FEDERAL RESERVE BANK *of* ATLANTA **WORKING PAPER SERIES**

SME Failures under Large Liquidity Shocks: An Application to the COVID-19 Crisis

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Working Paper 2020-21a
December 2020 (Revised August 2023)

Abstract: We study the effects of financial frictions on firm exit when firms face large liquidity shocks. We develop a simple model of firm cost-minimization that introduces a financial friction that limits firms' borrowing capacity to smooth temporary shocks to liquidity. In this framework, firm exit arises from the interaction between this financial friction and fluctuations in cash flow due to aggregate and sectoral changes in demand conditions, as well as more traditional shocks to productivity. To evaluate the implications of our model, we use firm level data on small and medium-sized enterprises (SMEs) in 11 European countries. We confirm that our framework is consistent with official failure rates in 2017−19, a period characterized by standard business cycle fluctuations in demand. To capture a large liquidity shock, we apply our framework to the COVID-19 crisis. We find that, absent government support, SME failure rates would have increased by 6.01 percentage points, putting 3.1 percent of employment at risk. Our results are consistent with the premise that financial frictions lead to inefficient exit as, without government support, the firms failing due to COVID have similar productivity and past growth to firms that survive COVID.

JEL classification: D2, E62, E65, G33, H81, L25

Key words: COVID-19, business failures, liquidity, small business

https://doi.org/10.29338/wp2020-21a

An earlier version of this paper circulated under the title "COVID-19 and SME Failures." The authors thank Ryan Decker, Philippe Martin, Brent Neiman, Xavier Ragot, David Sraer, David Thesmar, Jianlan Wang, colleagues at the International Monetary Fund, seminar participants at the Bank for International Settlements, the Organisation for Economic Co-Operation and Development, Banco Central de Chile, the San Francisco Fed, the 2019 "Conference on the Financial Consequences of the COVID-19 Pandemic" organized by the *Journal of Finance* and the Fama-Miller Center (Chicago Booth), the 2020 Federal Reserve System Macroeconomics Conference, and the ASSA 2021 meetings for useful comments. The views expressed here are those of the authors and do not necessarily reflect the views of the Federal Reserve System, the Federal Reserve Board of Governors, the Bank of Canada, or their staff.

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1 Introduction

Firm exit is an important contributor to macroeconomic boom-bust cycles. In the United States, 7.5 percent of firms exit annually, with both the level and cyclicality of this exit being primarily driven by small firms (Crane, Decker, Flaaen, Hamins-Puertolas, Kruz and Christopher, 2022). The high rate of firm exit, especially during recessionary periods, raises two key questions: are the "right" firms exiting during downturns (i.e are recessions "cleansing"); and if not, does government intervention, aimed at saving productive firms, instead prevent the failure of "weak" firms and risk creating "zombies"?[1]

Much of the existing theoretical firm dynamics literature is not well suited to tackle these questions. Most models of firm dynamics put firms' productivity and shocks to that productivity at the core of firms' exit decision, which generates higher exit rates from low productivity firms and "cleansing" recessions.[2] Many models therefore ignore both the types of frictions and shocks that contribute to "inefficient" exit and sluggish recoveries. Specifically, small and medium sized enterprises (SMEs) have been shown to be financially constrained in both the U.S. and Europe during normal times and crises (e.g. Caglio, Darst and Kalemli-Ozcan, 2021, Dinlersoz, Hyatt, Kalemli-Ozcan and Penciakova, 2019, Gopinath, Kalemli-Ozcan, Karabarbounis and Villegas-Sanchez, 2017). Moreover, besides shocks to productivity and disruptions in the credit market, firms also face changing demand conditions, especially during recessions.[3] Precisely the interaction between these firm financial frictions and changing market demand conditions could be important in explaining firm exit.

In this paper, we use a simple firm cost-minimization model, combined with firm balance sheet data, to study the impact of firm financial frictions on SME exit, under a variety of shocks that may affect firm liquidity. These shocks arise from changes in aggregate and sectoral demand and supply conditions. For instance, when consumer demand declines, firm cash flow falls. Firms fail when they cover their input costs and financial expenses because of the cash shortfall and limited access to new credit, even after shedding workers and/or closing temporarily. Firm failure therefore arises from the interaction between negative non-financial shocks to liquidity and firm financial frictions because firms, even when financial markets function normally, cannot fully smooth these shocks by borrowing from the financial sector.[4]

[1]See Caballero and Hammour, 1994, Caballero, Hoshi and Kashyap, 2008, Foster, Haltiwanger and Krizan, 2001, Klette and Kortum, 2003, and Samaniego, 2010 arguing for the "cleansing" effects of recessions. On the other hand, potentially inefficient firm exit arising from frictions, including financial constraints, may contribute to sluggish recoveries. See Ates and Saffie, 2021, Choi, 2013, Khan and Thomas, 2013, Kalemli-Ozcan, Laeven and Moreno, 2022, and Sedláček and Sterk, 2017.

[2]See Clementi and Palazzo, 2016, Lee and Mukoyama, 2018, Samaniego, 2008

[3]Note that models that incorporate financial frictions often focus on the effect of shocks in the credit market. See Ayres and Raveendranathan, 2021, and Khan and Thomas, 2013.

[4]This type of financial friction has been shown to be empirically relevant for SMEs (e.g. Caglio et al., 2021,

A key feature of our framework, relative to previous modeling exercises, is that it uses pre-crisis firm level *data* to summarize the initial distribution of firm health and profitability, and models how firms *adjust* their production decisions when faced with a series of both demand and supply shocks. With this approach, we can estimate the impact of shocks on individual firms, and consequently the effect of shocks on the distribution of surviving versus failing firms. Doing so enables us to tackle the question of whether the "right" firms fail. We can also evaluate counterfactual scenarios in which different degrees and forms of government support are implemented to evaluate the distributional, sectoral, and aggregate impact of policy on SME failure, as well as the associated costs and benefits. Consequently, we can answer whether policy saves more productive, growing firms or "weak" firms. Furthermore, our framework can be utilized in real time, providing a powerful tool for policymakers to gain potential insights on firm health quickly at the onset of a crisis.

Our starting point is a firm cost minimization model in which firms face a set of liquidity shocks (in the form of sectoral demand and supply shocks) that affect firm cash flow. Total demand for a firm's output in each sector is affected by both aggregate and sector-specific demand shocks. An aggregate demand shock captures changes in aggregate expenditures and affects all firms proportionately. A sector-specific demand shock reflects changes in the pattern of household spending resulting from changes in preferences for certain goods. On the supply side, prices are fixed and output is demand determined. Firms must adjust variable intermediate inputs (labor and materials) to meet demand, subject to labor supply shocks. Meeting demand in this constrained environment may lead to further cash flow deterioration, in which case firms may prefer to temporarily shut down rather than produce (*mothball*).[5]

In the model, firms fail if pre-shock cash balances plus current period cash flow are insufficient to cover the interest payments on pre-existing debt for the year.[6] Two aspects of our failure criterion are worth noting. First, while SMEs face financial constraints in terms of borrowing to smooth out the original shock, our exit criterion recognizes that they have *some* capacity to smooth cash flow in times of temporary stress. We allow firms to hold their existing debt levels constant and require them only to make interest payments on this debt. Moreover, by categorizing firms as failing only if their end-of-year cash balance is negative, we are implicitly assuming firms can obtain credit to remain liquid during temporary cash deficits, provided their remaining profits for the year are sufficient to allow full repayment of this credit. Second, the failure criterion is based on firm *illiquidity* as opposed to insolvency. Empirical evidence shows that SMEs face liquidity constraints that likely dominate solvency concerns during large liquidity shocks .[7] In such instances, promising (i.e. solvent) firms can

Dinlersoz et al., 2019, Gopinath et al., 2017

[5] As in Bresnahan and Raff (1991).

[6] This calculation is intended to approximate the end-of-year cash position of the firm.

[7] See Acharya and Steffen (2020).

fail along with weaker (i.e. insolvent) firms.

We use firm balance sheet and income statement data from Bureau van Djik's Orbis. We focus on SMEs in a sample of eleven European countries.[8] In the European Union, SMEs account for 99.8 percent of all employer firms, 59.4 percent of private sector employment, and 53.1 percent of gross output.[9] For each firm, we observe sales, labor and material costs, cash balances, and interest payments, which are used to estimate changes in cash flow. We also observe various metrics of firm health, such as labor productivity, revenue and employment growth, and leverage. Using the model and a sequence of shocks, for each firm we can estimate changes in cash flow and evaluate the failure criterion, as well as fully characterize and compare the labor productivity, growth, and leverage of failing versus surviving firms.

We first consider a "typical" year scenario for 2017-2019—years in which our sample of eleven European countries faced modest economic shocks. We combine country-specific aggregate and 1-digit sectoral shocks, calculated using Eurostat data, with the prior year's firm level Orbis data, and our model to predict firm failures.[10] The difference between our estimated failure rates and actual failure rates at the country-sector-year level is on average only 0.69 percentage points, which is less than 10 percent of the 8.94 percent average failure rate over the period. We also compare firms classified as failing versus classified as surviving on simple profitability and liquidity measures. Consistent with the predictions of both the empirical literature and modelling approaches where exit is based on solvency, we find that firms predicted to fail were less productive, grew slower, had less cash on hand, and were more leveraged than those predicted to survive.[11]

We then apply our framework to COVID-19, which was an unprecedented shock to a vast number of firms' cash flows. As such, it is precisely the type of situation where our framework can provide insight on the underlying sources of economic vulnerability and on the potential implications of various policy interventions. To model COVID-19, we assume that shocks hit at the end of February 2020 and the subsequent stringent social distancing period lasts 8 weeks.[12] During these 8 weeks, each sector in the economy is affected by four types of shocks: sector-specific demand shocks, reflecting changes in the pattern of spending away from social consumption; declines in overall spending due to precautionary savings and falls in income; productivity losses from shifting to remote work; and labor restrictions reflecting

[8]The countries in our sample are Czech Republic, Finland, France, Hungary, Italy, Poland, Portugal, Romania, Slovak Republic, Slovenia, and Spain.

[9]SME contribution to the economy is derived using Eurostat's Structural Business Statistics for the available set of sectors. Note that SMEs account for over 50 percent of output even when all the sectors of the economy are considered, as shown in Kalemli-Ozcan, Sorensen, Villegas-Sanchez, Volosovych and Yesiltas (2019).

[10]We calculate sectoral shocks at the 1-digit NACE level because it is the most disaggregated level at which sector data on sales and labor productivity are available for our sample of countries.

[11]See Clementi and Palazzo (2016), Foster et al. (2001) and Lee and Mukoyama (2015).

[12]This timing coincides with the lockdown period imposed in many of our sample countries.

lockdowns and workplace social distancing.[13] At the end of lockdown, sectoral supply shocks return to their pre-COVID levels, while aggregate demand evolves according to IMF quarterly projections and sector-specific demand reverts back to normal slowly.

To understand sources of vulnerability to the COVID-19 crisis, we first estimate failure rates absent government intervention. Under this baseline scenario, COVID would have raised overall SME failure rates by 6.01 percentage points (relative to a non-COVID 2020 scenario).[14] With excess failure rates above 19 percentage points, the most vulnerable sectors are Arts, Entertainment, & Recreation and Education. We find that most of the sectoral variation in failure rates results from large falls in sector-specific demand.[15] Vulnerability also varies considerably across countries. For instance, the excess failure rate in Romania is estimated at 2.37 percentage points (pp), 5.27 pp in France, and 10.30 pp in Italy. An important source of vulnerability in a country like Italy is that firms entered COVID with considerably lower cash balances and higher debt burdens than firms in other countries, like France, that faced similar shocks. Italian firms will therefore experience larger cash shortfalls than French firms in response to the same set of shocks. Because our financial friction limits the time firms have to recover cash deficits, Italian firms fail at a higher rate than French ones.

The baseline scenario highlights that many additional firms are at risk of failure due to COVID-19. Using firm level data, we investigate the characteristics of these firms. Specifically, we compare three groups of firms: "strong firms" that survive the baseline COVID-19 scenario; "weak firms" that would have failed even in the absence of COVID-19 (i.e. in the non-COVID scenario); and "viable firms" that only fail if COVID-19 occurs (i.e. survive the non-COVID scenario, but fail in the baseline COVID scenario). We find that "viable" firms are almost identical to "strong" firms in terms of past economic performance (labor productivity and past revenue growth). These "viable" firms are failing during COVID because they are cash poor and have high leverage, metrics on which they look very similar to "weak" firms.

The strong economic fundamentals of "viable" firms may provide a rationale for government support. Yet, policies with broad eligibility criteria risk channeling resources to "weak" firms. We evaluate the benefits and costs of various fiscal policies. Our benchmark is a hypothetical policy that bails out only "viable" firms. The policy costs 0.77 percent of GDP, lowers failure rates back to their non-COVID level, and helps preserve 3.1 percent of private sector employment. We compare this benchmark to several interventions that mimic polices implemented in practice, including interest, tax and rent rebates, cash grants, and government

[13] Labor restrictions and sector-specific demand shocks are measured by O*NET data on the ability to shift to remote work (following Dingel and Neiman (2020)), and reliance on face-to-face interactions respectively, and aggregate demand is assumed to follow quarterly IMF GDP growth projections made at the time.

[14] If firms cannot temporarily suspend operations or smooth through temporary within-2020 cash deficits, then we predict a 9.20 percentage point excess failure rate.

[15] Many of the sectors with high failure rates also face severe labor restrictions, but we find that their failure rates are similar if we shut down the labor restrictions channel in our model.

guaranteed loans (or pandemic loans).[16] We find that cash grants and pandemic loans provide the most relief, but are untargeted and costly. For example, the pandemic loan mobilizes 6.43 percent of GDP in government-guaranteed funding and saves 7.85 percent of firms and 4.02 percent of jobs, bringing failure rates *below* their non-COVID level.

Contrary to concerns that policies would disproportionately benefit "weak" firms, we find that both cash grants and pandemic loans primarily save "viable" firms, and are costly because they provided substantial funding to "strong" firms. Under the pandemic loan policy, for example, 4.92 percent of GDP (out of a total of 5.78 percent) is disbursed to "strong" firms while only 0.53 and 0.45 percent of GDP is channeled to "viable" and "weak" firms, respectively. Of the firms saved, 56 percent are "viable", while the remaining 44 percent are "weak". We also confirm that the saved "weak" firms tend to have lower labor productivity than saved "viable" firms, suggesting that in practice, policy prevents or delays the failure of some "weak", low-productivity firms.

We are related to several papers in the literature. Similar to Bornstein and Castillo-Martinez (2022), we emphasize financial frictions and liquidity shocks both at the firm and macro (sectoral) level, where macro (sectoral) shocks do not originate in the financial sector.[17] These authors have a general equilibrium model, where they add aggregate fluctuations to the influential framework of Cooley and Quadrini (2001), who introduce financial frictions at the firm level to the firm dynamics model of Hopenhayn (1992). Relative to these papers, our model is a simple partial equilibrium model that does not micro-found the financial friction. We combine our model with detailed firm level balance sheet data that helps us capture the importance of firm level financial frictions for firm exit under large liquidity shocks originating from aggregate and sectoral demand shocks. Our contributions are that with this framework we estimate SME failure rates at the firm, sector and country levels, and provide a characterization of the surviving and failing firms under typical year and crisis scenarios.

Moreover, our financial friction is consistent with the recent literature on earnings-based constraints, wherein firms hit by liquidity shocks have difficulty borrowing from the financial sector. Empirically, Lian and Ma (2020) show that over 80 percent of publicly listed firm debt in the U.S. is cash flow based. More importantly for us, as we focus on SMEs who are generally private companies, Caglio et al. (2021) show that earnings based constraints are even more important for SMEs in the United States. These firms tighten their financial constraint when there is a direct hit to their earnings. Ivashina, Laeven and Moral-Benito (2022) show that in

[16] According to OECD (2020) tax deferrals have been one of the most common policy support measures used by OECD governments and 22 OECD countries have implemented some form of rent deferral or waiver scheme. Cash grants and government guaranteed loans are also widely used. See ECB Economic Bulletin 6/2020 Focus

[17] Two other related papers are Arellano, Bai and Kehoe (2019) and Khan and Thomas (2013). The former studies the impact of firm uncertainty shocks on firm financing. The latter studies the effect of shocks originating in the financial intermediary sector. Our paper differs in that our liquidity shocks can also arise from the effect that negative demand shocks have on firms' cash flow, even in the absence of shocks to the financial sector.

Spain and Peru, cash flow loans drive the contraction during the Great Financial Crisis. On the theoretical front, Drechsel (forthcoming) shows that earnings-based constraints lead to larger business cycle amplification under shocks to cost of investment funding.

In terms of our COVID application, we also relate to several papers. Crane et al. (2022) study firm exit in the U.S. during COVID using alternative measures of exit because official measures are only available several with a few years of lag. Autor, Cho, Crane, Goldar, Lutz, Montes, Peterman, Ratner and Villar Vallenas (2022), study the empirical effects of "The Paycheck Protection Program (PPP)" in the U.S., which provided small businesses with roughly $800 billion dollars in uncollateralized, low-interest loans during the pandemic, almost all of which will be forgiven. Their result that the untargeted program ended up being highly regressive is consistent with our findings for European SMEs. Bartik, Cullen, Glaeser, Luca, Stanton and Sunderam (2020) also studies the same program with a model that justifies government support based on operational delays in bank funding as the financial friction.

