(a_{\tilde{\theta}}^{(q)}+a_{\tilde{\theta},\iota}^{(q)}\iota_{t-1}\right)\log\left(\widetilde{\theta}_t\right)+\left(a_{z}^{(q)}+a_{z,\iota}^{(q)}\iota_{t-1}\right)\log\left(z_t\right)\\&+\left(a_{\xi}^{(q)}+a_{\xi,\iota}^{(q)}\iota_{t-1}\right)\log\left(\xi_t\right)+\left(a_{G}^{(q)}+a_{G,\iota}^{(q)}\iota_{t-1}\right)\epsilon_t^G,\end{aligned}$$

and $A^{(q)}=\left(a_{\tilde{\theta}}^{(q)},a_{\tilde{\theta},\iota}^{(q)},a_{z}^{(q)},a_{z,\iota}^{(q)},a_{\xi}^{(q)},a_{\xi,\iota}^{(q)},a_{G}^{(q)},a_{G,\iota}^{(q)}\right)$ is the vector of coefficients of the forecast rule in $q$-th iteration.

The equilibrium type $\iota_t$ is determined by the best response functions implied by equation (92) and the history-dependence of equilibrium selection. Specifically, if $\iota_{t-1}=0$ (passive equilibrium in period $t-1$), we verify whether the passive equilibrium still exists in the current period $t$, i.e., $\iota_t=0$, by checking whether:

$$\arg\min_{\sigma_{I,t}\geq 0}c\left(\sigma_{I,t}\right)-\widetilde{\beta}\left[\phi+\left(\psi+\sigma_{I,t}\right)\psi\right]\xi_t\mathbb{E}_t\left[\widetilde{\tau}TSJV\left(z_{t+1},\xi_{t+1},\epsilon_{t+1}^G,\iota_t=0,\widetilde{\theta}_{t+1}\right)\right]=0 \tag{100}$$

$$\arg\min_{\sigma_{F,t}\geq 0}c\left(\sigma_{F,t}\right)-\widetilde{\beta}\left[\phi+\left(\psi+\sigma_{F,t}\right)\psi\right]\xi_t\mathbb{E}_t\left[\left(1-\widetilde{\tau}\right)TSJV\left(z_{t+1},\xi_{t+1},\epsilon_{t+1}^G,\iota_t=0,\widetilde{\theta}_{t+1}\right)\right]=0 \tag{101}$$

hold. If these conditions hold, $\iota_t=\iota_{t-1}=0$. Otherwise, $\iota_t=1$.

Analogously, if $\iota_{t-1}=1$ (active equilibrium in period $t-1$), we verify whether the active equilibrium still exists in the current period, i.e., $\iota_t = 1$, by checking whether:

$$\arg \min_{\sigma_{I,t} \geq 0} c\left(\sigma_{I,t}\right) - \widetilde{\beta}\left[\phi + \left(\psi + \sigma_{I,t}\right)\left(\psi + \sigma_{F,t}\right)\right] \xi_t \mathbb{E}_t \left[\widetilde{\tau} TSJV\left(z_{t+1}, \xi_{t+1}, \epsilon_{t+1}^G, \iota_t = 1, \widetilde{\theta}_{t+1}\right)\right] > 0 \tag{102}$$

$$\arg \min_{\sigma_{F,t} \geq 0} c\left(\sigma_{F,t}\right) - \widetilde{\beta}\left[\phi + \left(\psi + \sigma_{I,t}\right)\left(\psi + \sigma_{F,t}\right)\right] \xi_t \mathbb{E}_t \left[\left(1 - \widetilde{\tau}\right) TSJV\left(z_{t+1}, \xi_{t+1}, \epsilon_{t+1}^G, \iota_t = 1, \widetilde{\theta}_{t+1}\right)\right] > 0 \tag{103}$$

hold. If these conditions hold, $\iota_t = \iota_{t-1} = 1$. Otherwise, $\iota_t = 0$.

Given the forecast rule with $A^{(q)}$, we can solve for the value function $TSJV$, the policy function $\sigma$, and the transition rule $\iota$ with equation (99) and conditions (100)-(103) using value function iteration.

**Step 3: Solve for $\widetilde{TS}$ and $\theta$.** Equation (54) can be rewritten, for $i \in \{I, F\}$, as:

$$\begin{aligned} \widetilde{TS}\left(z_t, \xi_t, \epsilon_t^G, \iota_{t-1}, \widetilde{\theta}_t\right) = & -h - c\left(\sigma_{i,t}\right) \\ & + \beta \xi_t \mathbb{E}_t \begin{bmatrix} \left(1-\delta\right) \pi_{i,t} \Delta TS\left(z_{t+1}, \xi_{t+1}, \epsilon_{t+1}^G, \iota_t, \widetilde{\theta}_{t+1}\right) + \\ \left(\left(1-\delta\right) - \left(1-\tau\right) \theta_i^\alpha\left(z_t, \xi_t, \epsilon_t^G, \iota_{t-1}, \widetilde{\theta}_t\right)\right) \\ * \widetilde{TS}\left(z_{t+1}, \xi_{t+1}, \epsilon_{t+1}^G, \iota_t, \widetilde{\theta}_{t+1}\right) \end{bmatrix}, \end{aligned} \tag{104}$$

where we have used the fact that $\Delta TS_{i,t+1} = TS_{i,t+1} - \widetilde{TS}_{i,t+1}$ and $\mu_{i,t} = \theta_{i,t}^\alpha$.

