FEDERAL RESERVE BANK *of* ATLANTA WORKING PAPER SERIES

High Discounts and Low Fundamental Surplus: An Equivalence Result for Unemployment Fluctuations

Indrajit Mitra, Taeuk Seo, and Yu Xu

Working Paper 2021-22
September 2021

Abstract: Ljungqvist and Sargent (2017) (LS) show that unemployment fluctuations can be understood in terms of a quantity they call the "fundamental surplus." However, their analysis ignores risk premia, a force that Hall (2017) shows is important in understanding unemployment fluctuations. We show how the LS framework can be adapted to incorporate risk premia. We derive an equivalence result that relates parameters in economies with risk premia to those of an artificial economy without risk premia. We show how to use properties of the artificial economy to deduce how risk premia affect unemployment dynamics in the original economy.

JEL classification: E23, E24, E32, E44, J23, J24, J31, J41, J63

Key words: risk premia, fundamental surplus, time-varying discounts, unemployment fluctuations

https://doi.org/10.29338/wp2021-22

Please address questions regarding content to Indrajit Mitra, Federal Reserve Bank of Atlanta, 1000 Peachtree St. NE, Atlanta, GA 30309, indrajit.mitra@atl.frb.org; Taeuk Seo, Ross School of Business, University of Michigan, taeuks@umich.edu; or Yu Xu, Lerner College of Business and Economics, University of Delaware, yuxu@udel.edu.

1 Introduction

In a recent paper, Ljungqvist and Sargent (2017) (henceforth "LS") show that existing search-based models which amplify productivity shocks to generate realistic model-implied volatility of labor market variables can be understood in terms of the behavior of a quantity that they call the "fundamental surplus". LS (p. 2631) define the fundamental surplus $y - x^j$ as "a quantity that deducts from productivity y a value x^j that the invisible hand cannot allocate to vacancy creation, a quantity whose economic interpretation differs across models". In particular, they show that the elasticities of key labor market variables with respect to aggregate productivity are inversely proportional to the fundamental surplus, so that existing explanations for unemployment fluctuations are viewed as separate mechanisms for generating a small value of the fundamental surplus.

The LS framework applies to models in which firms ignore time-variation in discount rates in making hiring decisions. However, Hall (2017) shows that accounting for such time-variation in discount rates, where these discounts are imputed from asset prices in financial markets, are of first-order importance in understanding unemployment fluctuations. In this paper, we show how the LS approach can be adapted to account for time-varying discount rates and how risk premia changes the fundamental surplus. Our approach is flexible and accommodates rich specifications of discount rate processes that are featured in leading equilibrium asset pricing models such as habit, long-run risk, and time-varying disaster risk, which have been proposed to explain asset prices (see Campbell (2018) for discussions of these models).

We illustrate our approach using the canonical Diamond-Mortensen-Pissarides (DMP) search model with Nash bargaining which we augment to assume that all agents discount future payoffs using a time-varying discount rate process. We establish an equivalence between the equilibrium in this economy and that of an artificial economy without time-varying discount rates (so that the LS framework can be directly applied). Importantly, the artificial economy has an altered process for productivity y and some of the values of the structural parameters, in particular x^j (which is the value of leisure in the canonical DMP model). Our equivalence result provides an explicit dictionary which relates parameters and equilibrium quantities between the two economies. This dictionary provides a transparent way to understand the effect of various properties of time-varying risk premia on the fundamental surplus, and hence its implications for labor market fluctuations.

In constructing the artificial economy, we build on a tool from the finance literature, namely, risk-neutral valuation (see, e.g., Duffie 2001, chapter 6). To see how the logic of this tool applies to our setting, consider an economy in which a firm discounts future risky payoffs from hiring a potential worker at a rate that is greater than the risk-free interest rate. In other words, the risk premium is positive. The present value of these payoffs is therefore less than the expected value of the payoffs discounted at the risk-free rate. The idea of risk-neutral valuation is that the same present value of the potential worker is obtained in an artificial economy without risk premia in which the firm shifts the probability distribution by attaching higher (lower) probabilities to lower (higher) payoff outcomes than their true probabilities, computes the expected value of these payoffs under these artificial probabilities, and finally discounts the expected value at the risk-free rate.

Using the logic above, we show that the artificial economy has an altered process for the potential hire's productivity y in which the average productivity is depressed by an amount that is positively related to risk premia. In addition, time-varying discounts introduce a low frequency component to the productivity process of the artificial economy. We show that an increase in the discount rate in the original economy depresses the low frequency component of productivity in the artificial economy. Moreover, more volatile discount rates in the original economy translate into larger swings in the long run productivity of the artificial economy. We show how to use these properties of the artificial economy to deduce the impact of discount rates on unemployment dynamics in the original economy.