Our paper is structured as follows. Section 2 presents model. Section 3 introduces the Bureau van Djik Orbis firm level data for eleven European countries where we have good coverage of SMEs and reliable official data. Section 4 shows how well our framework approximates official firm failure rates in non-crisis years. Our COVID-19 application in Section 5, evaluates firm, sectoral and country vulnerability to the crisis, and assesses the cost and impact of various fiscal support measures. Section 6 presents robustness. Section 7 concludes.

2 The Model

In this section we introduce a tractable model that can be combined with firm level data to investigate the effects of liquidity shocks on firms. The model allows for a rich set of sectoral and aggregate demand and supply shocks, which can impact firm liquidity through their impact on cash flow. We focus here on the first-round, partial equilibrium effects of these shocks, emphasizing their impact on firm failure.

For each firm, we start off with economic conditions in a benchmark year, which will be informed by a large firm level dataset. Then we introduce a rich set of shocks, which are expressed as perturbations in economic conditions relative to the benchmark year. The set of shocks allow the modeler to capture a wide variety of scenarios and policy counterfactuals. In the model, firms solve a cost-minimization problem, subject to these shocks. Their optimal decisions are expressed as (non-linear) deviations from their decisions in the benchmark year.

2.1 Supply

The economy consists of $\mathcal{S}$ sectors. In each sector $s \in \mathcal{S}$ there is a mass $\mathcal{N}_s$ of firms, indexed by i. We take the initial mass of firms in each sector as given. We assume that each firm i in sector s produces according to the following sector-specific production function:

$$y_{is} = z_{is} f_s(k_{is}, A_s n_{is}, m_{is}) \tag{1}$$

In Eq. (1), y_{is} denotes gross output, k_{is} represents any fixed factor, including capital, entrepreneurial talent etc.., n_{is} is a labor input, and m_{is} denotes other variable inputs such as materials or intermediate inputs. A_s is a sector-specific labor-augmenting productivity, so that $A_s n_{is}$ is the effective labor supply in firm i, while z_{is} is a firm-specific productivity. Because our analysis is essentially static, we ignore time subscripts. We assume that, regardless of fixed factors, firms need both labor and intermediate goods to produce, so that $f_s(.,0,.) = f_s(.,.,0) = 0$.

We denote p_{is} as the price of output of firm i in sector s, w_s the wage rate per effective unit of labor, r_s the user cost for fixed factors and p_{ms} the price of other variable inputs. Factor prices only vary at the sector level. Prices, both for factors and output, are assumed constant in the short run, perhaps because of nominal rigidities.[18]

Some shocks can impose short run constraints on firms' production sets either in terms of input combinations available or in terms of productivity (A_s). For instance, in a natural disaster, some materials may be rationed, or, as occurred during the COVID-19 pandemic, firms may be forced to reduce the size of their labor force due to health-mandated lockdowns. We model this constraint at the firm level as follows:

$$h_{is}\left(n_{is}, m_{is}\right) \leq 0 \tag{2}$$

where we assume that the constraint $h_{is}(.,.)$ satisfies regularity conditions such that the problem of the firm is well-defined and convex.

2.2 Demand

Each firm within a given sector sells a differentiated variety. We assume that total demand has a nested-CES structure of the form:

$$D = \left[\sum_s \mathcal{N}_s \xi_s D_s^{(\eta-1)/\eta}\right]^{\eta/(\eta-1)} \tag{3}$$

[18] See Gourinchas, Kalemli-Ozcan, Penciakova and Sander (2021b) for the case of fully-flexible prices.

In Eq. (3), D denotes aggregate (real) demand, D_s is sectoral (real) demand, ξ_s is a sectoral demand shock, and η is the elasticity of substitution between sectors. For simplicity, we assume that sectors are initially symmetric, and set $\mathcal{N}_s \xi_s = 1, \forall s$. We also denote with a "prime" the value of variables in the scenario under consideration, so that ξ_s is the is the unobserved value of the sectoral demand in sector s in the benchmark year and ξ_s' is the new value in the scenario under consideration, with $\xi_s' < \xi_s$ when demand for sector s falls and $\xi_s' > \xi_s$ when it increases.

In turn, sectoral demand D_s satisfies:

$$D_s = \left(\frac{1}{\mathcal{N}_s} \int_0^{\mathcal{N}_s} d_{is}^{(\rho_s - 1)/\rho_s} \mathrm{d}i \right)^{\rho_s/(\rho_s - 1)} \tag{4}$$

where ρ_s is the sector-specific elasticity of substitution between varieties.

From Eqs. (3) and (4), the demand for variety i in sector s is given by:

$$d_{is} = \xi_s^{\eta} \left(\frac{p_{is}}{P_s} \right)^{-\rho_s} \left(\frac{P_s}{P} \right)^{-\eta} D, \tag{5}$$

where P_s denotes the average sectoral price index per unit of expenditure, and P the overall price level. They satisfy:[19]

$$P_s = \left(\frac{1}{\mathcal{N}_s} \int_0^{\mathcal{N}_s} p_{is}^{1-\rho_s} \mathrm{d}i \right)^{1/(1-\rho_s)} \quad ; \quad P = \left(\sum_s \xi_s^{\eta} \mathcal{N}_s P_s^{1-\eta} \right)^{1/(1-\eta)} \tag{6}$$

Because we assume that the price of individual varieties p_{is} and the mass of firms $\mathcal{N}_s$ are constant, sectoral price indices P_s given in Eq. (6) are also constant. The aggregate price index P, however, can change because of the demand shifters ξ_s.

We denote with a "hat" the ratio of variables relative to the benchmark period, e.g. $\hat{\xi}_s \equiv \xi_s'/\xi_s$. From Eq. (5), we can use hat algebra to express the change in demand relative to a benchmark period as:

$$\hat{d}_{is} = \hat{\xi}_s^{\eta} \hat{P}^{\eta - 1} \widehat{PD} \tag{7}$$

Under the assumption that the equilibrium is symmetric in the benchmark period, $P_s \mathcal{N}_s =$

[19] P_s is a sectoral price index per unit of expenditure. The usual Fischer-ideal price index is given by $\mathcal{N}_s P_s$ and aggregate expenditure equals $\sum_s \mathcal{N}_s P_s D_s$.

$P\mathcal{S}^{1/(\eta-1)}$, we can write:

$$\hat{P}^{\eta-1} = \left(\frac{P'}{P}\right)^{\eta-1} = \left(\frac{\sum_s \hat{\xi}_s^{\eta}(P_s\mathcal{N}_s)^{1-\eta}}{P^{1-\eta}}\right)^{-1} = \left(\frac{1}{\mathcal{S}}\sum_s \hat{\xi}_s^{\eta}\right)^{-1}$$

Putting the two previous equations together, we obtain the following expression for the change in demand relative to a benchmark period:

$$\hat{d}_{is} = \frac{\hat{\xi}_s^{\eta}}{\sum_\sigma \hat{\xi}_\sigma^{\eta}/\mathcal{S}}\widehat{PD} \tag{8}$$

Eq. (8) indicates that the total change in sectoral demand is a function of two drivers: a relative and an aggregate one. First, sectoral demand shocks ($\hat{\xi}_s$) reallocate a given level of aggregate expenditure *across* sectors. It is the relative pattern of sectoral demand shocks that matters, not their absolute level. For instance, suppose there is no change in aggregate demand so $\widehat{PD} = 1$ and the economy consists of two sectors with $\hat{\xi}_s < \hat{\xi}_{s'}$, then $\hat{d}_s < 1 < \hat{d}_{s'}$—one sector is in recession, and the other is in a boom. The elasticity of substitution across sectors η modulates the intensity of the sectoral demand shocks ($\hat{\xi}_s$). When goods are very substitutable (high η), small sectoral demand shocks lead to large demand responses. Conversely, when demand is very inelastic (low η) demand responses become more similar across sectors (in the limit of $\eta = 0$, we obtain $\hat{d}_{is} = \widehat{PD}$). Second, for a given pattern of sectoral demand shocks, all sectors respond proportionately to changes in aggregate demand. For instance, if all sectors are affected uniformly so that $\hat{\xi}_s = \hat{\xi}, \forall s$, then Eq. (8) indicates that total demand in all sectors is affected uniformly with $\hat{d}_{is} = \widehat{PD}$.

Define $\tilde{\xi}_s^{\eta} \equiv \hat{\xi}_s^{\eta}/(\sum_\sigma \hat{\xi}_\sigma^{\eta}/\mathcal{S})$. $\tilde{\xi}_s^{\eta}$ summarizes the impact of sector-specific demand shocks on total demand and satisfies $\sum_s \tilde{\xi}_s^{\eta}/\mathcal{S} = 1$. With this notation, each firm i in sector s experiences the same proportional change in demand relative to a benchmark period, given by:

$$\hat{d}_s = \tilde{\xi}_s^{\eta}\widehat{PD} \tag{9}$$

2.3 The Firm's Cost Minimization Problem

We evaluate scenarios over a short horizon. Consequently, we assume that the prices of goods and factors are taken as given and firms meet the demand they face. We further assume that labor cannot reallocate across firms or sectors in the short run, so workers who cannot work for their original place of employment are laid off.

Each firm minimizes variable costs by solving the following problem:

$$\min_{m'_{is},n'_{is}} \quad w_s n'_{is} + p_{ms} m'_{is} \tag{10}$$
$$z_{is} f(k_{is}, A'_s n'_{is}, m'_{is}) \geq d'_{is}$$
$$h_{is}(n'_{is}, m'_{is}) \leq 0$$

where the level of demand d'_{is} is given by Eq. (5).

We specialize the problem further by assuming that the production function $f_s(.)$ is Cobb-Douglas:

$$y_{is} = z_{is} k_{is}^{\alpha_s} (A_s n_{is})^{\beta_s} m_{is}^{\gamma_s} \tag{11}$$

where the (sector-specific) exponents α_s, β_s and γ_s sum to one.[20]

We also specialize the supply constraint as follows:

$$h_{is}\left(n'_{is}, m'_{is}\right) = \omega_s(n'_{is} - x_{ns} n_{is}) + (1 - \omega_s)(m'_{is} - x_{ms} m_{is}) \leq 0 \tag{12}$$

In this expression, n_{is} and m_{is} denote the level of employment and materials before the shocks, x_{ns} and x_{ms} denote the tightness of the labor and intermediate input constraint, defined at the sector level, while ω_s captures the relative importance of the labor and intermediate input constraints for firms in sector s. To illustrate, Eq. (12) can capture a situation in which firms in some sectors can only employ a fraction x_{ns} of their benchmark year employment level (n_{is}) by setting $\omega_s = 1$. In that case the constraint becomes $n'_{is} \leq x_{ns} n_{is}$. Alternatively, Eq. (12) can capture a situation in which firms in some sectors face supply-chain constraints and can only purchase a fraction x_{ms} of their benchmark period materials (m_{is}) by setting $\omega_s = 0$. In that case, the constrained becomes $m'_{is} \leq x_{ms} m_{is}$. Policymakers may have information on which sectors are constrained and on what inputs (i.e. data on ω_s, x_{ns} and x_{ms}). We assume that the supply constraint can bind only on one of the factors for any given sector, that is $\omega_s \in \{0, 1\}$.

It follows that we have three cases to consider: (a) when the supply constraint doesn't bind; (b) when it binds on labor and (c) when it binds on materials.

[20]Because we assume that capital k_{is} is fixed, the relevant part of this assumption is that production exhibits decreasing returns to labor and intermediate jointly, i.e. $\beta_s + \gamma_s < 1$.

2.3.1 When Supply is Not Constrained

If the supply constraint Eq. (12) does not bind, we can solve the above program for labor and materials demand. Manipulating the first-order conditions yields:

$$\hat{m}_{is} = \hat{n}_{is} = \hat{d}_{is}^{1/(\beta_s+\gamma_s)} \hat{A}_s^{-\beta_s/(\beta_s+\gamma_s)} = \left(\tilde{\xi}_s^{\eta} \widehat{PD}\right)^{1/(\beta_s+\gamma_s)} \hat{A}_s^{-\beta_s/(\beta_s+\gamma_s)} \equiv \hat{x}_s^c \tag{13}$$

Intermediate input and labor demand increase with output demand ($\tilde{\xi}_s^{\eta}\widehat{PD}$) and decrease with productivity ($\hat{A}_s$). This solution obtains as long as $\hat{n}_{is} \leq \hat{x}_{ns}$ and $\hat{m}_{is} \leq \hat{x}_{ms}$. Because only one of the supply constraints in Eq. (12) binds for any firms in any sector, inputs are unconstrained as long as:

$$\hat{x}_s^c \leq \hat{x}_s \equiv \omega_s \hat{x}_{ns} + (1-\omega_s)\hat{x}_{ms} \tag{14}$$

We can rewrite Eq. (13) and impose Eq. (14) to get the following expression:

$$\hat{x}_s^{(\beta_s+\gamma_s)} \hat{A}_s^{\beta_s} \geq \tilde{\xi}_s^{\eta} \widehat{PD} \tag{15}$$

This equation shows how supply and demand conditions help inform whether a firm is supply constrained. The left hand side of this expression captures the *supply side* of the model—the supply constraint, as well as the productivity shock. The exponent on the supply shocks is $\beta_s + \gamma_s < 1$ because adjustment in one variable input also forces an adjustment in the other one, with a total exponent $\beta_s + \gamma_s$. The right hand side captures the *demand side* of the model—the change in demand coming from sectoral or aggregate demand shifts. The inequality tells us for which firms the demand or supply side is the binding factor—demand constrains output and input use if the demand terms are lower than the supply terms, while supply constraints bind in the opposite case. Because all the variables in this expression are defined at the sectoral level, the threshold for binding supply vs. demand factors is also defined at the sectoral level.

Variable profits for unconstrained firms can be expressed as:

$$\pi'_{is} \equiv pd'_{is} - wn'_{is} - p_m m'_{is} = pd_{is}\left(\tilde{\xi}_s^{\eta}\widehat{PD} - (s_{ni} + s_{mi})\hat{x}_s^c\right) \tag{16}$$

where $s_{ni} = wn_{is}/pd_{is}$ and $s_{mi} = p_m m_{is}/pd_{is}$ denote respectively the firm's wage and material bills as a share of revenue in the period prior to the shock.[21]

[21] If the firm is behaving competitively and optimizing over its level of output prior to the shocks, $s_{ni} = \beta_s$ and $s_{mi} = \gamma_s$, but we don't need to impose these conditions. The firm may have market power or be demand determined prior. Our framework only imposes cost-minimization during the scenario under consideration.

2.3.2 When Labor Input is Constrained

Labor is constrained when $\omega_s = 1$ and $\hat{x}_s \equiv \hat{x}_{ns} < \hat{x}_s^c$. By manipulating the first-order conditions, we obtain:

$$\hat{n}_{is} = \hat{x}_{ns} \quad ; \quad \hat{m}_{is} = \left(\tilde{\xi}_s^{\eta}\widehat{PD}\right)^{1/\gamma_s} (\hat{A}_s \hat{x}_{ns})^{-\beta_s/\gamma_s} = \hat{x}_{ns}^{-\beta_s/\gamma_s} \hat{x}_s^{c(\beta_s+\gamma_s)/\gamma_s} > \hat{x}_{ns} \tag{17}$$

Compared to the unconstrained case, a binding labor supply reduces labor input and increases the use of materials. The lower is the output elasticity of materials γ_s, the stronger the response of materials when labor is constrained.

In the case of a constrained firm, variable profits are given by:

$$\pi'_{is} = pd_{is} \left(\tilde{\xi}_s^{\eta}\widehat{PD} - \hat{x}_s^c \left(s_{ni}\left(\frac{\hat{x}_{ns}}{\hat{x}_s^c}\right) + s_{mi}\left(\frac{\hat{x}_{ns}}{\hat{x}_s^c}\right)^{-\beta_s/\gamma_s}\right)\right) \tag{18}$$

Comparing this expression to Eq. (16), when labor is unconstrained, we observe that the lower use of labor tends to increase variable profits (the term $s_{ni}\hat{x}_s/\hat{x}_s^c$ decreases because $\hat{x}_{ns} < \hat{x}_s^c$), while the extra reliance on materials tends to lower profits (the term $s_{mi}(\hat{x}_{ns}/\hat{x}_s^c)^{-\beta_s/\gamma_s}$ increases). On net, and at unchanged demand, variable costs must increase when the firm is constrained. The increase in material costs is larger for firms in sectors with a relatively low output elasticity of materials (low γ_s) and a high output elasticity of labor (high β_s).