We also have, for $i \in \{I, F\}$, the free-entry condition implied by equation (49):

$$\chi = \beta \xi_t \tau \theta_i^{\alpha-1}\left(z_t, \xi_t, \epsilon_t^G, \iota_{t-1}, \widetilde{\theta}_t\right) \mathbb{E}_t \left[\widetilde{TS}\left(z_{t+1}, \xi_{t+1}, \epsilon_{t+1}^G, \iota_t, \widetilde{\theta}_{t+1}\right)\right]. \tag{105}$$

With $\Delta TS_{i,t}$, $\sigma_{I,t}$, $\sigma_{F,t}$ and $\iota_t$ being solved in step 2 (in particular, $\Delta TS_t = \widetilde{\tau} TSJV_t / \tau$), we can solve for the value function $\widetilde{TS}_{i,t}$ and the policy function $\theta_{i,t}$ approximately with equations (104) and (105) using value function iteration.

**Step 4: Simulate the model.** We simulate the model for 10,000 periods (disregarding the first 2,000 as a burn-in) with random draws of $\left\{z_t, \xi_t, \epsilon_t^G\right\}$. Then, we compute the realized equilibrium inter-firm market tightness ratio $\widetilde{\theta}_t$.

**Step 5: Update the forecast rule.** Based on the simulated data, we update the coefficient of the forecast rule $A^{(q)}$ with $A^{(q+1)}$ using ordinary least squares. If $A^{(q)}$ and $A^{(q+1)}$ are sufficiently close to each other, we stop the iteration. Otherwise, we return to step 2. The converged forecasting rule explains the fluctuations of $\widetilde{\theta}_t$ well, with an $R^2$ of 0.91.

# E Mixed-strategy Nash equilibria

This appendix discusses the role of mixed-strategy Nash equilibria in our model. We first establish the condition for the existence of a mixed-strategy Nash equilibrium in the DSS (the case with stochastic shocks is similar, but more cumbersome to derive). Then, we argue that such a mixed-strategy Nash equilibrium exists and is unique for the calibration in Section 5. However, this mixed-strategy Nash equilibrium is unstable: a small deviation from the mixed-strategy makes the system converge to the pure-strategy Nash equilibrium.

In a mixed-strategy setting, firms randomize their search intensity by choosing $\sigma = 0$ with probability $q$ and choosing $\sigma = \hat{\sigma}$ with probability $(1-q)$. The random choice is independent across firms. Due to the law of large numbers, the average search intensity in both sectors is $\overline{\sigma} = q \cdot 0 + (1-q)\,\hat{\sigma}$. For a single firm, the inter-firm matching probability is given by $\pi(\sigma) = \phi + \psi(\psi + \sigma)(\psi + \overline{\sigma})$. In the mixed-strategy Nash equilibrium, the inter-firm matching probability takes two values: $\pi(0) = \phi + \psi(\psi + \overline{\sigma})$ and $\pi(\hat{\sigma}) = \phi + (\psi + \hat{\sigma})(\psi + \overline{\sigma})$.

A mixed-strategy Nash equilibrium consists of a tuple $\{q, \hat{\sigma}\}$ with $\hat{\sigma} \in \left(0, \sqrt{1-\phi} - \psi\right)$ and $q \in (0,1)$. The tuple $\{q, \hat{\sigma}\}$ implies that single firms are indifferent between choosing $\sigma = 0$ and $\sigma = \hat{\sigma}$, i.e., $\widetilde{TS}(0) = \widetilde{TS}(\hat{\sigma})$. Since $\Delta TS(0) = TS - \widetilde{TS}(0)$ and $\Delta TS(\hat{\sigma}) = TS - \widetilde{TS}(\hat{\sigma})$, it holds that $\Delta TS(0) = \Delta TS(\hat{\sigma})$. We denote $\Delta TS(0) = \Delta TS(\hat{\sigma}) = \Delta TS$.