We illustrate our approach by applying our general framework to three, progressively more flexible, discount rate processes. We begin with the simplest case of a constant, positive risk premium. We show that in this case, the average productivity in the artificial economy is depressed by a constant amount that is proportional to the risk premium. Consequently, x^j measured in productivity units is magnified by a constant factor. This lowers the fundamental surplus, and, following the arguments of LS, increases the elasticity of aggregate labor market variables to productivity shocks.

The same logic applies to more general discount rate processes. We show that in the case of time-varying risk premia, the value of x^j measured in productivity units is magnified by a time-varying factor that is higher (lower) in states with higher (lower) risk premia. That is, high discounts in the original economy are equivalent to low fundamental surpluses in the artificial economy. The LS arguments then imply that the elasticity of aggregate labor market variables to productivity shocks is higher (lower) in states with higher (lower) risk premia. In our third example, we consider a discount rate process in which discount rate shocks themselves carry risk premia. We show that such a discount rate process can slow the recovery of unemployment rates following large increases in discount rates. It would be interesting to investigate if this mechanism contributes to the slow recovery of unemployment rates following recessions documented in Hall and Kudlyak (2020).

Related literature. Our paper provides a tool to link two stands of the literature. The first strand of the literature is a response to the observation in Shimer (2005) that the canonical labor search model of Diamond-Mortensen-Pissarides (DMP) predicts a volatility of the U.S. unemployment rate that is too small compared to that in the data. Theories in this literature build on the labor search framework and provide potential explanations for the amplification of primitive productivity shocks. Instead of listing all these theories, we point our reader to Table 1 of Ljungqvist and Sargent (2017) who show that all of these theories use different mechanisms to lower the fundamental surplus. All of these models ignore the role of risk premia and assume that agents in these models discount future payoffs at the risk-free rate.

The second strand of the literature has argued that fluctuations in the discount rate imputed from asset prices in financial markets can help explain labor market fluctuations. The fundamental premise in these models relies on the existence of time variation in financial market discount rates. There is a large existing literature in finance which establishes evidence for this premise (see, e.g., Cochrane (2011) and Campbell 2018, chapter 6). Examples of papers which explore the implications of time-varying discount rates for labor markets include Mukoyama (2009), Hall (2017), Kilic and Wachter (2018), Mitra and Xu (2020), and Kehoe et al. (forthcoming).

Borovicka and Borovickova (2018) is related to our paper. As in our paper, agents in their framework discount future cashflows with an assumed time-varying discount rate process. By requiring the model to match the observed hiring process, they derive conditions that must be satisfied by a firm's cashflow process from hiring a worker. Our goal is different. Our equivalence result provides explicit expressions which show how various discount rate processes translate into different productivity processes in the artificial economy (for a given cashflow process in the original economy). This helps us understand how different properties of discount rate processes affect the fundamental surplus.

2 The Environment

We first describe a continuous time version of the canonical matching model with Nash bargaining (Pissarides, 1985, 2000) in Section 2.1. In this discussion, we stick to the assumption in the canonical model that agents discount future cashflows at the risk-free rate.

We call this the economy without risk premia. Next, in Section 2.2, we assume that agents discount future cashflows using a stochastic discount factor. We call this the economy with risk premia. Later, in Section 3, we show how the economy with risk premia can be mapped to an artificial economy without risk premia. Specifically, the artificial economy obeys the same system of equations as the economy without risk premia in Section 2.1, but has an altered productivity process and structural parameter values.

We make use of two probability measures. The first is the physical probability measure $\mathcal{P}$ that describes the objective likelihood of outcomes. The second is the risk-neutral probability measure $\mathcal{Q}$, described in detail below, that embeds the effect of risk premia on discount rates. To avoid confusion between probability measures, we denote by $\mathbb{E}^{\mathcal{M}}$ the expectation taken with respect to measure $\mathcal{M} \in \{\mathcal{P}, \mathcal{Q}\}$. Similarly, $std^{\mathcal{M}}$, $Var^{\mathcal{M}}$ and $Cov^{\mathcal{M}}$ denote standard deviations, variances, and covariances computed under measure $\mathcal{M}$, respectively.