2.3.3 When Materials Input is Constrained

The case of constrained materials is entirely symmetric and described here for completeness. This case arises when $\omega = 0$ and $\hat{x}_s \equiv \hat{x}_{ms} < \hat{x}_s^c$. In that case:

$$\hat{m}_{is} = \hat{x}_{ms} \quad ; \quad \hat{n}_{is} = \left(\tilde{\xi}_s^{\eta}\widehat{PD}\right)^{1/\beta_s} (\hat{A}_s \hat{x}_{ms})^{-\gamma_s/\beta_s} = \hat{x}_{ms}^{-\gamma_s/\beta_s} \hat{x}_s^{c(\beta_s+\gamma_s)/\beta_s} > \hat{x}_{ms} \tag{19}$$

while variable profits are given by:

$$\pi'_{is} = pd_{is} \left(\tilde{\xi}_s^{\eta}\widehat{PD} - \hat{x}_s^c \left(s_{ni}\left(\frac{\hat{x}_{ms}}{\hat{x}_s^c}\right)^{-\gamma_s/\beta_s} + s_{mi}\left(\frac{\hat{x}_{ms}}{\hat{x}_s^c}\right)\right)\right) \tag{20}$$

2.4 Temporary Business Shutdowns—"Mothballing"

In the case where production costs are excessive, we allow firms to prevent large falls in their cash flows by allowing them to shut down temporarily (i.e. mothballing their operations, see

Bresnahan and Raff (1991)). In that case, $y_{is} = n_{is} = m_{is} = \pi_{is} = 0$. While the firm still has to cover its fixed costs and financial expenses, this option is particularly relevant for firms that face severe supply constraints—either on labor or materials—that would force them to substitute—at excessively high cost—with the other available inputs. Formally a firm will choose to mothball if its variable profits are negative:

$$\pi'_{is} < 0 \Leftrightarrow \begin{cases} \tilde{\xi}_s^{\eta}\widehat{PD} < \hat{x}_s^c \left(s_{ni}\left(\frac{\hat{x}_{ns}}{\hat{x}_s^c}\right) + s_{mi}\left(\frac{\hat{x}_{ns}}{\hat{x}_s^c}\right)^{-\beta_s/\gamma_s}\right) & \text{if } \omega_s{=}1 \text{ and } \hat{x}_s^c{>}\hat{x}_{ns} \text{ (Labor Constrained)} \\ \tilde{\xi}_s^{\eta}\widehat{PD} < \hat{x}_s^c \left(s_{ni}\left(\frac{\hat{x}_{ms}}{\hat{x}_s^c}\right)^{-\gamma_s/\beta_s} + s_{mi}\left(\frac{\hat{x}_{ms}}{\hat{x}_s^c}\right)\right) & \text{if } \omega_s{=}0 \text{ and } \hat{x}_s^c{>}\hat{x}_{ms} \text{ (Materials Constrained)} \\ \tilde{\xi}_s^{\eta}\widehat{PD} < (s_{ni}+s_{mi})\hat{x}_s^c & \text{if } \hat{x}_s^c{\le}\hat{x}_s \text{ (unconstrained)} \end{cases} \tag{21}$$

where, as above, $\hat{x}_s = \omega_s \hat{x}_{ns} + (1-\omega_s)\hat{x}_{ms}$.

For constrained firms, direct inspection of Eq. (21), reveals that mothballing is more likely when labor supply is constrained ($\omega_s = 1$) and firms have a low materials output elasticity γ_s relative to the labor output elasticity β_s, or conversely, when material supply is constrained and firms have a low labor output elasticity relative to the materials output elasticity.

For unconstrained firms we can substitute $\hat{x}_s^c$ using Eq. (13) to get the following expression in terms of shocks:

$$\hat{A}_s^{\beta_s} < (s_{ni}+s_{mi})^{\beta_s+\gamma_s}\left(\tilde{\xi}_s^{\eta}\widehat{PD}\right)^{1-\beta_s-\gamma_s} \tag{22}$$

Eq. (22) shows that when $\beta_s + \gamma_s < 1$ (i.e. there are diminishing returns to variable inputs), unconstrained firms will shut down when total demand ($\tilde{\xi}_s^{\eta}\widehat{PD}$) is excessively *high* or productivity is low.

2.5 Evaluating Business Failures

To evaluate business failure, we assume that firms follow a simple decision rule—they remain in business as long as their initial cash balances and operating cash flow over a given assessment period are sufficient to cover their financial expenses. Otherwise, they fail.

In the remainder of this section, we formalize this liquidity based failure criterion. We begin by showing how to link the expressions for variable profits in the scenario under consideration (π'_{is}) to firm cash flow in that scenario (CF'_{is}). We then discuss how to use firm cash flow to evaluate whether a firm is illiquid. In the process, we also address two complications. First, how to deal with important missing variables in typical balance sheet data—such as fixed costs or taxes. Second, how to apply our framework in a mixed frequency context—such as when firm balance sheet data is available at an annual frequency, but shocks are measurable at a higher frequency.

We start by defining operating cash flow in some period t as:

$$CF_{is,t} \equiv pd_{is,t} - wn_{is,t} - pm_{is,t} - F_{is,t} - T_{is,t} = \pi_{is,t} - F_{is,t} - T_{is,t} \tag{23}$$

where the $pd_{is,t}$ represents revenues, $wn_{is,t}$ represents wages, and $pm_{is,t}$ the intermediate input bill. $F_{is,t}$ represents any costs associated with fixed factors (rent, utilities, etc.), including capital costs ($r_{s,t}k_{i,s,t}$), and $T_{is,t}$ denotes business taxes. The last expression writes operating cash flow in terms of the variable profits ($\pi_{is,t}$), minus payments to fixed factors and taxes.

Cash flow in the scenario under consideration is therefore:

$$CF'_{is} = \pi'_{is} - F'_{is} - T'_{is}$$

As long as fixed costs and taxes are unchanged between the benchmark year and the scenario under consideration ($F'_{is} = F_{is}$ and $T'_{is} = T_{is}$), we can difference them out by considering the change in cash flows from CF to CF' (i.e. from the observed to the predicted cash flows).[22] An advantage of this approach is that it does not require information on fixed cost or taxes in the benchmark year, which may not always be available in balance sheet data.

$$\begin{aligned} CF'_{is} &= \pi'_{is} - F_{is} - T_{is} \\ &= CF_{is} + (\pi'_{is} - \pi_{is}) \end{aligned} \tag{24}$$

The predicted cash flow (CF'_{is}) is then obtained by substituting our estimated variable profits π'_{is} using Eqs. (16), (18) or (20), depending on whether the firm is unconstrained, labor constrained or material constrained.

When implementing the framework, it may happen that balance sheet data is available at one frequency (e.g. annual) and shocks at another (e.g. weekly). To account for this, we let the time period t be denoted by a touple $t = (y, \tau)$ where $y \in \mathcal{Y} \equiv \{y_1, \ldots, y_n\}$ denotes years and $\tau \in \mathcal{T} \equiv \{1, \ldots, \bar{T}\}$ denotes subperiods within each year (e.g. weeks, months, quarters). In the general case, the cash flow condition becomes:

$$CF'_{is,y,\tau} = \frac{CF_{is,y_0}}{\bar{T}} + \left(\pi'_{is,y,\tau} - \frac{\pi_{is,y_0}}{\bar{T}}\right) \tag{25}$$

[22] For short horizons such as one year, this is likely a reasonable assumption. Rental contracts often fix rent for several years, and many business taxes are paid in the following calendar year. Therefore, from a liquidity perspective the taxes a business needs to pay in year t are likely determined in year $t-1$ and will not change if an unexpected shock occurs in year t.

where CF_{is,y_0} and π_{is,y_0} represent annual cash flow and profits from the benchmark year, respectively; and $\pi'_{is,y,\tau}$ represents profits in the scenario under consideration in year y and subperiod τ, given by:

$$\pi'_{is,y,\tau} = \begin{cases} \frac{pd_{is,y_0}}{\bar{T}} \left(\tilde{\xi}^{\eta}_{s,y,\tau} \widehat{PD}_{y,\tau} - \hat{x}^c_{s,y,\tau} \left(s_{ni,y_0} \left(\frac{\hat{x}_{ns,y,\tau}}{\hat{x}^c_{s,y,\tau}} \right) + s_{mi,y_0} \left(\frac{\hat{x}_{ns,y,\tau}}{\hat{x}^c_{s,y,\tau}} \right)^{-\frac{\beta_s}{\gamma_s}} \right) \right) & \text{if } \omega_{s,y,\tau}=1 \text{ and } \hat{x}^c_{s,y,\tau} > \hat{x}_{ns,y,\tau} \text{ (Labor Constrained)} \\ \frac{pd_{is,y_0}}{\bar{T}} \left(\tilde{\xi}^{\eta}_{s,y,\tau} \widehat{PD}_{y,\tau} - \hat{x}^c_{s,y,\tau} \left(s_{ni,y_0} \left(\frac{\hat{x}_{ms,y,\tau}}{\hat{x}^c_{s,y,\tau}} \right)^{-\frac{\gamma_s}{\beta_s}} + s_{mi,y_0} \left(\frac{\hat{x}_{ms,y,\tau}}{\hat{x}^c_{s,y,\tau}} \right) \right) \right) & \text{if } \omega_{s,y,\tau}=0 \text{ and } \hat{x}^c_{s,y,\tau} > \hat{x}_{ms,y,\tau} \text{ (Materials Constrained)} \\ \frac{pd_{is,y_0}}{\bar{T}} \left(\tilde{\xi}^{\eta}_{s,y,\tau} \widehat{PD} - (s_{ni,y_0} + s_{mi,y_0}) \hat{x}^c_{s,y,\tau} \right) & \text{if } \hat{x}^c_{s,y,\tau} \leq \hat{x}_{s,y,\tau} \text{ (unconstrained)} \end{cases} \tag{26}$$

Next, denote initial (benchmark year) cash balances $\mathcal{Z}_{is,y_0}$ and annual financial expenses, defined as interest payments due on the firms' debt, $\iota L_{is,y_0}$. Let $\mathcal{T}_i \subset \mathcal{T}$ denote the subperiods within the year when interest payments are due. Then, define the cash position in each period $t = (y, \tau)$ of the scenario as:

$$\mathcal{Z}_{is,y,\tau} = \mathcal{Z}_{is,0} + \sum_{y'<y} \sum_{\tau' \leq \bar{T}} \left(CF'_{is,y',\tau'} - \iota L_{is,y_0} \frac{1}{|\mathcal{T}_i|} \mathbb{1}_{\tau' \in \mathcal{T}_i} \right) + \sum_{\tau' \leq \tau} \left(CF'_{is,y,\tau'} - \iota L_{is,y_0} \frac{1}{|\mathcal{T}_i|} \mathbb{1}_{\tau' \in \mathcal{T}_i} \right) \tag{27}$$

where $|\mathcal{T}_i|$ represents the size of the set $\mathcal{T}_i$. Note that allowing for the set $\mathcal{T}_i$ to differ from $\mathcal{T}$ allows for interest payments to occur at a lower frequency than shocks.

Finally, let $\mathcal{F} = (Y_f, T_f) \subset (\mathcal{Y}, \mathcal{T})$ denote a set of "assessment periods" where firm failures are assessed. Firms survive if:

$$\mathcal{Z}_{is,y,\tau} \geq 0, \quad \forall (y, \tau) \in \mathcal{F} \tag{28}$$

and fail otherwise.

Note that in all of our scenarios, we consider a single year comprised of 52 weeks (i.e. $y_n = 1$ and $\bar{T} = 52$). It is worth noting that how the assessment period is defined matters. For example, if a weekly assessment period is chosen ($\mathcal{F} = \{(1, \tau)\}_{\tau=1}^{52}$), then we would require firms to have positive cash balance in every week in order to survive. In contrast, if an annual assessment period is chosen ($\mathcal{F} = (1, 52)$), firms can experience periods of illiquidity within the year and only fail if they cannot cover their financial expenses at the end of the year.

In our baseline implementation, we consider an annual assessment period. A weekly assessment period would impose an excessively strict failure condition on businesses, preventing them from engaging in common strategies such as delaying the payment of receivables, running down input inventories or accessing very short term debt or credit lines, and even temporarily halting operations. An annual assessment period captures the ability of SMEs to take advantage of these options without explicitly modeling them, while also capturing that they cannot be used indefinitely.

Two caveats are worth noting regarding our failure criterion. First, while Eq. (28) has the advantage of simplicity, it assume that firms with a cash flow shortfall in any assessment period $(y, \tau) \in \mathcal{F}$ cannot access credit markets to borrow new funds. Note however, that this criterion also assumes that existing debt levels can be maintained, but that firms are constrained in each assessment period $(y, \tau) \in \mathcal{F}$ when it comes to obtaining *additional* funds. This is not unrealistic for SMEs as shown in Caglio et al. (2021). Selecting a sparse set of assessment periods $\mathcal{F}$ (for instance, if $y_n = 1$ and $\bar{T} = 52$ and $\mathcal{F} = (1, 52)$) implicitly allows the firm to smooth cash flow shortfalls between assessment periods while also imposing that the ability to smooth cash flow (via perhaps unused credit lines) is temporary.

A second caveat is that we ignore the role of bankruptcy courts. In theory, as long as a business remains viable, the failure to repay creditors in the short run does not mean that it ceases to operate. Instead, business liabilities should optimally be restructured under bankruptcy proceedings. In practice, however, there is substantial variation in bankruptcy regimes across countries. In the U.S. for example, there is automatic stay and lenders lend based on future cash flow during the restructuring process. However, this is mostly for the larger corporations, but it is less well suited for SMEs. Moreover, bankruptcy courts in many countries may not be able to efficiently preserve viable businesses in the middle of a large downturn if a wave of small business failures congests the courts.

3 Taking the Model to the Data

To bring the model to the data, we construct empirical counterparts to the sector-specific ($\tilde{\xi}^{\eta}_{s,y,\tau}$) and aggregate ($\widehat{PD}_{y,\tau}$) demand shocks, and sectoral supply ($\{\hat{x}_{ns,y,\tau}, \hat{x}_{ms,y,\tau}, \omega_{s,y,\tau}\}$) and productivity ($\hat{A}_{s,y,\tau}$) shocks.[23] We also estimate sector-specific output elasticities ($\beta_{s,y,\tau}, \gamma_{s,y,\tau}$). Together with benchmark year, firm level factor shares ($s_{n,is,y_0}, s_{m,is,y_0}$) and sales ($p_{is,y_0} d_{is,y_0}$), we construct a counterfactual change in cash flow. With data on the firm's cash balances ($\mathcal{Z}_{is,y_0}$), financial expenses ($\iota L_{is,y_0}$) and cash flow (CF_{is,y_0}) in the benchmark year, we then evaluate Eq. (28) to determine which businesses fail.

Because the construction of shocks varies based on the specific application, we defer the details of shock construction to the sections describing each application. Our source for the firm level data is common to all applications.

[23] Note that because we directly assess the change in sectoral demand according to Eq. (8), and not the underlying shock to preferences $\tilde{\xi}_{s,y,\tau}$, we do not need to make an assumption about the elasticity of substitution η. This is already encoded in our measure of $\tilde{\xi}^{\eta}_{s,y,\tau}$.

Firm Level Data: We use Orbis, a firm level data set from BvD-Moody's that covers both private and publicly listed firms. Orbis data are collected by BvD from various sources, including national business registries, and are harmonized into an internationally comparable format. The Orbis database covers more than 200 countries and over 200 million firms. The longitudinal dimension and representativeness of Orbis data vary from country to country, depending on which firms are required to file information with business registries.

In our analysis, we focus on a set of eleven countries. The countries included are Czech Republic, Finland, France, Hungary, Italy, Poland, Portugal, Romania, Slovak Republic, Slovenia and Spain. As described in Table A.2 in the appendix, we have good coverage of aggregate revenues for the countries in our sample, both for all firms and SMEs.

We evaluate SME failures because these firms account of a large fraction of economic activity and are particularly vulnerable to liquidity shocks . Across our sample of countries, SMEs account for 62.68 percent of employment, 61.34 percent of payroll, 65.52 percent of revenue, and 65.90 percent of total assets.[24] These SMEs are especially exposed to liquidity shocks because they tend to have lower cash balances, be bank-dependent, and have limited ability to draw on credit lines.

We use data on firm revenue, wage bill, material cost, number of employees, net income, depreciation, cash balance and financial expenses.[25] Cash flow is calculated as the sum of net income and depreciation, less financial profits. The analysis focuses on non-financial SMEs.[26]

Estimating Output Elasticities: We also use Orbis data to estimate labor and material elasticities (β_s and γ_s) at the 2-digit NACE level for each country. Taking into account our modeling assumption that labor and intermediate inputs are variable inputs, and recent critiques of the key identifying assumptions of popular production function estimation techniques, we estimate elasticities as the weighted average of the firm revenue share of input expenditures (e.g., labor cost share of revenue and material cost share of revenue), where the weights are

[24] SME shares are based on the cleaned Orbis data used in analysis. Aggregation is done over our sample of countries. The SME shares are first calculated at the country level and aggregated across countries using country GDP for weighting. The contribution of SMEs to the aggregate economy in the official data mimics the numbers here based on Orbis, as shown in detail in Kalemli-Ozcan et al. (2019).

[25] We winsorize all of the level variables used for analysis at the 99.9th percentile. Note also that, in principle, inital cash balances $\mathcal{Z}_{is}$ could include overdraft facilities or undrawn credit lines. Unfortunately, Orbis does not contain any information on these so we use intial cash balances for $\mathcal{Z}_{is}$ and present several exercises where we allow for firms to access additional funds until the end of the year.