According to equation (59):

$$\Delta TS = z^{ss} p^{ss} + \beta\left[\left(1 - \delta - \tilde{\delta}\right) - (1-\delta)\,\pi(0)\right]\Delta TS, \tag{106}$$

where $c(0) = 0$. From equation (56), the single firm's total surplus with zero search effort is:

$$\widetilde{TS}(0) = -h + \beta\left[(1-\delta)\,\pi(0)\,\Delta TS + \left((1-\delta) - (1-\tau)\,\theta^{\alpha}\right)\widetilde{TS}(0)\right]. \tag{107}$$

Analogously, the single firm's total surplus by choosing $\hat{\sigma}$ search effort satisfies:

$$\widetilde{TS}(\hat{\sigma}) = -h - c(\hat{\sigma}) + \beta\left[(1-\delta)\pi(\hat{\sigma})\Delta TS + ((1-\delta)-(1-\tau)\theta^{\alpha})\widetilde{TS}(\hat{\sigma})\right]. \tag{108}$$

Since $\widetilde{TS}(0) = \widetilde{TS}(\hat{\sigma})$ in the mixed-strategy Nash equilibrium, combining equations (107) and (108) delivers:

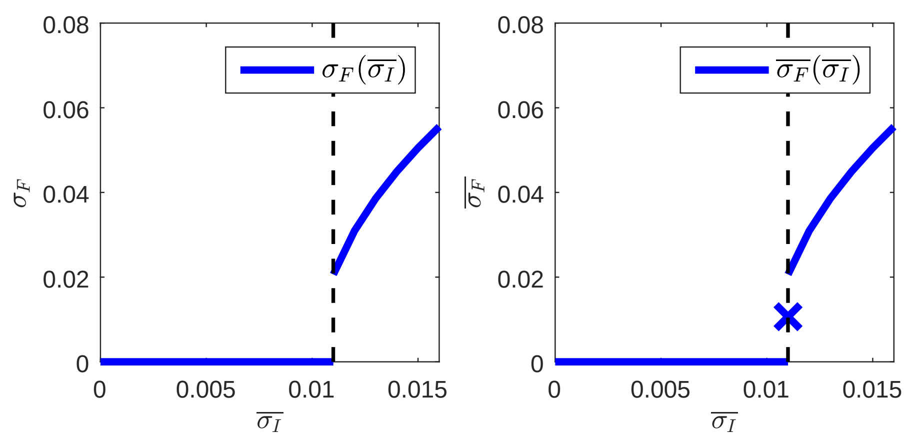
$$c(\hat{\sigma}) = \beta(1-\delta)(\pi(\hat{\sigma}) - \pi(0))\Delta TS. \tag{109}$$

Finally, according to the first-order condition for $\{\sigma_{I,t}, \sigma_{F,t}\}$ in equation (67):

$$\beta(1-\delta)(\psi + \overline{\sigma})\Delta TS = c_0 + c_1\hat{\sigma}^{\nu}. \tag{110}$$

In sum, we have three equations (i.e., equations (106), (109), and (110)) and three unknowns (i.e., $\hat{\sigma}$, $q$, $\Delta TS$). The mixed-strategy Nash equilibrium exists if the system of equations has a solution for the three unknowns. Using the calibration in Section 5, the mixed-strategy Nash equilibrium is $q = 0.3425$, $\hat{\sigma} = 0.0164$, and $\Delta TS = 2.7417$. The average search intensity $\overline{\sigma}$ is $(1-q)\times\hat{\sigma} = 0.0107$.

**Figure 20:** Best response in the mixed-strategy Nash equilibrium




The left panel in Figure 20 displays the firm's optimal search effort in sector $F$ as a function of $\overline{\sigma}_I$. The firm chooses a positive search effort if $\overline{\sigma}_I > 0.0107$ (i.e., for values to the right of the vertical dashed line). The firm chooses a zero search effort if $\overline{\sigma}_I < 0.0107$ (i.e., for values to the left of the vertical dashed line). The firm is indifferent between choosing $\sigma = 0.0164$ and $\sigma = 0$ if $\overline{\sigma}_I = 0.0107$ (i.e., if sector $I$ uses the mixed-strategy $q_I = 0.3425$, $\hat{\sigma}_I = 0.0164$).

The right panel in Figure 20 plots $\overline{\sigma}_F$ as a function of $\overline{\sigma}_I$. The firm chooses a positive search effort if $\overline{\sigma}_I > 0.0107$ (i.e., for values to the right of the vertical dashed line). The firm would choose a zero search effort if $\overline{\sigma}_I < 0.0107$ (i.e., for values to the left of the vertical dashed line). If $\overline{\sigma}_I < 0.0107$ (i.e., if sector $I$ uses the mixed-strategy $q_I = 0.3425$, $\hat{\sigma}_I = 0.0164$), a fraction of 0.3425 firms chooses $\sigma = 0$, while the rest of the firms choose $\sigma = 0.0164$, which implies $\overline{\sigma}_F = 0.0107$ (i.e., the cross marker).