2.1 Economy *without* risk premia

Workers and firms. There is a representative household consisting of a unit measure of workers and a large mass of firms. Firms hire workers to produce output. Log output per worker y_t follows

$$dy_t = \kappa_y(\mu_t - y_t)dt + \sigma_y dB_t^{y,\mathcal{P}}, \tag{1a}$$

$$d\mu_t = -\kappa_\mu \mu_t dt + \sigma_\mu dB_t^{\mu,\mathcal{P}}, \tag{1b}$$

where $B_t^{y,\mathcal{P}}$ and $B_t^{\mu,\mathcal{P}}$ are standard Brownian motions under the physical probability measure $\mathcal{P}$. The productivity process (1) has the following interpretation. First, productivity y_t is subject to Brownian shocks $dB_t^{y,\mathcal{P}}$ with volatility σ_y and mean reverts towards the value μ_t at a speed of $\kappa_y > 0$. The value μ_t can be interpreted as long run productivity towards which current productivity y_t trends towards. Long run productivity μ_t is normalized to have a zero mean, $\mathbb{E}^{\mathcal{P}}[\mu_t] = 0$. It mean reverts towards zero at a speed of $\kappa_\mu \geq 0$ and is subject to

Brownian shocks $dB_t^{\mu,\mathcal{P}}$ with volatility $\sigma_\mu \geq 0$. For simplicity, we assume that short run and long run productivity shocks, $dB_t^{\mu,\mathcal{P}}$ and $dB_t^{\mu,\mathcal{P}}$, are uncorrelated.[1]

The literature typically analyzes the special case of the productivity process (1) in which long run productivity $\mu_t = 0$ is constant (see, e.g., Shimer 2005). We consider the more general process (1) because, as we will show in Section 3, unemployment fluctuations in an economy with risk premia and a constant long-run productivity correspond to that in an artificial economy without risk premia but with a time-varying long run productivity process.

There is free entry of firms. Firms attempt to hire unemployed workers by posting vacancies, and each vacancy must be maintained at a flow cost of c. An unemployed worker becomes employed when matched to a firm and produces output according to the process (1). Employed workers are paid wages at rate w_t with the residual profit flowing to the matched firm at rate $e^{y_t} - w_t$. This split is determined according to a generalized Nash bargaining rule in which workers' have bargaining power $\beta \in (0, 1)$. Matched firm-worker pairs separate at rate s; employed workers become unemployed following separations. Unemployed workers obtain a value of leisure z (interpreted as the value from the combination of unemployment benefits and nonmarket activity).

Matching. Denote by v_t and u_t the total number of vacancies posted and the unemployment rate, respectively, at time t. Their ratio $\theta_t \equiv v_t/u_t$ is labor market tightness. The instantaneous rate at which new firm-worker matches are formed is given by $m(u_t, v_t)$, where the matching function $m(u, v)$ is assumed to be homogenous of degree one. The job finding rate for unemployed workers is $f(\theta_t) = m(u_t, v_t)/u_t$, and the vacancy filling rate for firms is $q(\theta_t) = m(u_t, v_t)/v_t$. In our numerical exercises, we follow Ljungqvist and Sargent (2017) and specialize to the Cobb-Douglas matching function $m(u, v) = Au^\alpha v^{1-\alpha}$ so that $f(\theta) = A\theta^{1-\alpha}$

[1] The discrete time analog of the process (1) is the VAR(1) process $y_{t+1} = (1-\rho_y)\mu_t + \rho_y y_t + \sigma_y \varepsilon_{t+1}^{y,\mathcal{P}}$ and $\mu_{t+1} = \rho_\mu \mu_t + \sigma_\mu \varepsilon_{t+1}^{\mu,\mathcal{P}}$ with $\varepsilon_{t+1}^{y,\mathcal{P}}$ and $\varepsilon_{t+1}^{\mu,\mathcal{P}}$ being uncorrelated and following standard Normal distributions.

and $q(\theta) = A\theta^{-\alpha}$. The law of motion for the unemployment rate is

$$\frac{du_t}{dt} = s(1-u_t) - f(\theta_t)u_t. \tag{2}$$

Value functions. Let J and V denote the value of a filled and unfilled vacancy to the firm, respectively, and W and U denote the value of employment and unemployment to a worker, respectively. These values discount future cashflows at the risk-free rate r_f and satisfy:

$$\begin{aligned}
J_t &= \mathbb{E}_t^{\mathcal{P}}\left[\int_t^{\tau_{sep}} e^{-r_f u}\left(e^{y_u} - w_u\right) du + e^{-r_f(\tau_{sep}-t)}V_{\tau_{sep}}\right], && \text{(3a)}\\
V_t &= \mathbb{E}_t^{\mathcal{P}}\left[-\int_t^{\tau_{match}} e^{-r_f u} c\, du + e^{-r_f(\tau_{match}-t)}J_{\tau_{match}}\right], && \text{(3b)}\\
W_t &= \mathbb{E}_t^{\mathcal{P}}\left[\int_t^{\tau_{sep}} e^{-r_f u} w_u\, du + e^{-r_f(\tau_{sep}-t)}U_{\tau_{sep}}\right], && \text{(3c)}\\
U_t &= \mathbb{E}_t^{\mathcal{P}}\left[\int_t^{\tau_{match}} e^{-r_f u} z\, du + e^{-r_f(\tau_{match}-t)}W_{\tau_{match}}\right], && \text{(3d)}
\end{aligned}$$

where τ_{sep} denotes the (random) time when a match separates and τ_{match} denotes the time when a vacancy gets matched to an unemployed worker.