[26] Additional data construction details: we focus on firms in NACE 1-digit sectors A, B, C, D, E, F, G, H, I, J, L, M, N, P, Q, R, S. We impose standard cleaning steps that check the internal consistency of balance sheet data. We exclude firms that do not report on the line items needed in order to calculate total assets and total liabilities. We exclude financial and insurance activities (K), public administration and defense (O), activities of households as employers (T), and activities of extraterrestrial organizations and bodies (U). We also exclude sub-sectors 78 and 81 in the Administration (N) because they have very large labor cost shares which together with our labor constraint generates unrealistically high failure rates and cash shortfalls. We exclude companies owned by public authorities and firms that were previously recorded by Orbis as bankrupt, dissolved, or illiquid.

given by firm revenue.[27] Due to the lack of price data, the elasticities we estimate are revenue, rather than output, elasticities. The mean and standard deviation of the labor and material elasticities are reported in Table A.1.

4 Applying the Framework to Typical Years

With our first application, we show that our framework produces failure rates in line with the observed data, and that the characteristics of failing firms are consistent with findings in the literature. Specifically, we consider a scenario that is equivalent to a one year ahead forecast of firm failure rates in our sample of countries, wherein we define benchmark years as 2016-2018 and predict firm failures in 2017-2019. We refer to these scenarios as a "typical year" scenarios. We then compare our predicted failure rates to those obtained from official sources, and compare the characteristics of failing firms to those emphasized in the existing literature.

Calibrating Shocks: We calibrate shocks using data from OECD.Stat and Eurostat to measure the perturbations in economic conditions around each benchmark year. This means, for example, that when we forecast firm failures in 2017, we measure shocks as changes in economic conditions between 2016 and 2017. All sectoral shocks are measured at the one-digit NACE level, which is the finest level of granularity for which data are consistently available across sectors in our sample of countries.

Fig. A.1 depicts the average total demand and sectoral productivity shocks across countries between 2017 and 2019. The total demand shock is composed of aggregate demand and sector-specific demand shocks. The aggregate demand shock ($\widehat{PD}$) is measured as the cumulative quarterly change in real GDP in each country. The sector-specfic demand shock ($\tilde{\xi}^{\eta}_s$) is constructed by first obtaining annual sectoral revenue growth for each country, and then normalizing the revenue growth to be consistent with aggregate demand, Eq. (8), by constructing $\tilde{\xi}^{\eta}_s = \hat{\xi}^{\eta}_s / (\sum_\sigma \hat{\xi}^{\eta}_\sigma / \mathcal{S})$. The sectoral productivity shock ($\hat{A}_s$) is measured as the annual growth in output per worker for each country. Finally, we assume the input constraints are inactive ($\hat{x}_{ns} = \hat{x}_{ms} = \infty$) because there were no notable supply bottlenecks or labor market disruptions in our set of countries during the years under consideration. Note that we abstract from firm level, idiosyncratic shocks.

Forecasting Firm Failures: We use firm level data from Orbis in each benchmark year to forecast firm failures. Starting with the benchmark year cash position of each firm, we use our model equations and calibrated shocks to simulate each firm's cash flow over the subsequent year. We combine the estimated cash flow with our liquidity criterion (Eq. (28)) to predict

[27] See Ackerberg, Caves and Frazer (2015), Gandhi, Navarro and Rivers (2012), Levinsohn and Petrin (2003), and Wooldridge (2009). Our approach is similar to that of Blackwood, Foster, Grim, Haltiwanger and Wolf (2021) for variable inputs and is an alternative to the parametric approach of Gandhi et al. (2012).

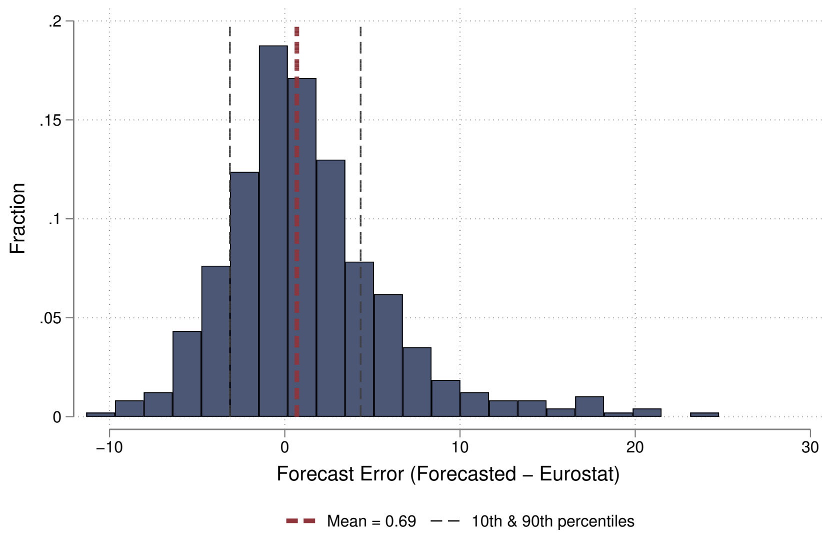
which firms fail. Our "typical year" scenario makes three assumptions, described in Section 2. We estimate weekly cash flow in order to exploit within-year variation in shocks, but evaluate the liquidity criterion at the end of each year to allow firms to smooth cash flow during the year. We allow firms to temporarily mothball in periods of low profitability. And we assume firms can maintain existing debt levels, but must pay the interest due on this debt monthly.

Comparing Forecasted versus Official Failure Rates: Fig. 1, Fig. 2 and Table 1 show that the "typical year" implementation of our framework produces failure rates broadly in line with the observed data. Fig. 1 pools all countries over the 2017-2019 period, and shows the full distribution of forecast errors (estimated - actual failure rates) at the 1-digit sector level. Overall, 80 percent of our forecast errors are within four percentage points of the true value, with a mean forecast error of 0.69 percentage points (pp) and mean absolute error of 2.29 pp.[28] Note, the bulk of the extreme forecast errors come from Portugal and Romania, which is confirmed in Table 1. Dropping these two countries lowers our mean forecast error to 0.45 pp and the mean absolute error to 2.12 pp. Moreover, Table 1 shows that outside of Portugal and Romania, our framework captures the cross-country variation in failure rates well, with a forecast error of less than one pp in over half of our sample. Fig. 2 further shows that our framework matches the sectoral patterns in failure rates with reasonable accuracy.

It is worth noting that our framework can, in theory, accommodate very granular shocks—down to the firm level. For this exercise, we used the most consistently reliable sectoral shock data, at the finest level of granularity availble—1-digit sector level. Yet, despite the use of less granular shocks, our framework captures well the average pattern of failure rates at both the country and sector levels.

[28] Average failure rates for our sample of countries over this period is 8.94 percent suggesting a moderate bias of 7.7 percent (0.69/8.94). All averages are weighted using (country x sector x year) GVA.

Figure 1: Forecast Errors at the Country x Sector Level (2017-2019)



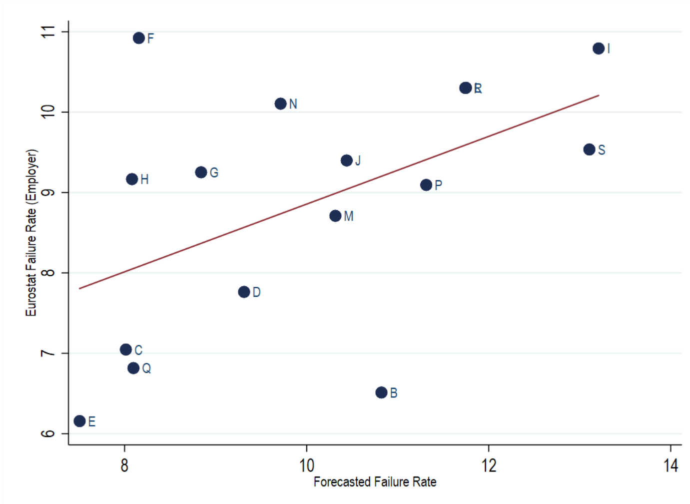
Notes: Eurostat failure rates are obtained from the Structural Business Statistics for employer businesses at the (country x 1-digit NACE x year) level. Failure rates are forecasted by combining Orbis firm level balance sheet data with sector-specific demand and labor productivity shocks calculated using Eurostat national accounts at the 1-digit NACE level, and aggregate demand shocks measured as quarterly GDP growth from OECD.Stat. The liquidity criterion is evaluated for each firm at the end of the year. This histogram shows the distribution of forecast errors at the (country x 1-digit NACE sector x year) level.

Table 1: Failure Rates Comparison at the Country Level (2017-2019)

	(1) Eurostat	(2) Forecasted	(3) Forecast Error
Czech Republic	7.46	7.46	0.01
Finland	10.62	10.07	-0.55
France	9.84	9.46	-0.38
Hungary	9.86	9.48	-0.39
Italy	7.41	9.77	2.36
Poland	12.49	11.94	-0.55
Portugal	7.69	13.08	5.39
Romania	8.65	13.24	4.59
Slovak Republic	9.19	10.05	0.86
Slovenia	8.63	6.89	-1.74
Spain	8.36	8.32	-0.04
Weighted Average	8.97	9.67	0.69

Notes: Eurostat failure rates are obtained from the Structural Business Statistics for employer businesses at the (country x 1-digit NACE x year) level. Failure rates are forecasted by combining Orbis firm level balance sheet data with sector-specific demand and labor productivity shocks calculated using Eurostat national accounts at the 1-digit NACE level, and aggregate demand shocks measured as quarterly GDP growth from OECD.Stat. The liquidity criterion is evaluated for each firm at the end of the year. The table shows (1) official Eurostat and (2) forecasted failure rates, as well as the (3) the forecast error (i.e. Forecasted-Eurostat failure rate) at the country level. The (country x sector x year) observations are first aggregated to the (county x year) level using sectoral GVA as weights. The observations are then aggregated to the country level by taking a simple average over time (2017-2019). The cross-country average is calculated using GDP for weighting.

Figure 2: Forecasted versus Actual Failure Rates at the Sector Level (2017-2019)



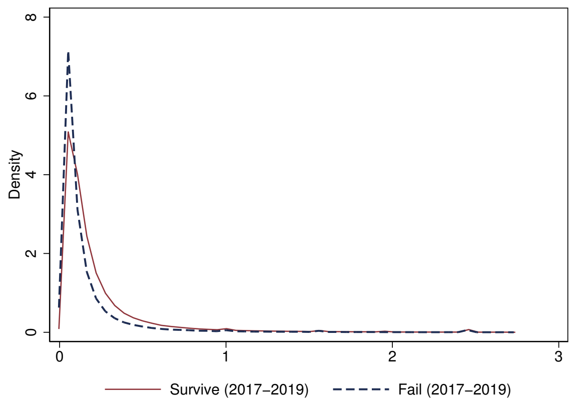
Notes: Eurostat failure rates are obtained from the Structural Business Statistics for employer businesses at the (country x sector x year) level. Failure rates are forecasted by combining Orbis firm level balance sheet data with sector-specific demand and labor productivity shocks calculated using Eurostat national accounts at the 1-digit NACE level, and aggregate demand shocks measured as quarterly GDP growth from OECD.Stat. The liquidity criterion is evaluated for each firm at the end of the year. The figure plots official Eurostat versus estimated failure rates at the sector level. The (country x sector x year) observations are first aggregated to the (sector x year) level using cross-country sectoral GVA as weights. The observations are then aggregated to the sector level by taking a simple average over time (2017-2019).

Characterizing Failing and Surviving Firms: An advantage of our framework is that we can investigate differences in firm characteristics between firms predicted to fail and those predicted to survive in any given year. Fig. 3 compares the distributions of labor productivity, past revenue growth, initial cash-to-assets ratio, and short-term leverage for failing and surviving firms in 2017-2019. First, given our liquidity based criterion, we find that firms with relatively low cash-to-assets ratios and high leverage are predicted to fail. We also find that failing firms tend to have lower labor productivity and growth.

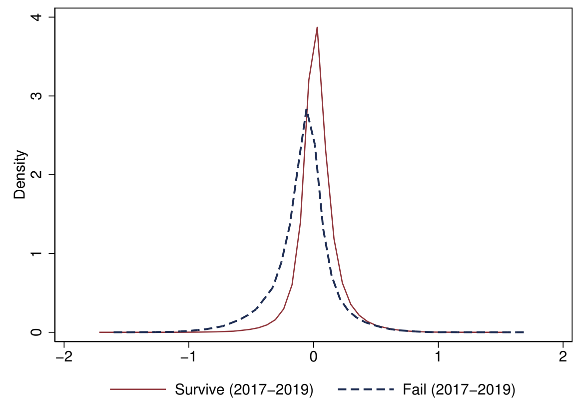
The weakness of failing firms is further investigated in Table 2, where firms predicted to fail are reported to be on average smaller in terms of revenue and employment and younger than surviving firms. Moreover, firms predicted to fail are those that shrank and were unprofitable in previous periods. Taken together, these findings suggest that our liquidity criterion matches stylized data facts and predictions of firm dynamics models regarding exiting firms.[29] Our findings also match the differences in firm performance between failing and surviving firms — a difference often used to justify solvency based failure criteria in firm dynamics models.

[29]See Albuquerque and Hopenhayn (2004); Arellano et al. (2019); Ayres and Raveendranathan (2021); Cooley and Quadrini (2001); Foster, Grim and Haltiwanger (2016); Lee and Mukoyama (2015); Tian (2018).

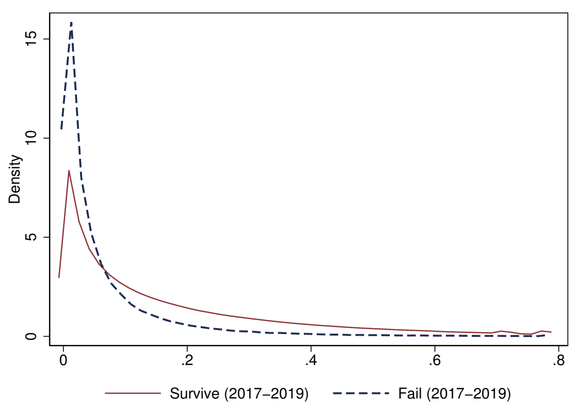
Figure 3: Distributions of Survivors vs. Failures (2017-2019)



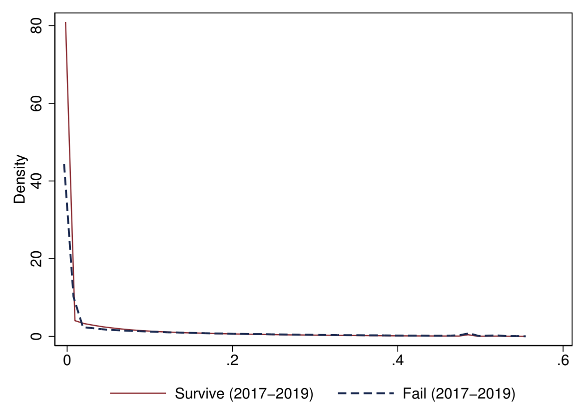
(a) Log Labor Productivity



(b) Revenue Growth Rate (3-year Average, %)



(c) Cash/Total Assets (%)



(d) Short-term Leverage (%)

Notes: Depicted are the distributions of (a) log labor productivity (sales per worker), (b) revenue growth rate in percent, (c) beginning of period cash-to-total assets ratio and (d) short-term leverage (defined as short-term loans/initial total assets) of firms who we predict will survive or fail in 2017-2019. Note that a firm that fails in 2019 will be classified as surviving in 2017 and 2018 and as failing in 2019.

Table 2: Summary Statistics (Median): Survivors vs. Failures (2017-2019)

	(1) Survive	(2) Fail
Number of Employees	11.26	8.85
Revenue (Millions USD)	1.83	1.12
Employment Growth	0.54	-0.52
Revenue Growth	1.70	-5.89
Firm Age	14.46	12.20
Labor Productivity	0.15	0.10
EBITDA/Total Assets	9.50	-11.22
Cash/Total Assets	10.36	2.15
Short-term Loans/Total Assets	0.86	0.92

Notes: Table reports the median of variables of interest, separately for firms who we predict will (1) survive or (2) fail in 2017-2019. Note that a firm that fails in 2019 will be classified as surviving in 2017 and 2018 and as failing in 2019.

5 Applying Framework to COVID-19

During economic crises, two concerns often prevail—whether productive firms can survive without government intervention and whether government support will save productive, or instead, already weak firms. With our approach, we can address both concerns directly. To illustrate how our framework can be applied in a crisis context, we use COVID-19 as a laboratory. The COVID-19 crisis is the perfect setting to implement our framework because the combination of an unprecedented reallocation of demand across sectors and severe lockdowns and put enormous pressure on firms' cash flows and many firms' liquidity. This forced governments to react swiftly with policies that disbursed funds to struggling firms. We first describe our calibration of sectoral and aggregate shocks. We then evaluate how vulnerable firms in our set of countries were to COVID-19 shocks, and describe the characteristics of firms predicted to fail. Finally, we evaluate the cost and impact of policy support.

5.1 Calibrating COVID-19 Shocks

In our COVID-19 scenarios, we define shocks as perturbations in economic conditions caused specifically by the COVID-19 pandemic, relative to conditions in a benchmark year. In order to highlight how our framework can be deployed quickly at the onset of a crisis, we calibrate our shocks using information available at the early stages of the COVID-19 crisis—June 2020.

Essential versus Non-essential Sectors: We first separate sectors, at the 4-digit NACE level, into essential and non-essential, based on the U.S. Department of Homeland Security Guidance on the Essential Critical Infrastructure Workforce.[30] While the DHS does not provide a list of industry codes that are deemed to be essential, we classify sectors based on the information provided regarding the types of workers and activities considered as part of essential critical infrastructure. Among essential workers are those working in public health, public safety, food supply chain, energy infrastructure, transportation and logistics, critical manufacturing, hygiene products and services, among others.