Figure 20 shows that the mixed-strategy Nash equilibrium is unstable: a decrease in $\overline{\sigma}_I$ induces all firms in sector $F$ to search with zero intensity and the system converges to the pure-strategy Nash equilibrium with zero search intensity (i.e., passive equilibrium). Similarly, an increase in $\overline{\sigma}_I$ induces all firms in sector $F$ to search with positive intensity; hence, the system converges to the pure-strategy Nash equilibrium with positive search intensity (i.e., active equilibrium).

# F Simulations based on shocks to productivity

In this appendix, we complete our discussion of the effects of technology shocks in the model. Figure 21 plots the ergodic distribution of selected variables for the case where we only have AR(1) shocks to technology, $z_t$ (for transparency, we eliminate the discount factor shocks). As outlined in the paper, persistent exogenous disturbances to the technological process fail to move the system to a different equilibrium, the equilibrium is always active, and the ergodic distributions of the variables of interest are unimodal.

For completeness, in Figure 22, we plot the ergodic distribution of selected variables for the case where we have both shocks to technology, $z_t$ and to the discount factor, $\xi_t$. We recover bimodality, but this feature is induced by the shocks to $\xi_t$ and their ability to switch equilibria. The main effect of the shocks to productivity is to spread out the ergodic distribution in Figure 12 in the main text (only shocks to $\xi_t$) around its two modes.

Figure 23 shows the GIRFs to a range of persistent negative productivity shocks when the economy starts from the active DSS. Negative productivity shocks are unable to generate a shift in equilibrium even when their magnitude gets very large. In each case, the costly search intensity falls after the productivity shock, and then gradually recovers. The effect of a productivity shock on the labor market tightness ratio and the unemployment rate is also transitory. The mechanism is that the gain of matching with a partner ($TS - \widetilde{TS}$) in the active equilibrium

**Figure 21:** Ergodic distribution with AR(1) shocks to $z_t$

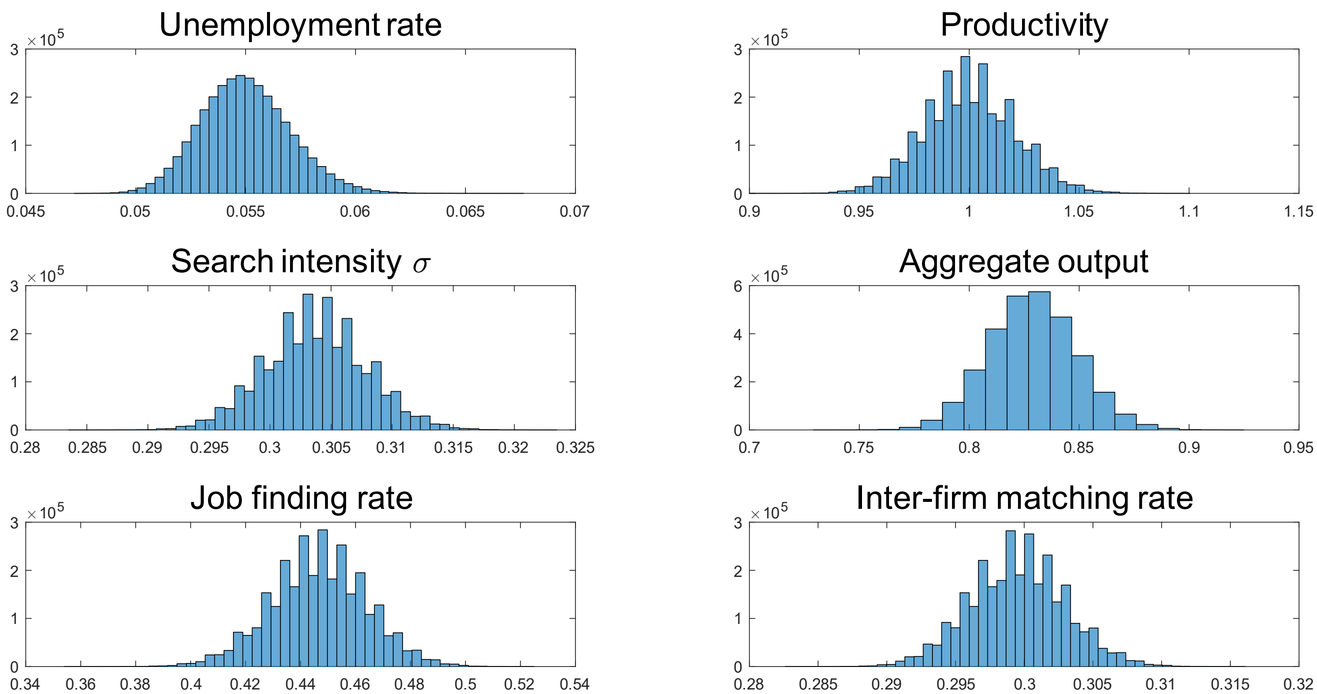
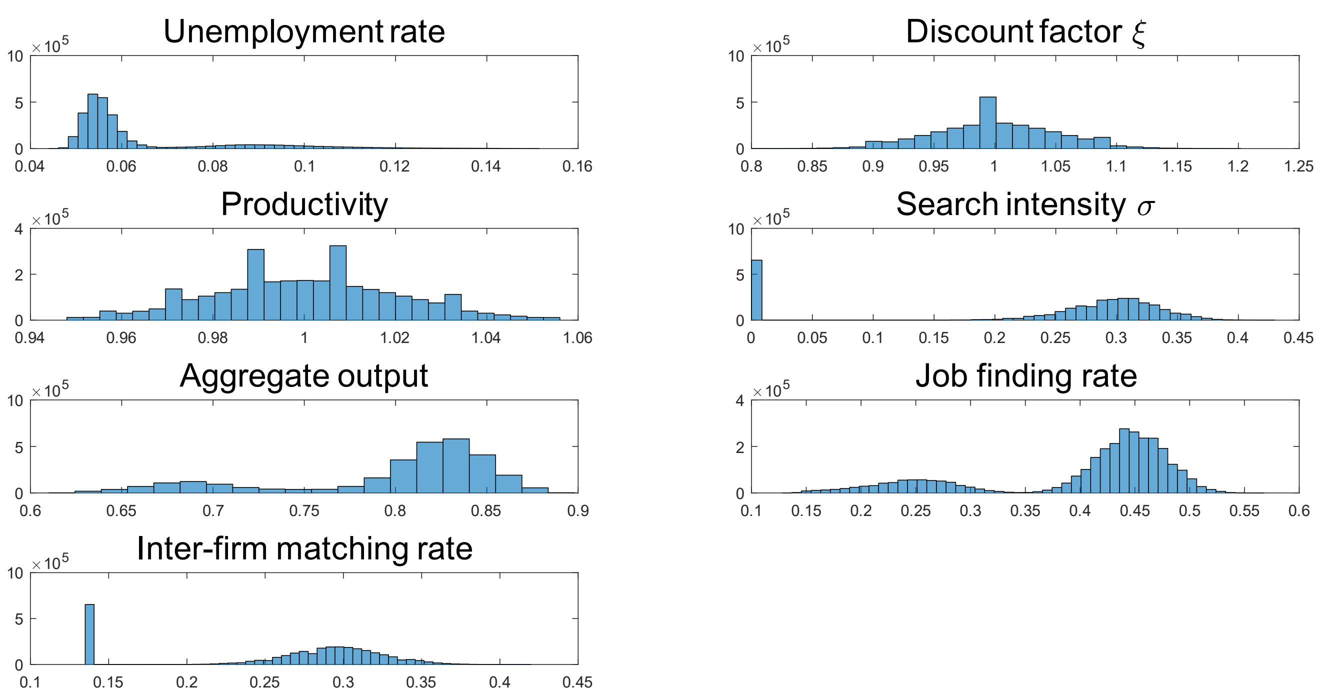
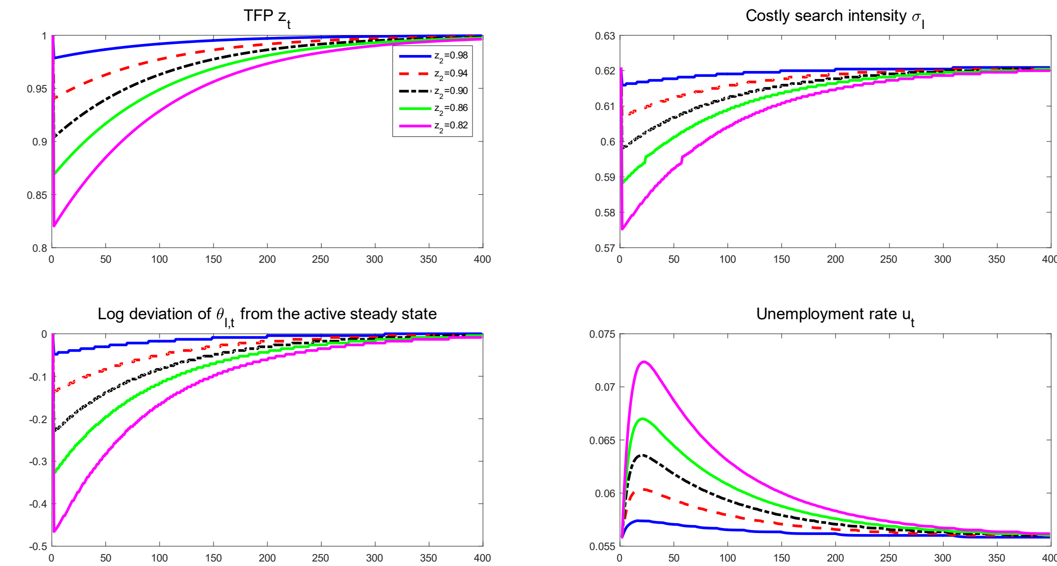



**Figure 22:** Ergodic distribution with *i.i.d.* shocks to $\xi_t$ and AR(1) shocks to $z_t$




is inelastic with the change in productivity. This result is similar to the intuition in Shimer (2005), who points out that the gain of matching with a worker $TS$ is inelastic with the change in productivity in a canonical DMP model. As a result, the existence condition for the active equilibrium in equation (35) keeps holding: if we start at the active DSS, firms find it desirable to search actively for a partner even when productivity is low.