Sectoral Input Shock: In the context of COVID-19, an important constraint facing firms was that workplace restrictions limited the number of workers that could be used on site. We therefore focus on a labor supply constraint where in Eq. (12) we set $\omega_s = 1$ and $\hat{x}_s \equiv \hat{x}_{ns}$:

$$n'_{is} - \hat{x}_s n_{is} \leq 0$$

Because in the benchmark (pre-COVID) year, the labor supply constraint was inactive, the sectoral labor supply shock ($\hat{x}_s$) captures by how much firms are forced to reduce their labor

[30] See CISA's Guidance on the Essential Critical Infrastructure Workforce.

force due to lockdowns and workplace social distancing requirements.

To calibrate the labor supply shock, we follow Dingel and Neiman (2020) and measure the feasibility of remote work by industry. To construct the measure, we start with the "work context" and "generalized work activities" surveys conducted by the Occupational Information Network (O*NET). We classify occupations into those that can be performed remotely versus those that cannot, based on characteristics such as reliance on being outdoors, interacting with patients, repairing and inspecting structures and equipment, controlling machines, handling and moving objects, among others. We then use information from the U.S. Bureau of Labor Statistics (BLS) on the prevalence of each occupation by NAICS industry. Using a cross-walk between NAICS and NACE codes, we arrive at the fraction of employees that can perform their work remotely by 4-digit NACE industry.

In constructing the sectoral labor supply shock ($\hat{x}_s$), we assume that firms in non-essential sectors can produce with at most the fraction of workers they can shift to remote work, and that firms in essential sectors face no such restriction. The left panel of Fig. 4 illustrates the severity of the labor supply shock at the 1-digit NACE level.[31] The Accommodation & Food Service and Arts, Entertainment & Recreation sectors are among the most affected, while essential infrastructure sectors, including Electricity and Water & Waste, remain largely unaffected.

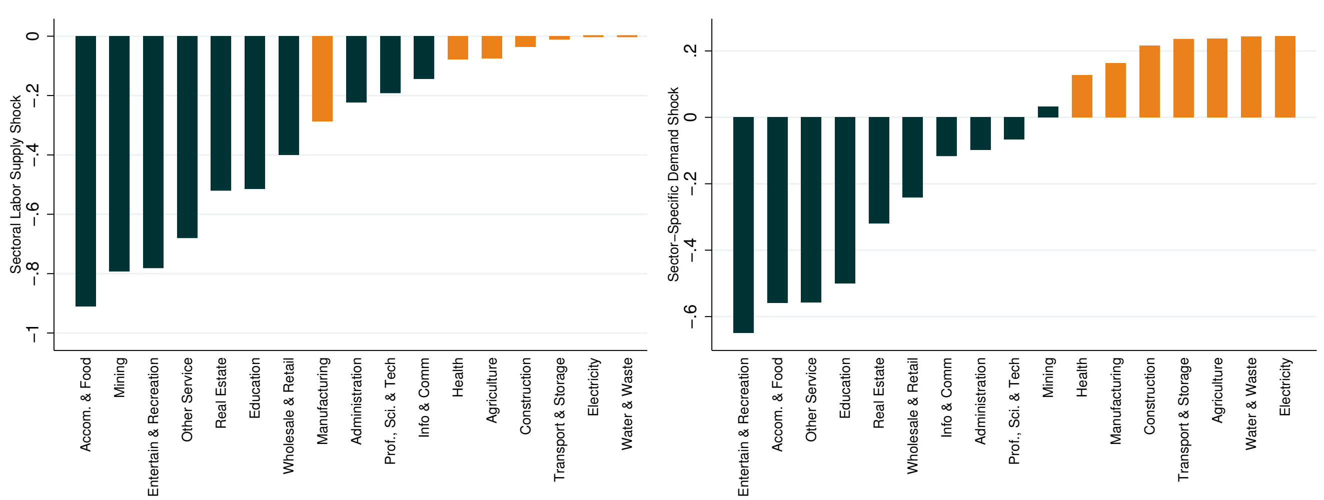
Sector-Specific Demand Shock: The sector-specific demand shock ($\tilde{\xi}^{\eta}_s$) measures how much the COVID-19 pandemic reallocates demand across sectors, relative to a benchmark (pre-COVID) year. Because the pandemic affected the ability and willingness of consumers to interact in person, we calibrate the shock using information on whether industries are customer facing. Specifically, using O*NET surveys, we classify occupations based upon reliance on face-to-face interactions. We consider occupations as highly reliant on face-to-face interactions when working with external customers or in physical proximity, caring for others, working with the public, and selling to others are deemed important. As with the sectoral labor supply shock, we aggregate occupation-level data to arrive at an estimate of the fraction of employees reliant on face-to-face interactions at the 4-digit NACE level.

We assume that under COVID-19 the raw sector-specific demand shock ($\hat{\xi}^{\eta}_s$) is one in essential sectors and one minus the fraction of customer facing employees in non-essential industries. We then normalize the raw sectoral demand shocks to be consistent with aggregate demand, Eq. (8), by constructing $\tilde{\xi}^{\eta}_s = \hat{\xi}^{\eta}_s / (\sum_{\sigma} \hat{\xi}^{\eta}_{\sigma} / \mathcal{S})$. The right panel of Fig. 4 illustrates the size of the sector-specific demand shock at the 1-digit NACE level. The figure illustrates that COVID-19 reallocates expenditure from highly affected non-essential sectors such as Arts,

[31]To construct Fig. 4, we aggregate to the 1-digit level by first averaging 4-digit NACE shocks to the 1-digit level in each country and then using the GVA sector share of each country to aggregate 1-digit sector shocks across countries.

Entertainment, & Recreation to non-affected essential sectors including Water & Waste.[32]

Figure 4: Shocks by Sector: Baseline COVID-19 Scenario



(a) Sectoral Labor Supply Shock

(b) Sector-Specific Demand Shock

Notes: Depicts the COVID-19 (a) sectoral labor supply and (b) sector-specific demand shocks by 1-digit NACE sector. Shocks are first aggregate from the 4-digit NACE to 1-digit NACE level by taking a simple average across 4-digit sectors within each country. The GVA sector share of each country is used to aggregate 1-digit sector shocks across countries. Sectors composed mainly of non-essential industries are depicted in blue and those composed mainly of essential industries are depicted in orange

Aggregate Demand Shock: The aggregate demand shock measures the change in aggregate expenditures ($\widehat{PD}$) due the COVID-19 pandemic, relative to the benchmark year. While not explicitly modelled in our framework, these aggregate expenditures likely react to COVID lockdowns and other COVID shocks via income and precautionary savings channels. We can implicitly capture these effects by calibrating the change in aggregate expenditures using a measure that accounts for the effects of supply shocks. We therefore calibrate aggregate demand shocks using quarterly country GDP growth predictions, constructed by the IMF for the June 2020 World Economic Outlook Report. These early forecasts account for the likely reaction of aggregate income to *all* COVID shocks.

Sectoral Productivity Shock: Many on-site workers in the benchmark (pre-COVID) year were forced to shift to remote work during the COVID-19 pandemic. The sectoral productivity shock ($\hat{A}_s$) captures possible declines in productivity due to this transition.

We assume sectoral productivity is a weighted average of the productivity of on-site and

[32]Within each country $\sum_s \tilde{\xi}_s^{\eta}/\mathcal{S} = 1$ holds. However, Fig. 4 aggregates sector-specific demand shocks at the 1-digit NACE level across countries using the gross value added sector share of each country. Consequently, the sector-specific demand shocks depicted in the figure do not sum to one.

remote workers:

$$A_s = A_s^{work}\theta_s + A_s^{home}(1-\theta_s) \qquad \text{Before COVID,} \tag{29}$$
$$A'_s = A_s^{work'}\theta'_s + A_s^{home'}(1-\theta'_s) \qquad \text{COVID-19,}$$

where θ_s is the fraction of on-site workers, A_s^{work} is productivity of workers on-site and A_s^{home} is productivity of remote workers in each sector.

If we assume that A_s^{work} and A_s^{home} are constant (i.e. do do not change because of COVID), then we can express the sectoral productivity shock as:

$$\hat{A}_s = \frac{\theta'_s + \frac{A_s^{home}}{A_s^{work}}(1-\theta'_s)}{\theta_s + \frac{A_s^{home}}{A_s^{work}}(1-\theta_s)}. \tag{30}$$

We assume that firms in essential sectors are not forced to shift to remote work. Consequently, in essential sectors, $\theta'_s = \theta_s$ and $\hat{A}_s = 1$. Because firms in non-essential sectors can only employ remote workers during the lockdown period ($\theta'_s = 0$), Eq. (30) collapses to:

$$\hat{A}_s = \frac{\frac{A_s^{home}}{A_s^{work}}}{\theta_s + \frac{A_s^{home}}{A_s^{work}}(1-\theta_s)}. \tag{31}$$

To calibrate the sectoral productivity shock in non-essential sectors, we first use data from the 2018 American Community Survey (ACS) to calculate θ_s as the share of remote workers pre-COVID, by industry. Absent any good data on the relative productivity of on-site and remote workers, we opt to calibrate $A_s^{home}/A_s^{work} = 0.8$. This implies that $\hat{A}_s = 0.8$ (i.e. a 20 percent decline) is the maximum reduction in sectoral productivity, which would occur in a sector with no remote work before COVID and 100 percent remote work during the crisis.

5.2 Evaluating a Baseline COVID-19 Scenario

We first examine the vulnerability of countries, sectors, and firms to the COVID-19 crisis by evaluating a baseline scenario, absent government support. We model COVID-19 as a lockdown occurring for 8 weeks beginning in week 9 of 2020. During this lockdown, the sectoral labor supply ($\hat{x}_s$) and productivity ($\hat{A}_s$) and total demand ($\hat{d}_s = \tilde{\xi}_s^{\eta}\widehat{PD}$) shocks are active. After the lockdown ends, sectoral labor supply and productivity shocks return to benchmark year levels. Total demand continues to evolve throughout the year, with the aggregate demand component evolving according to IMF projections, and the sector-specific demand shock decaying according to an AR(1) process with quarterly persistence of 0.5. The evolution of total demand captures the subdued demand that persisted even after stay-at-home order were lifted because of continued uncertainty and fear of infection.

We use 2018 firm level Orbis data to measure benchmark (initial) firm sales, input cost shares, cash flow, cash balances, and financial expenses.[33] In the baseline scenario, we again make three assumptions, described in Section 2. First, cash flows are estimated weekly to reflect the evolution of COVID-19 shocks throughout 2020; but the liquidity criterion is only evaluated at the end of 2020 to capture that firms can smooth cash flow over the course of the year. Second, firms are allowed to temporarily mothball. Third, because financial markets functioned well throughout 2020, we assume that firms have access to financing such that they may maintain their pre-existing debt levels and need only pay the interest due on this debt. We classify firms as failing if by the end of 2020, they have insufficient cash flow and cash balances to cover their financial expenses.[34]

5.2.1 Estimating Aggregate SME Failure Rates

Table 3 reports our baseline, aggregate results. Column (1) reports the predicted 2020 failure rate in the absence of COVID-19 (non-COVID scenario) and serves as a useful benchmark. The non-COVID failure rate is calculated as a "typical year" scenario.[35] Column (2) reports the end of 2020 estimated SME failure rate under the baseline COVID-19 scenario. Column (3) reports the difference between the two (Δ), and represents the excess SME failures in 2020. Throughout the remainder of the text, the excess failure rate is our preferred metric and is defined as the difference between a COVID-19 scenario and the non-COVID scenario. We find that the COVID-19 crisis results in a 6.01 percentage point excess SME failure rate.

Table 3: Aggregate SME Failure Rate

	(1) Non-COVID	(2) COVID	(3) Δ
Average	9.53	15.55	6.01

Notes: Reports the estimated (1) non-COVID and (2) baseline COVID failure rates, and (3) the excess failure rate (Δ = baseline COVID - non-COVID). Failure rates are first calculated at the 1-digit NACE level and aggregated to the country level using 2018 sector GVA as weights. Failure rates are aggregated across countries using GDP as weights.

Our baseline results allow firms two degrees of freedom to limit the impact of temporary

[33] In our baseline scenario we use 2018 data because it was the most recent and complete balance sheet data available in June 2020.

[34] In a companion piece Gourinchas, Kalemli-Ozcan, Penciakova and Sander (2021a), we investigate the effects of COVID-19 and the wind-down of policy support on failures in 2021.

[35] Specifically, we predict end of 2020 failures by combining 2018 firm level data and shocks that are calibrated using realized quarterly GDP growth (aggregate demand), growth in sectoral revenue (sector-specific demand), and growth in sectoral labor productivity between 2018 and 2019. Effectively we assume that if COVID had never occurred, 2020 failure rates would be identical to our estimated failure rates for 2019.

shocks on their viability: the ability to mothball and to access additional credit during 2020. Table 4 documents the impact on excess SME failure rates when we restrict firm access to these tools. Column (1) repeats our baseline excess failure rate and columns (2) through (4) show how removing the ability to mothball and/or to take a year to correct cash deficits affects excess failure rates. First, in column (2) we no longer allow firms to mothball—instead they must stay open and meet demand regardless of the cost. Many firms may have faced this constraint, having signed contracts prior to the pandemic that committed them to deliver output by a certain date. In this case, excess failure rates increase by almost one percentage point above our baseline. Column (3) shows excess failure rates when firms must have enough cash on hand to meet expenses in every week. This scenario raises excess failure rates by two percentage points above the baseline. Finally, column (4) shows the effects of *both* not allowing firms to mothball and requiring them to be liquid in every week of 2020. Under these stringent production and funding assumptions, excess failure rates would be around 9.20 percentage points, or 3.19 percentage points above our baseline estimates. Taken together, these results suggest that assumptions about firms' ability to mothball or obtain financing have considerable effects on excess failure rates. Yet, regardless of the exact assumptions made, excess failures rates in the COVID scenario remain high.

Table 4: Excess Failure Rates (Δ) under Extensions

	(1) Baseline	(2) No Mothballing	(3) Annual + Weekly + Mothball	(4) Weekly + No Mothball
Average	6.01	6.91	8.09	9.20

Notes: Reports the excess failure rates (Δ = baseline COVID - non-COVID) under—(1) baseline scenario: annual liquidity criterion evaluation and firms are allowed to mothball; (2) annual liquidity criterion and no mothballing; (3) weekly evaluation of the liquidity criterion and mothballing allowed; (4) liquidity criterion evaluated weekly and no mothballing. Failure rates are first calculated at the 1-digit NACE level and aggregated to the country level using 2018 sector GVA as weights. Failure rates are aggregated across countries using GDP as weights.

5.2.2 Exploring Sources of Sectoral and Country Heterogeneity

Considerable heterogeneity underlies our average estimate of a 6.01 percentage point excess SME failure rate—the excess failures were much higher in some country-sectors and much lower in others. Because our framework estimates failures at the firm level, we can study how individual firms with different initial financial conditions respond to shocks. This allows us to evaluate sources of heterogeneity in sector and country outcomes and to compare characteristics of failing firms in COVID to those that fail in a typical year.

Sectoral Exposure to Shocks: Table 5 confirms that there is considerable variation across sectors in excess failure rates under COVID-19. Columns (1) and (2) report the non-COVID and baseline COVID-19 SME failure rates, respectively. Column (3) reports the excess failure rate (Δ). Given their customer orientation and limited scope for remote work, some service sectors, such as Accommodation & Food Service or Arts, Entertainment & Recreation, experience excess failure rates exceeding 10 percentage points. In stark contrast, majority-essential 1-digit sectors (henceforth referred to as "essential sectors" and highlighted in gray), including Construction and Transport & Storage, that face small sectoral supply shocks and higher sector-specific demand, experience less than 3 percentage point excess SME failure rates.[36] Finally, sectors with fewer essential workers, but relatively low total demand shocks and/or high scope for remote work (Professional, Scientific & Technical Services) are moderately affected, experiencing excess failure rates between 5 and 10 percentage points.

Table 5: Sector SME Failure Rates

	(1) Non-COVID	(2) COVID	(3) Δ
Agriculture	8.65	9.64	0.98
Mining	9.59	14.72	5.13
Manufacturing	8.46	10.38	1.92
Electric, Gas & Air Con	9.21	9.33	0.12
Water & Waste	7.80	7.33	-0.47
Construction	7.52	7.62	0.10
Wholesale & Retail	8.74	17.62	8.87
Transport & Storage	8.63	10.20	1.56
Accom. & Food Service	12.63	25.94	13.31
Info. & Comms	10.12	13.80	3.68
Real Estate	11.43	17.41	5.97
Prof., Sci., & Technical	10.54	17.33	6.79
Administration	8.02	19.05	11.02
Education	11.06	30.55	19.49
Health & Social Work	8.32	10.91	2.59
Arts, Ent., & Recreation	12.14	31.51	19.37
Other Services	13.32	28.20	14.88

Notes: Reports the estimated (1) non-COVID and (2) baseline COVID failure rates, and (3) the excess failure rate (Δ = baseline COVID - non-COVID). Sector failure rates are first calculated at the 1-digit NACE level for each country, and then aggregated across countries using (country x sector) GVA as weights. 1-digit sectors where the majority of 4-digit sectors are classified as essential are highlighted in gray.