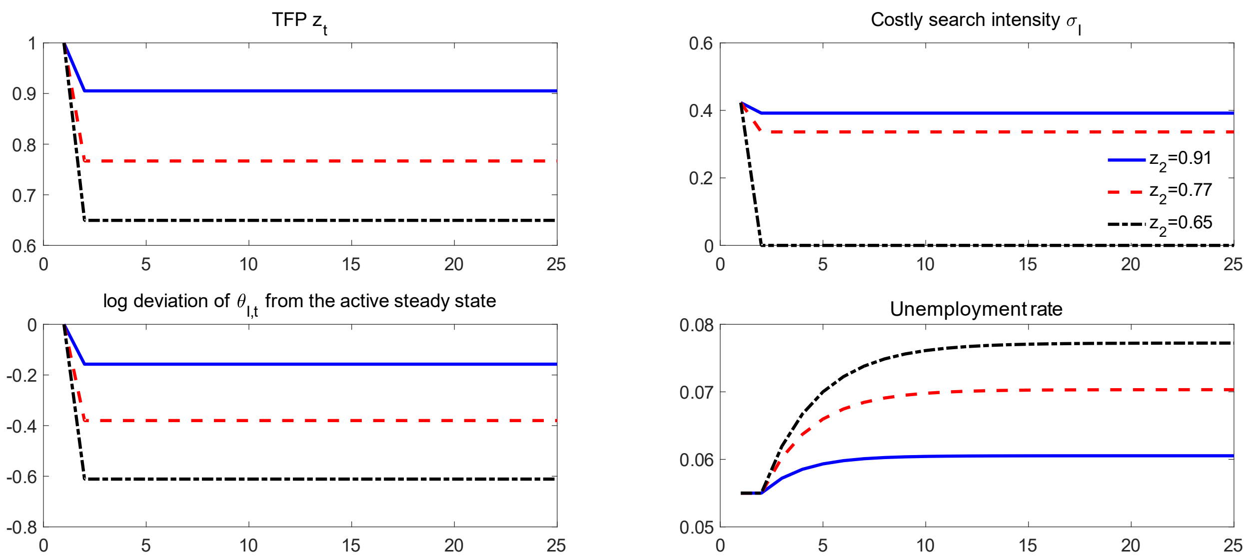
**Figure 23:** GIRFs to a negative productivity shock




We also experiment with permanent changes in productivity. In period $t = 1$, the economy starts from the active DSS with positive search intensity, and in period $t = 2$ a permanent fall in productivity hits the economy. This permanent shock may shift the equilibrium of the system by affecting the expected gain of match $\mathbb{E}_t(J_{i,t+1} - \widetilde{J}_{i,t+1})$. For example, in an economy in the active equilibrium, a sufficiently large fall in $z_t$ decreases the expected gain from joint venture formation and moves the system to the passive equilibrium.

We use the model to assess the magnitude of the fall in $z_t$ needed to move the system from the active to the passive equilibrium. Figure 24 shows the GIRFs of selected variables to a 19% (solid line), 23% (dashed line) and 35% (dot-dashed line) permanent decline in productivity ($z_t$), respectively. The first two shocks are unable to move the system to the active equilibrium because the expected gain from inter-firm matching is relatively inelastic to permanent changes in productivity. Productivity shocks induce $\widetilde{J}_{i,t+1}$ and $J_{i,t+1}$ to comove, leading to a weak response of $\mathbb{E}_t(J_{i,t+1} - \widetilde{J}_{i,t+1})$ to the shock. As we mentioned above, this finding is consistent with Shimer (2005). In comparison, a sufficiently large productivity shock of 35% moves the economy to the passive equilibrium with zero search intensity. This analysis suggests that a permanent productivity shock is unlikely to move the system between equilibria unless the shock is massive.

**Figure 24:** GIRFs to a negative permanent productivity shock




*Note*: Each panel shows the response of a variable to a permanent negative productivity shock ($z$) with magnitudes of 0.19 (solid line), 0.23 (dashed line) and 0.35 (dot-dashed line).

# G Volatility of shocks

Figure 25 plots the ergodic distribution of endogenous variables with *i.i.d.* shocks to $\xi$ in the case of high volatility. Figure 26 repeats the same exercise, but in the case of low volatility. In both cases, we see the bimodal distributions that we discussed in the main text and the long left tail of output when the volatility of $\xi_t$ is high.

**Figure 25:** Ergodic distribution with *i.i.d.* shocks to $\xi$, high volatility

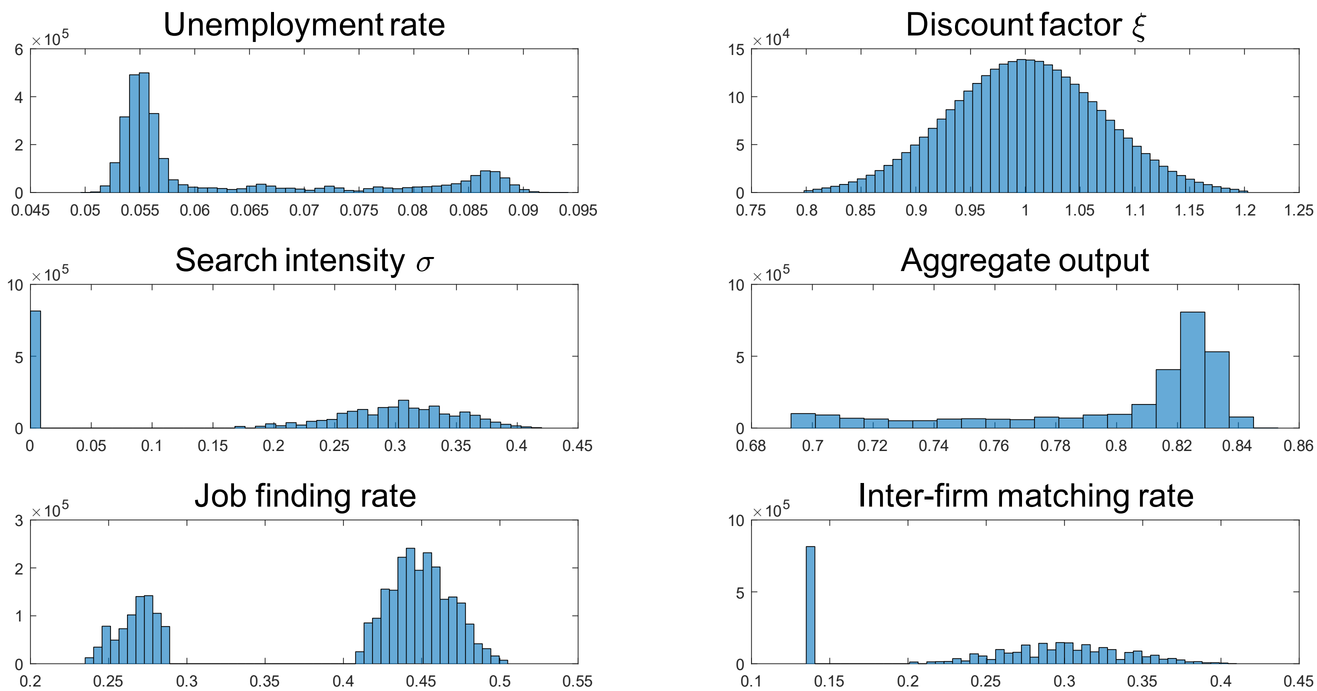
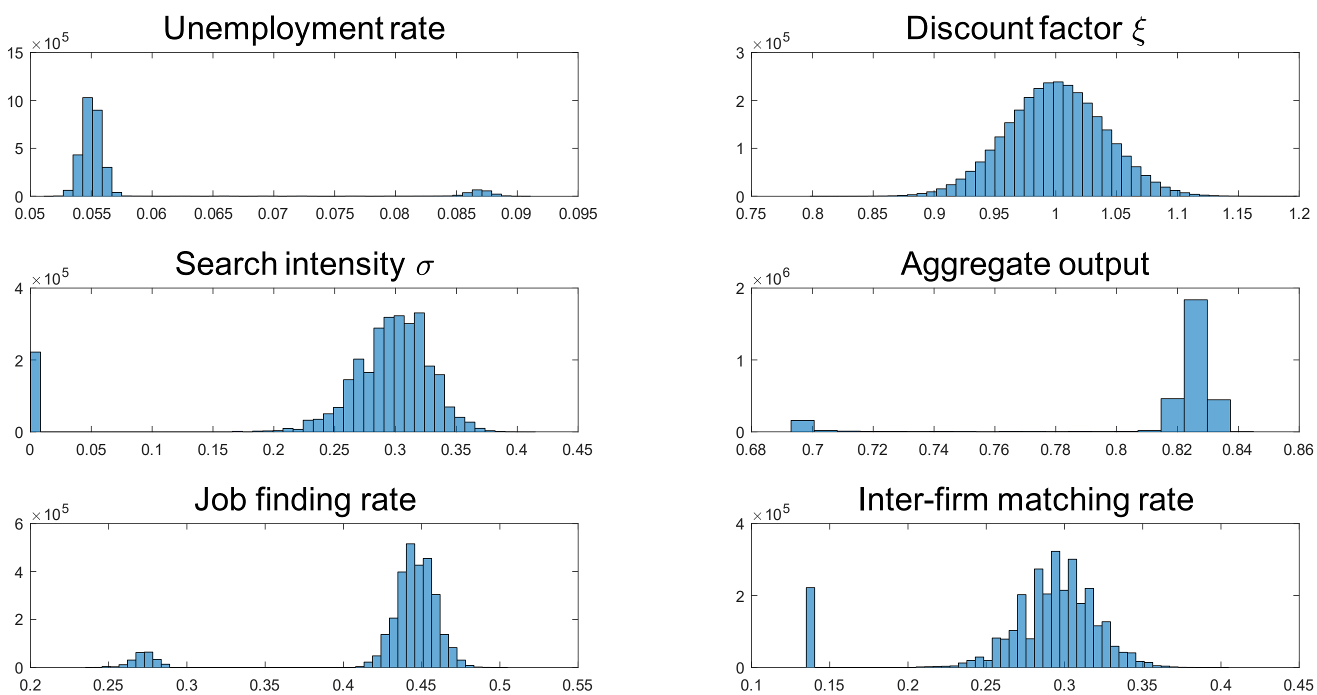