To better understand which COVID-19 shocks drive the observed cross-sector variation, Table 6 evaluates changes in excess failure rates under five alternative scenarios that differ in the composition of COVID-19 shocks. The first column only includes the aggregate demand shock ($\widehat{PD}$). The second column includes both sectoral and aggregate demand shocks (or total demand shock, $\widehat{PD\tilde{\xi}_s^{\eta}}$). The third includes both aggregate demand and sectoral labor supply shocks ($\widehat{PD}, \hat{x}_s$). The fourth includes total demand and sectoral labor supply shocks ($\widehat{PD\tilde{\xi}_s^{\eta}}, \hat{x}_s$). The last is our baseline, which adds sectoral productivity shocks to the fourth column.

Column (1) shows that when only the aggregate demand shock is included, excess failure rates range from 0.05 percentage points in Mining to 7.23 percentage points in Transportation & Storage. Because all sectors in a country face identical aggregate demand shocks, this heterogeneity must stem from differences in firm financial health across sectors. By this metric,

[36]Note that in some essential sectors, total demand can *rise* in COVID-19 and this can lead to lower failure rates than in a normal year—see Water & Waste.

Transport & Storage is ex-ante one of the most financially vulnerable sectors. This ex-ante vulnerability can arise from, for example, low cash balances and/or high debt levels, which increase the likelihood that declines in cash flow lead to liquidity shortages.

The addition of sector-specific demand shocks to the aggregate demand shock (col. 2) either exacerbates or mitigates underlying sectoral vulnerability, thus resulting in higher excess failure rates in some sectors and lower excess failure rates in others. In an already vulnerable sector, like Administration, even a modest negative sector-specific demand shock leads to a large rise in excess failure rates. Meanwhile, according to column (1) Transport & Storage is the most ex-ante vulnerable sector and Arts, Entertainment & Recreation among the least. Yet, because sector specific demand falls most in customer-oriented service sectors, like Art, Entertainment & Recreation, and increases in essential sectors, like Transport & Storage, excess SME failure rates in column (2) rise in Arts, Entertainment, & Recreation far above those in Transport & Storage.

Adding the sectoral labor supply shock to the *aggregate* demand shock (col. 3) heavily impacts non-essential, labor-intensive sectors that cannot easily transition to remote work, such as Accommodation & Food Service. The pronounced rise in excess SME failure rates in these sectors occurs because a small aggregate demand shock, relative to a more severe labor supply shock, leads to a high fraction of firms becoming labor constrained. For these firms to meet demand, they must make a costly substitution away from labor, which deteriorates their cash flow and leads to a liquidity shortage.[37] Meanwhile, labor-intensive sectors with higher capacity for remote work, such as Information & Communications, experience a smaller rise in excess failure rates. Sectors composed of essential sub-sectors, such as Construction and Transport & Storage, are exposed to small labor supply shocks and therefore experience only a small rise in excess failure rates.

The addition of sector-specific demand shocks to aggregate demand and sectoral labor supply shocks (col. 4) is informative about which shock—sectoral labor supply or sector-specific demand—is more binding for sectors. In some sectors, like Accommodation & Food Service and Mining, the addition of the sector-specific demand shock does not raise excess failure rates much above those in in column 3, pointing to the importance of sectoral labor supply shocks. In contrast, the sector-specific demand shock appears more important than the sectoral labor supply shock in a sector like Arts, Entertainment, & Recreation. Comparing columns (4) to (5) shows the effects of the productivity shock on sectoral excess failure rates, which in this case is modest.

Country-Specific Factors: Other than the evolution of $\widehat{PD}$, our baseline COVID-19 scenario features identical shocks for all firms that operate in the same sector, irrespective of country.

[37] While the worst affected can mothball during the lockdown, they still face cash flow reductions while closed due to fixed costs.

Table 6: Excess Failure Rate (Δ) Comparison (Alternative Shock Combinations)

	(1) $\widehat{PC}$	(2) $(\widehat{PC}\tilde{\xi}_s)$	(3) $\widehat{PC} + \hat{x}_s$	(4) $(\widehat{PC}\tilde{\xi}_s), \hat{x}_s$	(5) Baseline
Agriculture	0.73	0.38	1.26	0.97	0.98
Mining	0.05	0.41	4.12	4.84	5.13
Manufacturing	1.04	0.75	2.13	1.97	1.92
Electric, Gas & Air Con	1.12	0.07	1.12	0.07	0.12
Water & Waste	3.60	0.49	3.60	0.49	-0.47
Construction	1.81	-0.33	1.85	-0.34	0.10
Wholesale & Retail	2.18	8.76	2.86	8.56	8.87
Transport & Storage	7.23	1.21	7.24	1.22	1.56
Accom. & Food Service	0.09	7.85	10.27	11.79	13.31
Info. & Comms	1.77	3.15	1.92	3.15	3.68
Real Estate	1.60	6.04	0.97	6.03	5.97
Prof., Sci., & Technical	3.40	6.80	3.14	6.71	6.79
Administration	4.28	9.35	4.46	9.35	11.02
Education	2.35	19.01	12.73	19.01	19.49
Health & Social Work	2.11	2.50	3.48	2.50	2.59
Arts, Ent., & Recreation	1.88	18.58	10.60	18.82	19.37
Other Services	0.07	14.56	7.35	14.87	14.88
Average	2.16	5.36	3.71	5.72	6.01

Notes: Reports the excess failure rate (COVID - non-COVID) under five scenarios—(1) aggregate demand shock only ($\widehat{PD}$); (2) aggregate demand and sector-specific demand shocks ($\widehat{PD}\tilde{\xi}^{\eta}_s$); (3) aggregate demand and sectoral supply shocks ($\widehat{PD}, \hat{x}_s$); (4) total demand and supply shocks ($\widehat{PD}\tilde{\xi}^{\eta}_s, \hat{x}_s$) and; (5) the baseline ($\widehat{PD}\tilde{\xi}^{\eta}_s, \hat{x}_s, \hat{A}_s$). Sector excess failure rates (Δ) are first calculated at the 1-digit NACE level for each country, and then aggregated across countries using (country x sector) GVA as weights. The last row is the sector GVA weighted average. 1-digit sectors where the majority of 4-digit sectors are classified as essential are highlighted in gray.

Nonetheless, as Table 7 documents, there is considerable cross-country heterogeneity in excess SME failure rates (Δ, col. 3), ranging from 2.37 percentage points in Romania to 10.30 percentage points in Italy.

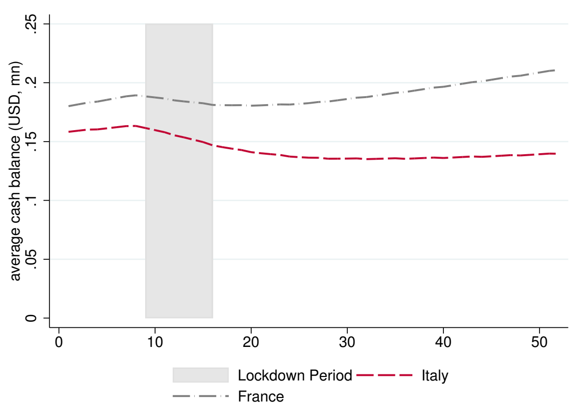
To better understand the sources of heterogeneity, in Fig. 5 we compare France and Italy. Under our baseline scenario, Italy's excess SME failure rate is 5.03 percentage points higher than France's. Fig. 5 makes clear the importance of both industrial composition and overall firm financial health in explaining the differential impact of COVID-19 across these two countries. The figure depicts the weekly evolution of (a) average firm cash balances divided by initial total assets, (b) total demand shocks, (c) sectoral supply shocks and (d) fraction of firms that are labor constrained.

Table 7: Country-Level SME Failure Rates

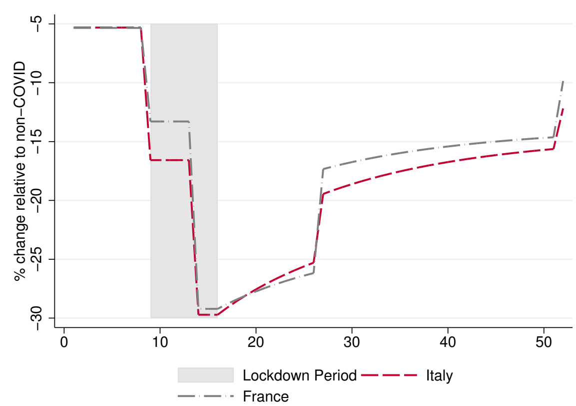
	(1) Non-COVID	(2) COVID	(3) Δ
Czech Republic	7.36	9.92	2.56
Finland	10.17	14.34	4.18
France	10.15	15.42	5.27
Hungary	8.86	11.63	2.77
Italy	9.24	19.54	10.30
Poland	11.88	17.39	5.50
Portugal	12.15	16.17	4.02
Romania	11.90	14.28	2.37
Slovak Republic	9.27	12.29	3.02
Slovenia	6.36	9.34	2.98
Spain	7.51	11.26	3.75

Notes: Reports the estimated (1) non-COVID and (2) baseline COVID failure rates, and (3) the excess failure rate (Δ = baseline COVID - non-COVID). Country level results represent the weighted average of 1-digit NACE failure rates, where weights are given by sector GVA.

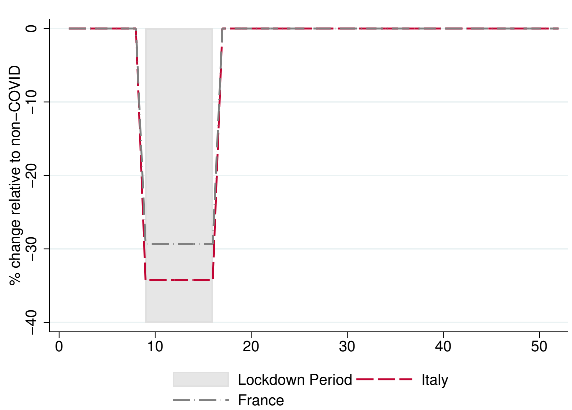
Figure 5: Weekly Evolution of Variables of Interest (Country)



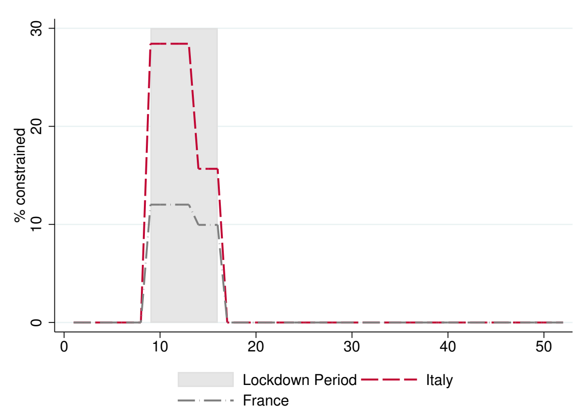
(a) Cash Balance/Initial Total Assets



(b) Total Demand Shock



(c) Sectoral Labor Supply Shock



(d) Fraction of Firms Constrained

Notes: Figures show the weekly evolution of (a) average firm cash balance divided by initial total assets, (b) total demand shock (interaction between sector-specific demand and aggregate demand shock), (c) sectoral labor supply shock, and (d) fraction of firms constrained. In each week, country-level variables represent the weighted average of 1-digit NACE variables, where weights are given by 2018 sector GVA.

While firms in a given sector face the same sectoral shocks regardless of the country they are in, the country *averages* of these shocks can vary based on differences in industrial composition. Total demand evolves similarly in France and Italy, as does the sectoral labor supply shock. However, because more Italian firms are in sectors facing relatively modest sector-specific demand shocks but stringent workplace restrictions, a higher fraction of firms become labor constrained. This means that Italian firms face higher costs during the lockdown than French firms. The largest difference between the two countries is firms' initial cash-to-assets ratio. Italian firms begin COVID with less cash, relative to their total assets, than French firms, which makes them more likely to fail under COVID.

5.2.3 Examining Firm Level Heterogeneity

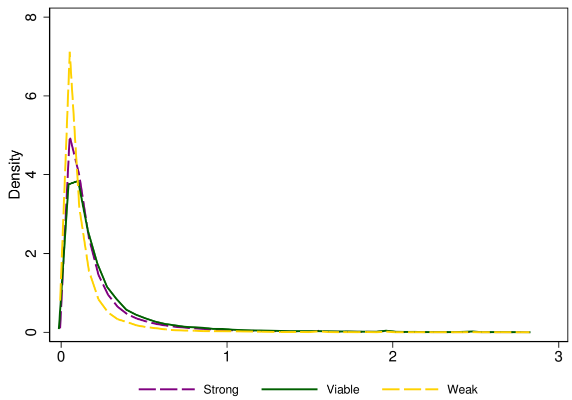
In Section 4, we showed that in typical years, failing firms have lower labor productivity, profitability, revenue growth, and cash balances than surviving firms. In our baseline COVID scenario, many more firms fail than in a typical year. These high excess SME failure rates raise the question of whether the additional failing firms continue to differ considerably from surviving firms.

To shed light on this question, we divide firms into three groups. The first group is "strong" firms that remain liquid through the end of 2020 in our baseline COVID-19 scenario. We then split firms that fail in the baseline COVID scenario into two subgroups—"weak" firms that would fail even if COVID never occurred (i.e. under the non-COVID scenario) and "viable" firms that survive the non-COVID scenario but fail in the baseline COVID scenario. Note that these firm groups are defined based on their survival under the *baseline* COVID scenario, relative to the non-COVID scenario. As such, their composition is invariant to the policy counterfactuals that we evaluate in the next section.

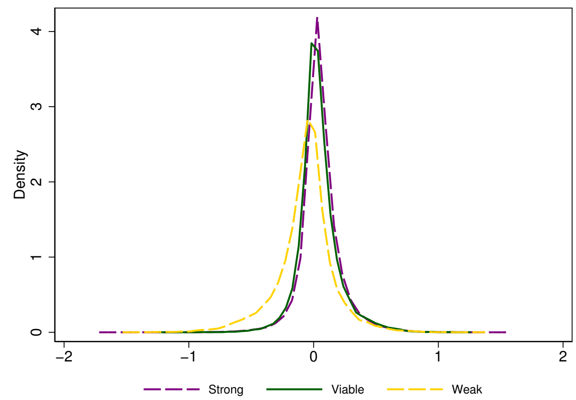
Fig. 6 and Table 8 compare the three firm groups. Panel (a) of Fig. 6 shows that, surprisingly, "viable" firms have higher labor productivity than *both* "strong" and "weak" firms. Panel (b) reports the average of past revenue growth, and again "viable" and "strong" firms look similar. Panels (c) and (d) show the cash-to-assets ratio and short-term leverage distributions, respectively. Here, "viable" firms look more similar to "weak" firms—they have lower cash balances and higher short-term leverage than "strong" firms. Table 8 further shows that, like "strong" firms, "viable" firms are profitable; but, like "weak" firms, are smaller and younger than "strong" firms.

Taken together, it appears that "viable" firms are likely to fail in the baseline COVID scenario due to their low cash buffers and high financial obligations. Given their strong labor productivity, profitability, and past growth, there is potentially a case to be made for preventing the failure of these firms.

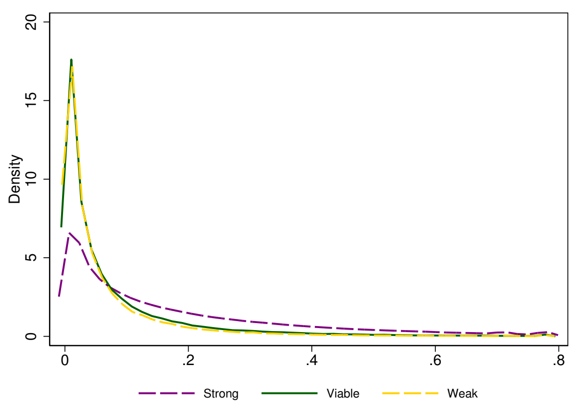
Figure 6: Distributions of Variables by Firm Types in Baseline COVID Scenario



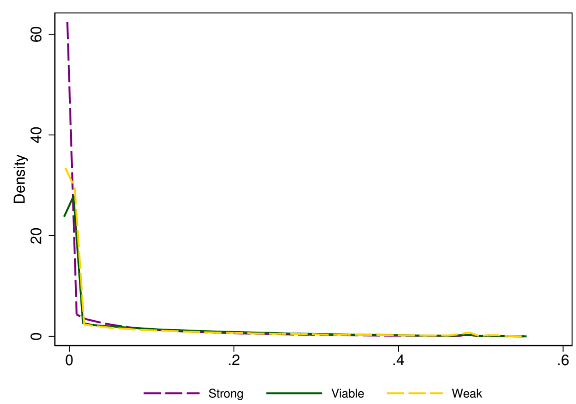
(a) Log Labor Productivity



(b) Revenue Growth Rate (3-year Average, %)



(c) Cash/Total Assets (%)



(d) Short-term Leverage (%)

Notes: Depicted are the distributions of (a) log labor productivity (defined as sales per worker), (b) revenue growth rate in percent, (c) beginning of period cash-to-total assets ratio and (d) short-term leverage (defined as short-term loans/initial total assets) of firms that we estimate fall into one of three groups: "strong" that survive both under the non-COVID and baseline COVID scenarios; "viable" that fail under baseline COVID only; and "weak" that fail under both non-COVID and baseline COVID.