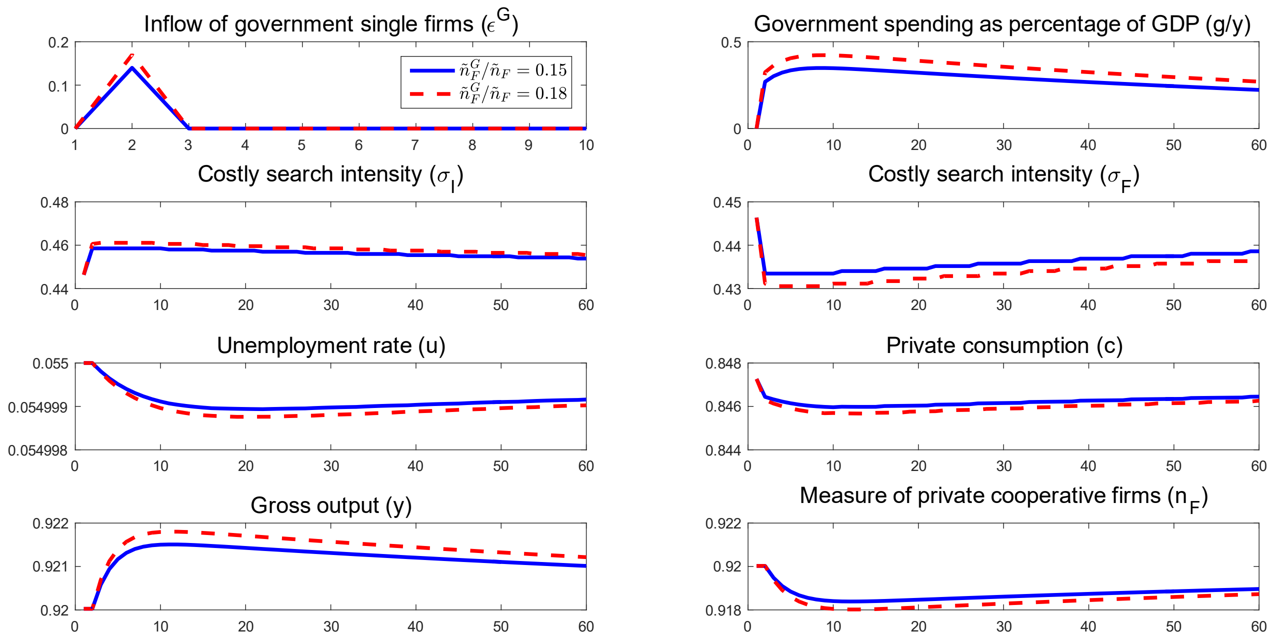

**Figure 26:** Ergodic distribution with *i.i.d.* shocks to $\xi$, low volatility



# H GIRFs to government spending shock in the active equilibrium

This appendix studies the effect of government spending shocks when the economy starts from the active equilibrium. Figure 27 shows the response in the level of selected variables to a 15% (the solid line) and an 18% (the dashed line) government spending shock. Since the economy is already in the active equilibrium, the effects of the fiscal expansion are limited and transitory.

**Figure 27:** GIRFs to positive government spending shock in the active equilibrium




*Note*: Each panel shows the response of a variable to a one-period 15% (solid line) and 18% (dashed line) increase in government spending.