Table 8: Summary Statistics (Median): Strong, Viable, and Weak Firms

	(1) Strong	(2) Viable	(3) Weak
Number of Employees	12.44	9.86	9.88
Revenue (Millions USD)	2.23	1.83	1.30
Employment Growth	0.72	0.49	-0.60
Revenue Growth	3.56	2.16	-4.11
Firm Age	15.79	12.67	12.65
Labor Productivity	0.16	0.17	0.11
EBITDA/Total Assets	9.73	5.75	-12.94
Cash/Total Assets	10.86	2.92	2.08
Short-term Loans/Total Assets	1.02	1.76	1.02

Notes: Table reports the median of variables of interest, separately for firms that we estimate to be (1) "strong" that survive both under the non-COVID and baseline COVID scenarios, (2) "viable" that fail under baseline COVID only and (3) "weak" that fail under both non-COVID and baseline COVID.

5.3 Evaluating Policy Counterfactuals

Our baseline scenario indicates which countries, sectors and types of firms are particularly vulnerable to the COVID-19 shocks in the absence of government support. We now consider several policy counterfactuals to highlight how our framework can be used to study the cost and impact of policy alternatives, as well as gauge the types of firms that these policies save.

We implement policy support in 2020 as a lump-sum cash injection to firms:

$$CF'_{is} = CF_{is} + (\pi'_{is} - \pi_{is}) + P'_{is}$$

where P'_{is} represents funds coming from policy support. We allow for policy support to be a function of firm balance sheet variables *in the benchmark year* to avoid accounting for the impact policy may have on firm choices during COVID. Notice that the method by which resources are transferred to firms (e.g. tax rebates or government guaranteed loans) is irrelevant to firms in 2020, the period which our exercise covers. To avoid failure, all that matters to a firm is the injection of additional resources (or reduction in expenses due) it receives (or owes). The form of the policy support (grant vs. loan) will however affect its net cost to government.

We make several assumptions when implementing the policy counterfactuals. First, we assume that aggregate demand evolves in exactly the same manner as in our baseline COVID scenario. To the extent that saving some firms and preserving some jobs may raise aggregate demand, the numbers presented here likely understate the overall effect of policy support. Further, because we assume perfectly rigid prices and wages, we do not capture worker reallocation effects or the possible impact of policy support on such reallocation. Finally, an assumption underlying our discussion of policy performance during COVID is that governments prefer channeling funds to firms that fail specifically due to the COVID-19 shock ("viable" firms) than to firms that either do not need the support ("strong" firms) or would have failed even if COVID-19 had not occurred ("weak" firms).

5.3.1 Evaluating Fiscal Policy Scenarios

For each policy we consider, Table 9 shows the costs and benefits of saving SMEs. The first column shows the percent of all firms saved by each policy, which we define as the difference between the excess failure rate in the baseline COVID-19 scenario and the excess failure rate when each policy is implemented. The second column shows jobs saved under each policy, as a fraction of total employment. The third column reports the amount of wages saved, which we define as the total labor compensation that is preserved under each policy, as a share of GDP. These numbers take into account that saved firms may choose to operate at lower scale—employing fewer workers and paying less in labor compensation—than in pre-COVID.[38] Finally, the fourth column reports the funds disbursed to firms by each policy, expressed as a fraction of GDP.

To benchmark the performance of policies implemented in practice, the first row of Table 9 considers a hypothetical policy that bails out every firm that fails specifically because of the COVID-19 crisis (i.e. "viable" firms). Under this policy, each "viable" firm receives the minimum amount required to leave it with a zero cash balance at the end of 2020. While this policy is feasible in our framework, the identity of "viable" firms and their cash deficits are not observable in practice. Nonetheless, we find this policy to be a useful benchmark because it approximates the level of resources that would be required if governments wanted to *fully* mitigate the impact of COVID-19 on "viable" firm failures.

Table 9: The Impact and Costs of Various Policy Scenarios

	(1) Firms Saved (% Firms)	(2) Jobs Saved (% Employed)	(3) Wages Saved (% GDP)	(4) Funds Disbursed* (% GDP)
Benchmark Policy	7.28	3.10	1.70	0.77
Financial Expenses Waiver	1.67	0.66	0.38	1.43
Tax Waiver	2.21	0.80	0.34	1.61
Rent Waiver	4.14	2.27	1.18	3.42
Cash Grant	4.74	2.63	1.35	2.63
Pandemic Loans	7.85	4.02	2.12	6.43

Notes: Because Orbis does not cover the universe of firms, we calculate aggregate costs by scaling the total costs in Orbis by the inverse of the coverage ratio of Orbis (based on 2018 value added for policy costs, total remuneration for wages saved, and employment at the 1-digit NACE level). The numbers presented here are GDP-weighted averages across countries.
* Unlike the other policies, the funds disbursed under the pandemic loan policy do not equal the fiscal cost, which depends on the rate of repayment and the distribution of losses between the government and the banking sector.

Our benchmark policy illustrates that, provided sufficient information, the fiscal cost of

[38] These jobs and wages saved numbers pertain specifically to jobs and wages saved in 2020 because firm failures were prevented. They may understate the long-run jobs and wages saved if saved firms return to their previous scale as they recovered from the COVID-19 shock.

saving "viable" SMEs could be modest. With an overall disbursements of 0.77 percent of GDP, the benchmark policy preserves 7.28 percent of firms, 3.10 percent of jobs, and 1.70 percent of GDP in wages.[39][40] Moreover, each dollar disbursed by this policy generates $2.20 in direct aggregate demand (1.70/0.77) in the form of wages saved. We call this ratio the *fiscal-bankruptcy multiplier*. This multiplier is not a traditional Keynesian multiplier; it reflects that businesses may be inefficiently shut down as a consequence of the pandemic, and that fiscal resources deployed to preserve "viable" businesses help increase overall output and employment.[41]

The next five rows of Table 9 consider a set of alternative policies that better reflect the policy responses implemented by countries. Rather than focus on the policies of any particular country, we focus on policy interventions that together span most types of policies implemented by governments. Policy responses have varied considerably across countries but have tended to take the form of cheaper debt refinancing, loan guarantees, expense rebates, and size-based grants.

The first set of policies rebate to firms their financial expenses (row 2 of Table 9), taxes (row 3) or rent (row 4) at the beginning of lockdown through the end of 2020.[42] The financial expenses and tax rebates have in common that they can be implemented at moderate cost, but have modest benefits. For example, under the financial expenses rebate, 1.67 percent of firms are saved, at a cost of 1.43 percent of GDP. The fiscal bankruptcy multiplier is low at 0.38/1.43 = 0.27. Meanwhile, waving rents is a bit costlier, at 3.42 percent of GDP and saves more firms, 4.14 percent.[43] Yet, the fiscal-bankruptcy multiplier remains low at 1.18/3.42 = 0.35.

The last two policies considered are injections of new funds rather than rebates. The first of these is a cash grant that disburses to firms their average 2018 weekly wage bill during the 8 weeks of lockdown.[44] Importantly, because the payments are lump-sum, assessed on the basis of the wage bill in the benchmark year, they do not affect the current cost of labor or firms'

[39] 7.28 percent of firms are viable despite excess failure rates being only 6.01 percent (Table 3). This difference is accounted for by the existence of 1.27 percent of firms that fail in our non-COVID scenario yet *survive* COVID because some (essential) sectors faced higher demand during COVID. The positive demand shocks helped save these firms from otherwise failing in 2020. These firms are classified as strong firms and offset some of the rise in excess failure rates.

[40] Note that Orbis does not cover the full universe of firms. To compute columns (2), (3) and (5) in Table 9, we calculate sectoral coverage rates by comparing 1-digit sectoral Orbis employment and labor costs to the equivalent OECD data for each country. We then scale by the inverse of the coverage ratio to get representative numbers for each country by sector pair.

[41] Traditional fiscal multipliers would differ—one dollar in fiscal resources used to preserve viable businesses may increase overall output by more (or less) than 0.77 dollars. We ignore these general equilibrium considerations in this paper and focus on the first-round effects of the fiscal interventions.

[42] Note that the financial expenses rebate is an extreme version of policies that guarantee existing firm loans or refinance them at lower interest rates.

[43] Orbis does not include any information on firm rents. Therefore, we estimate firm rent expenses by assuming that the ratio of rent to cost-of-goods-sold is constant within 1-digit sectors and use data from Compustat to calculate these ratios.

[44] This grant therefore equals 8/52=15.4 percent of the 2018 wage bill of the firm. Cash transfers of this form are discussed in an early policy note from April 2020, Drechsel and Kalemli-Ozcan (2020).

employment decisions. These cash grants have a much larger impact than the rebate policies on business failures, jobs and wages saved, though generally at a higher cost. The grant saves 4.74 percent of firms, 2.63 percent of jobs and 1.35 percent of GDP in wages, but at an overall cost of 2.63 percent of GDP.[45] The fiscal-bankruptcy multiplier is 0.51—each dollar of fiscal resources preserves 0.51 cents in direct aggregate demand.

The final policy we consider is a program of public loan guarantees for SMEs (e.g. pandemic loans), broadly similar to those implemented by several Euro-area countries.[46] Because most of the countries we focus on belong to the Euro-area, this policy is especially relevant. To remain consistent with how the policy was designed in Europe, we assume that zero interest and principal is due in 2020. Consequently, from the perspective of 2020 outcomes, the relevant aspects of the loan guarantees is the new injection of funds that help some SMEs survive the year. Other than affecting the policy's net cost, repayment terms and interest beyond 2020 have no effect on our analysis.[47]

This policy is the most generous, providing 6.43 percent of GDP in funding to SMEs.[48] It has a dramatic impact on failure rates, bringing them *below* their pre-COVID levels and saving 4.02 percent of jobs.[49] At first glance, the fiscal bankruptcy multiplier, in terms of wages saved relative to funds disbursed, appears low at 2.12/6.43=0.33. However, as we will discuss later, because this policy is a loan, the fiscal bankruptcy multiplier once repayment is accounted for could easily be much higher.

5.3.2 Evaluating which Firms Get Saved

Our analysis shows that real world policies, including cash grants and pandemic loans, can be effective at saving firms; but at costs that far exceed those required under our targeted benchmark policy. It remains to be seen which types of firms benefited most from these real world policies, both in terms of firms saved and money disbursed.

Table 10 decomposes the effects of a subset of policies on "strong", "weak" "viable" firms. We focus our attention on the cash grants and pandemic loan policies, and include our benchmark policy for comparison. Column (1) of Table 10 pertains to "strong" firms, columns (2)

[45] Several sectors (e.g. the financial sector and the government sector) are not included in our analysis, which may help explain why the overall policy costs of this cash grant appear small.

[46] Under the terms of this program, firms are eligible to borrow up to the larger of 25 percent of their 2018 revenues, or twice their 2018 wage-bill, during each week of lockdown. They are not required to pay interest or repay any principal in 2020. See ECB Economic Bulletin 6/2020 Focus for details.

[47] Our companion paper, Gourinchas et al. (2021a) explores the implications of repayment of this program on firm failures in 2021.

[48] This amount represents funds disbursed by the banking sector and not a policy cost. The policy cost will depend on the repayment rate and the distribution of losses between the government and banking sector.

[49] We assume funds are directly channeled from banks to firms, whereas in real-life these type of programs suffered several setbacks and delays due to frictions in banking intermediation.

and (3) to "weak" firms, and columns (4) and (5) to "viable" firms. Columns (2) and (4) show the failure rates under each policy for the "weak" and "viable" firms.[50] For instance, under our benchmark policy, all "weak" firms fail because they do not receive any support, while the failure rate of "viable" firms falls to zero.[51] Columns (1), (3) and (5) show the funds disbursed to each group and column (6) the total amounts disbursed, all as a percent of GDP.

Table 10 highlights two features of the cash grant and pandemic loan policies. First, despite concerns that policies would primarily benefit "weak" firms, we find that the majority of firms saved are "viable". The pandemic loan policy (cash grant) policy saves 60 (38) percent of all "viable" firms, which account for 59 (55) percent of all saved firms.[52] The pandemic loan (cash grant) policy does also inefficiently save 42 (24) percent of all "weak" firms, which account for the remaining 45 (41) percent of saved firms. Table 11 further shows that approximately 58 percent of the jobs saved (1.53/2.63) and wages saved (0.78/1.35) from the cash grants can be attributed to retaining workers at "viable" firms. The same figures for the pandemic loan policy are 55 and 54 percent, respectively.

Second, as shown in Table 10, despite concerns that most resources would flow to "weak" firms, the majority of fiscal resources flow to "strong" firms that do not need the support. The pandemic loans (cash grant) policy disbursed 6.43 (2.63) percent of GDP in funding to firms. The total cost of saving "viable" firms is 0.53 (0.19) and "weak" firms is 0.45 (0.19) percent of GDP. Note that though the actual cost of bailing out "weak" firms is small, saving them remains inefficient because they are likely to struggle and fail after fiscal support ends. The remainder of funds are directed towards "strong" firms. The cash grant policy disburses over 2 percent of GDP to "strong" firms. Though the pandemic loan is even less efficient in terms of disbursements, providing 5.45 percent of GDP to "strong" firms, one potential advantage is that these funds may be recovered in the future. If the 5.45 percent of GDP distributed to "strong" firms were to be fully recovered by repayments, the overall cost of the policy would fall to 0.98 percent of GDP and the fiscal bankruptcy multiplier would rise to 2.12/0.98= 2.16—a fiscally cost-efficient policy.

Table 10 and Table 11 show that the pandemic loan and cash grant policies were untargeted across firm types. Focusing on the pandemic loan policy, Fig. 7 investigates whether there is any selection *within* firm type. Specifically, we compare the labor productivity (panels (a) and (b)) and initial cash-to-assets ratio (panels (c) and (d)) of all versus saved "viable" (panels (a)

[50] We do not show a column for failure rates of strong firms because these are zero by definition.

[51] Weak firms comprise 8.26 percent of all firms, which is less than the 9.53 percent of firms we estimate would fail in a non-COVID 2020 scenario (Table 3). As discussed above, the remaining 1.27 percent of firms that fail in our non-COVID scenario *survive* COVID because they are in sectors receiving positive demand shocks.

[52] Firms saved by each policy can be calculated by subtracting the failure rate in each policy from the total number of firms in each subgroup (8.26 percent for weak firms and 7.28 percent for viable firms). For example, due to the cash grant policy, 8.26-6.30=1.96 percent of all firms were weak and saved. Therefore 1.96/8.26 = 23 percent of all weak firms were saved by the cash grant policy.

Table 10: The Distribution of Policy Support by Firm Type

	Firms that Survive COVID (Strong Firms)	Firms Bankrupt Regardless of COVID (Weak Firms)		Firms Bankrupt Only in COVID Scenario (Viable Firms)		Total
	(1)	(2)	(3)	(4)	(5)	(6)
	Funds Disbursed* (% GDP)	Failure Rate (% Firms)	Funds Disbursed* (% GDP)	Failure Rate (% Firms)	Funds Disbursed* (% GDP)	Funds Disbursed* (% GDP)
Benchmark Policy	0.00	8.26	0.00	0.00	0.77	0.77
Cash Grant	2.24	6.30	0.19	4.51	0.19	2.63
Pandemic Loans	5.45	4.75	0.45	2.94	0.53	6.43

Notes: Because Orbis does not cover the universe of firms, we calculate aggregate costs by scaling the total costs in Orbis by the inverse of the coverage ratio of Orbis (based on 2018 value added at the 1-digit NACE level). The numbers presented here are GDP-weighted averages. * Unlike the other policies, the funds disbursed under the pandemic loan policy do not equal the fiscal cost, which depends on the rate of repayment and the distribution of losses between the government and the banking sector.

Table 11: Wages, Jobs and Loans Saved by Firm Type

	Firms Bankrupt Regardless of COVID (Weak Firms)			Firms Bankrupt Only in COVID Scenario (Viable Firms)		
	(1)	(2)	(3)	(4)	(5)	(6)
	Jobs Saved (% Emp)	Wages Saved (% GDP)	Policy Cost* (% GDP)	Jobs Saved (% Emp)	Wages Saved (% GDP)	Policy Cost* (% GDP)
Benchmark Policy	0.00	0.00	0.00	3.11	1.70	0.77
Cash Grant	1.10	0.57	0.19	1.53	0.78	0.19
Pandemic Loans	1.80	0.96	0.45	2.23	1.16	0.53

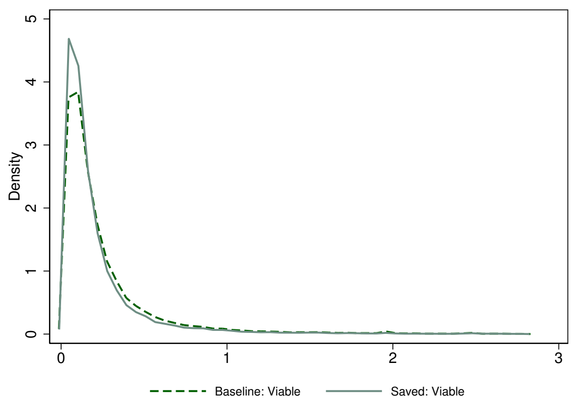
Notes: Because Orbis does not cover the universe of firms, we calculate aggregate costs by scaling the total costs in Orbis by the inverse of the coverage ratio of Orbis (based on 2018 value added at the 1-digit NACE level). The numbers presented here are GDP-weighted averages. * Unlike the other policies, the funds disbursed under the pandemic loan policy do not equal the fiscal cost, which depends on the rate of repayment and the distribution of losses between the government and the banking sector.

and (c)) and "weak" (panels (b) and (d)) firms. We see some evidence that saved "viable" firms have higher labor productivity, relative to the whole group; but see no such difference in the initial cash-to-assets ratio. Meanwhile, saved "weak" firms look virtually identical to the rest of their group in both labor productivity and the initial cash-to-assets-ratio.

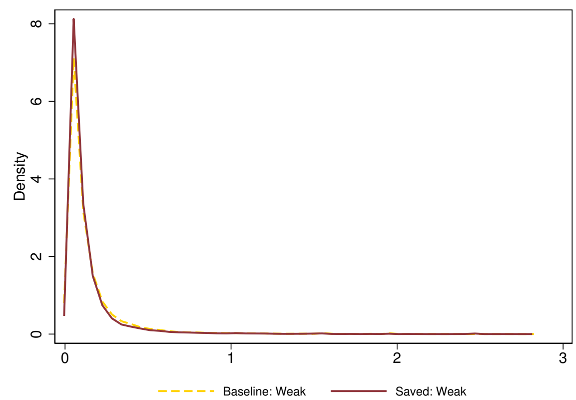
This section highlights how our framework can be used to provide insights on the cost and impact of various fiscal policies. While some were concerned that fiscal support would disproportionately benefit "weak" firms, our framework highlights a more nuanced message. Because policymakers lack full information and were pressed to respond quickly in the midst of the crisis, untargeted and costly policies were implemented. Through the lens of our framework, we predict that while these policies save many "viable" firms, they also inefficiently save some "weak" firms and disburse the vast majority of funds to "strong" firms.

Our findings therefore suggest that policy design is critical. Policymakers have options that may help reduce their overall fiscal burden. Take the pandemic loan policy as an example. The fiscal burden of this policy is lessened because "strong" firms are likely to repay, but a risk remains because some "viable" firms may not be able to repay loans. Instead, policymakers could couple immediate support with a mechanism by which fiscal authorities recoup some of the relief in future years from the best performing survivors—for example., via an excess profit tax (see Blanchard, Philippon and Pisani-Ferry (2020), Drechsel and Kalemli-Ozcan (2020), and Hanson, Stein, Sunderman and Zwick (2020) for similar recommendations).

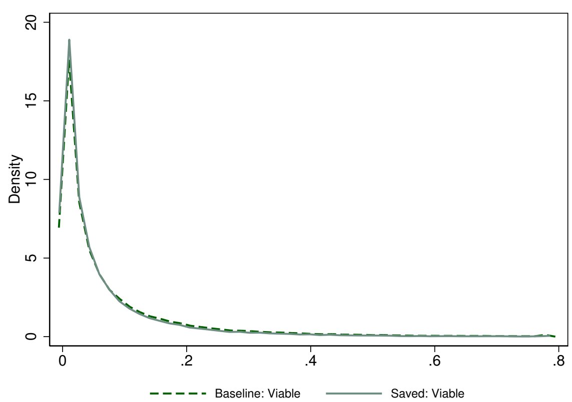
Figure 7: Baseline vs. Pandemic Loan Scenarios: Distributions



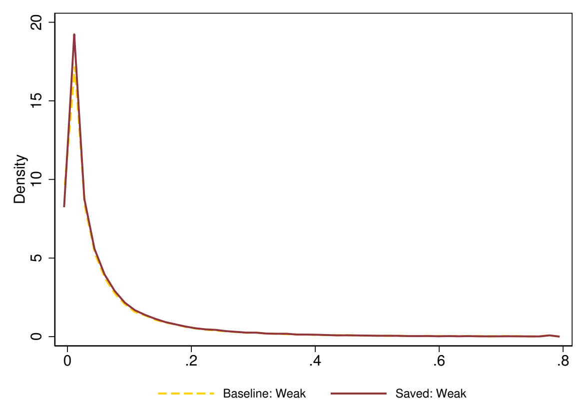
(a) Log Labor Productivity: Viable & Saved



(b) Log Labor Productivity: Weak & Saved



(c) Cash/Total Assets (%): Viable & Saved



(d) Cash/Total Assets (%): Weak & Saved

Notes: Depicted are the distributions of (a) log labor productivity for all viable firms under the baseline scenario and viable firms saved under the pandemic loan scenario, (b) log labor productivity for all weak firms under the baseline scenario and weak firms saved under the pandemic loan scenario, (c) initial cash-to-assets ratio for all viable firms under the baseline scenario and viable firms saved under the pandemic loan scenario, (d) initial cash-to-assets ratio for all weak firms under the baseline scenario and weak firms saved under the pandemic loan scenario.

6 Evaluating Whether Additional Data Changes the Message

Our baseline predictions rely on information available at the early stages of the crisis. In Table 12, we evaluate the accuracy of these predictions by comparing our baseline excess failure rates (col. 1) to estimates derived using (a) shocks calibrated with data that became available in later phases of the crisis (cols. 2-4), and (b) more recent firm level data (cols. 5-6).

Table 12: Excess Failure Rates when Additional Data is Incorporated

	(1) Baseline	(2) Variable Lockdown Intensity	(3) OECD GDP growth rates (2020)	(4) Both Columns (2) and (3)	(5) Updated Orbis (2018)	(6) Updated Orbis (2019)
Czech Republic	2.56	1.84	2.62	1.33	3.13	2.95
Finland	4.18	1.70	4.60	3.35	4.05	4.20
France	5.27	4.80	4.42	4.72	4.03	3.47
Hungary	2.77	1.53	2.56	1.59	2.93	1.50
Italy	10.30	9.90	8.40	8.62	10.17	10.13
Poland	5.50	4.20	5.94	3.93	5.60	5.73
Portugal	4.02	4.33	3.71	4.93	3.95	4.02
Romania	2.37	1.57	2.36	1.69	2.33	1.32
Slovak Republic	3.02	2.07	2.78	1.39	2.95	.
Slovenia	2.98	2.81	2.85	2.39	3.00	3.03
Spain	3.75	4.81	3.37	5.08	3.77	3.65
Average	6.01	5.69	5.19	5.41	5.66	5.43

Notes: Reports the estimated excess failure rates under (1) the baseline scenario; (2) scenario that incorporates the OxCGRT and Google Mobility data and calculates sectoral shocks at the country-sector-week level; (3) uses official OECD 2020 GDP growth rate for aggregate demand shocks; (4) incorporates data used in both columns (2) and (3); (5) uses updated Orbis firm-level 2018 balance sheet data; and (6) uses Orbis firm-level 2019 balance sheet data. Country level results represent the weighted average of 1-digit NACE failure rates, where weights are given by 2018 sector gross value added.

First, in column (2) we evaluate the effect of replacing our single 8 week lockdown period with sectoral shocks that are allowed to vary over the course of 2020 with country-specific lockdown intensity. Specifically, we use two series that were produced during the pandemic—the Oxford Government Response Tracker's (OxCGRT) stringency index and Google mobility data—to generate country-specific, weekly measures of lockdown intensity.[53] Because the OxCGRT index tracks government containment measures, we map it to our sectoral labor supply ($\hat{x}_s$) and productivity ($\hat{A}_s$) shocks. The Google mobility data tracks shopping activity, which we map to our sector-specific demand shock ($\tilde{\xi}_s$). We normalize both indexes to vary from 0 to 1 and interact them with the appropriate shocks to obtain new shocks that vary by country, sector and week. Column (2) of Table 12 reports that the average excess failure rate falls by only 0.32 percentage points, and for most countries the change in excess failure rates remains below one percentage point.

Second, column (3) shows the effects of updating ($\widehat{PD}$) with realized GDP, instead of IMF forecasts. Column (4) reports the excess failure rates from incorporating both country-specific variable lockdowns and up-to-date GDP. In both cases, average excess failure rates are 0.60

[53] The lockdown stringency index can be obtained from Oxford Government Response Tracker and the mobility data from Google's COVID-19 Mobility Reports.

to 0.82 percentage points below our baseline. Moreover, for almost all countries, our baseline and column (4) excess failure rates remain quantitatively similar.

Finally, by late 2022 we can update firm level Orbis data, which is subject to reporting lags, in two ways—reevaluate 2018 data with a more complete set of reporting firms; and use available 2019 data. The results are shown in columns (5) and (6), respectively. In both cases, excess failure rates remain remarkably similar to our baseline for all countries. We conclude that our baseline results, using data available in June 2020, are qualitatively (and quantitatively) robust to new data becoming available.

7 Conclusion

In this paper, we introduce a framework to study the impact of firm financial frictions on SME failures in the presence of shocks to firms' liquidity. We provide insights on the vulnerability of firms, sectors, and the aggregate economy to these shocks. Our framework also allows for a nuanced evaluation of the costs and impacts of fiscal interventions.

Our framework consists of a tractable, but flexible, model of firm cost minimization in which firms face an array of aggregate and sectoral demand and supply shocks, and have limited capacity to borrow in order to fund temporary cash deficits. Firms fail when, as a result of shocks, they are unable to cover input costs and financial expenses. We combine the model with detailed firm level balance sheet data that enables us to characterize a baseline distribution of firm outcomes prior to any scenario. Each scenario is modeled as a perturbation around this baseline, arising from shocks to aggregate and sectoral consumer demand, as well as sectoral productivity, labor and/or materials.

Using firm level data for SMEs in a sample of 11 European countries, we first use our framework to implement a "typical" year scenario, in which firms face modest shocks. We find that, in 2017-2019, the mean forecast error at the country-sector level is only 0.69 percentage points. We also show that firms predicted to fail are less productive and profitable, grow slower, have less cash on hand, and are more leveraged than those predicted to survive, which is consistent with predictions from the empirical and theoretical literature.

We then apply our framework to COVID-19 to illustrate the impact of a large cash flow shock on SME failures. First, we consider a baseline scenario, absent government support, and estimate a 6.01 percentage point excess SME failure rate. We highlight the importance of the interaction between exposure to sectoral shocks and firm financial constraints in explaining the observed heterogeneity in cross-sector and cross-country excess SME failure rates. We also show that "viable" firms are similar to "strong" firms in terms of labor productivity, profitability, and growth, but similar to "weak" firms in that they are cash poor and highly leveraged.

In short, firms with good fundamentals can fail in crises and recessions.

We then evaluate the costs and benefits of various fiscal support measures. We find that while cash grants and pandemic loans save many SMEs from failure, they do so in an untargeted fashion and at a high cost. Both policies primarily save "viable" firms, but also inefficiently save some "weak" firms, though at a low fiscal cost. Moreover, contrary to concerns that most resources would be spent on "weak" firms, we find that the vast majority of funds disbursed are channeled to "strong" firms that do not need the support. In a nutshell, directing support primarily to "viable" firms is difficult in practice.

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Appendices

A Additional Tables and Figures

Labor and material elasticities (β_s and γ_s) are calculated at the 2-digit NACE level for each country. Table A.1 reports the cross-country mean and standard deviation of these elasticities at the one-digit NACE level.

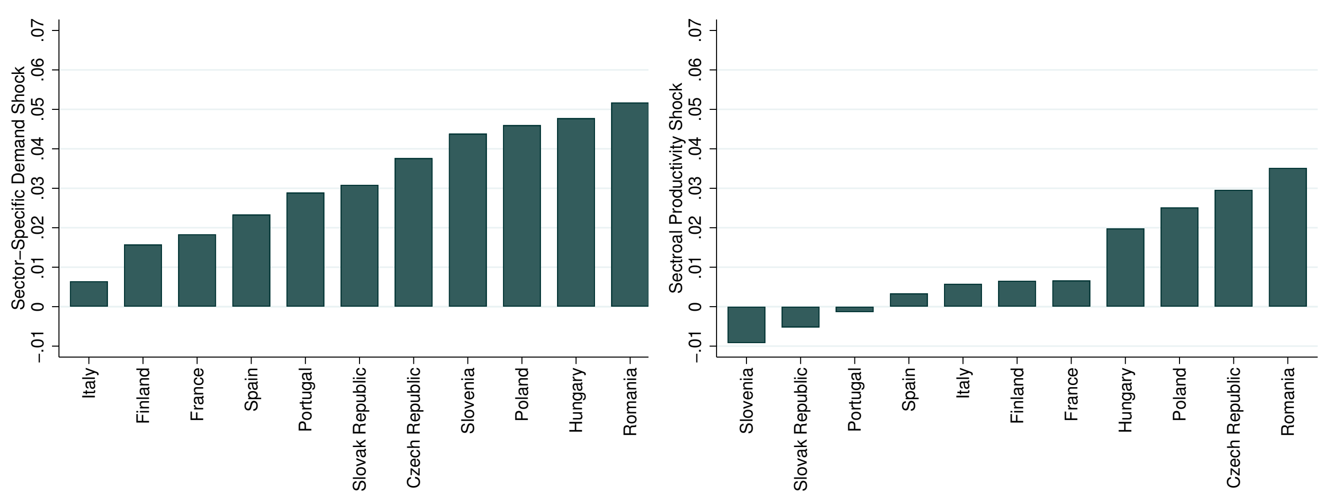
Table A.1: Output Elasticities

	Labor (β_s)		**Materials (γ_s)**	
	Mean	SD	Mean	SD
Agriculture	0.18	0.07	0.46	0.13
Mining	0.23	0.21	0.27	0.16
Manufacturing	0.17	0.07	0.54	0.13
Electric, Gas & Air Con	0.06	0.02	0.58	0.09
Water & Waste	0.22	0.08	0.26	0.14
Construction	0.19	0.07	0.33	0.09
Wholesale & Retail	0.08	0.03	0.76	0.05
Transport & Storage	0.21	0.10	0.22	0.14
Accom. & Food Service	0.26	0.05	0.33	0.14
Info. & Comms	0.23	0.10	0.17	0.10
Real Estate	0.14	0.04	0.33	0.09
Prof., Sci., & Technical	0.27	0.11	0.26	0.15
Administration	0.35	0.24	0.19	0.15
Education	0.42	0.11	0.14	0.11
Health & Social Work	0.46	0.12	0.16	0.09
Arts, Ent., & Recreation	0.23	0.11	0.16	0.12
Other Services	0.30	0.19	0.35	0.17

Notes: Elasticities are calculated at the 2-digit NACE level as the weighted average of the labor cost share of revenue (β_s) and material cost share of revenue (γ_s), where the weights are given by firm revenue. These elasticities are calculated for countries where labor and material costs are reported separately (Czech Republic, Finland, France, Hungary, Italy, Poland, Portugal, Romania, Slovakia, Slovenia, and Spain). The table reports the cross-country mean and standard deviation of the elasticities at the 1-digit NACE level.

Fig. A.1 depicts the average of (a) total demand and (b) sectoral productivity shocks at the country level for our typical year scenario (2017-2019).

Figure A.1: Shocks by Country: Typical Year Scenario (2017-2019)



(a) Total Demand Shock

(b) Sectoral Productivity Shock

Notes: Depicts the typical year scenario (2017-2019) (a) total demand and (b) sectoral productivity shocks by country. The height of each bar represents the simple average of the shock across sector-years in each country.

Table A.2 reports the aggregate revenue coverage for the countries in our sample, both for all firms and SMEs specifically in 2018. SMEs are defined as firms with less than 250 employees in both data sources, OECD and Orbis. Using raw Orbis data, our coverage ranges from 34.0 percent in France to 55.7 percent in Italy.[54] Focusing on SMEs, our coverage ranges from 33.1 percent in France to 66.7 percent in Slovak Republic. Even after imposing additional data requirements for analysis, such as availability of intermediate costs, our data cover at least 30 percent of the aggregate revenue of SMEs in our sample of countries.

[54]To obtain coverage rates we sum up all firm (and, separately, SME) revenue in Orbis by 1-digit NACE sector and merge it with 1-digit NACE sector total (and SME) revenue reported in the OECD's SDBS Business Demography Indicators. Keeping sectors covered in the Orbis and OECD data (for most countries the covered sectors are B, D, D, E, F, G, H, I, J, L, M,and N), we then aggregate the Orbis and OECD data to the country level and calculate the coverage rates for all firms and SMEs.

Table A.2: Orbis Coverage (2018)

	% of OECD Revenue	
	All Firms	SMEs
Czech Republic	49.1	37.0
Finland	53.2	52.9
France	34.0	33.1
Hungary	43.7	39.9
Italy	55.7	64.7
Poland	39.6	36.2
Portugal	52.1	62.6
Romania	51.5	37.5
Slovak Republic	50.5	66.7
Slovenia	46.1	53.6
Spain	47.5	62.1

Notes: OECD revenue (all firms and SMEs) in 2018 is obtained from the Structural Business Statistics Database. The SBSD provides data for a subset of sectors—for most countries the covered NACE 1-digit sectors are B, C, D, E, F, G, H, I, J, L, M, and N. Only sectors covered in both the OECD and Orbis data are used in calculating coverage statistics. To calculate coverage, Orbis revenue (all firms and SMEs) is summed and divided by the total revenue (all firms and SMEs) reported by OECD. The coverage rates are computed using cleaned Orbis data. Additional cleaning is done to generate the analysis data, including conditioning on variables needed to compute the failure condition. SMEs are defined as firms with less than 250 employees in both OECD and Orbis data.