The Markovian solution to the system (3), $J_t = J(y_t, \mu_t)$, $V_t = V(y_t, \mu_t)$, $W_t = W(y_t, \mu_t)$, and $U_t = U(y_t, \mu_t)$, is given by the solution to the following system of partial differential equations (PDEs):

$$\begin{aligned}
r_f J(y,\mu) &= e^y - w(y,\mu) + \mathscr{L}^{\mathcal{P}} J(y,\mu) + s\left[V(y,\mu) - J(y,\mu)\right], && \text{(4a)}\\
r_f V(y,\mu) &= -c + \mathscr{L}^{\mathcal{P}} V(y,\mu) + q(\theta(y,\mu))\left[J(y,\mu) - V(y,\mu)\right], && \text{(4b)}\\
r_f W(y,\mu) &= w(y,\mu) + \mathscr{L}^{\mathcal{P}} W(y,\mu) + s\left[U(y,\mu) - W(y,\mu)\right], && \text{(4c)}\\
r_f U(y,\mu) &= z + \mathscr{L}^{\mathcal{P}} U(y,\mu) + f(\theta(y,\mu))\left[W(y,\mu) - U(y,\mu)\right], && \text{(4d)}
\end{aligned}$$

where we have made use of the fact that equilibrium wages $w = w(y,\mu)$ and tightness $\theta = \theta(y,\mu)$ are functions of y and μ, and $\mathscr{L}^{\mathcal{P}}$ is defined by

$$\mathscr{L}^{\mathcal{P}}[\kappa_y, \sigma_y, \kappa_\mu, \sigma_\mu]F(y,\mu)$$

$$\equiv \kappa_y(\mu - y)\partial_1 F(y,\mu) + \frac{1}{2}\sigma_y^2 \partial_{11} F(y,\mu) - \kappa_\mu \mu \partial_2 F(y,\mu) + \frac{1}{2}\sigma_\mu^2 \partial_{22} F(y,\mu) \tag{5}$$

for a given function $F(y,\mu)$, with ∂_1 and ∂_{11} denoting first and second partial derivatives with respective to the first argument, respectively, and ∂_2 and ∂_{22} denoting first and second partial derivatives with respective to the second argument, respectively. The equivalence between the value functions (3) and the PDEs (4) is a consequence of the Feynman-Kac formula (see, e.g., Duffie 2001, Appendix E).

Equilibrium. In equilibrium, free entry for vacancy creation implies $V(y,\mu) = 0$; from equation (4b), it follows that

$$c = q(\theta(y,\mu))J(y,\mu). \tag{6}$$

Nash bargaining splits the match surplus

$$S(y,\mu) \equiv J(y,\mu) - V(y,\mu) + W(y,\mu) - U(y,\mu) \tag{7}$$

such that the firm obtains $J(y,\mu) - V(y,\mu) = (1-\beta)S(y,\mu)$ and the worker obtains $W(y,\mu) - U(y,\mu) = \beta S(y,\mu)$; it follows that wages equal $w(y,\mu) = \beta e^y + (1-\beta)z + \beta c\theta(y,\mu)$.

The equilibrium can be characterized through a single PDE for the match surplus:

$$(r_f + s)S(y,\mu) = e^y - z - \frac{\beta c}{1-\beta}\theta(y,\mu) + \mathscr{L}^{\mathcal{P}} S(y,\mu), \tag{8a}$$

$$c = (1-\beta)q(\theta(y,\mu))S(y,\mu), \tag{8b}$$

where $\mathscr{L}^{\mathcal{P}}$ is defined in equation (5). Equation (8a) follows from substituting the PDE system (4) into definition (7) for the match surplus, and equation (8b) follows from the free entry condition (6).

All equilibrium quantities can be recovered from the match surplus $S(y,\mu)$ after it is computed. For example, equation (8b) implies that equilibrium tightness $\theta(y,\mu)$ is equal to $q^{-1}\left(c/\left[(1-\beta)S(y,\mu)\right]\right)$.

2.2 Economy *with* risk premia

The economy with risk premia is identical to the economy without risk premia described in Section 2.1 in all but two aspects. The first difference is that labor productivity in the economy with risk premia follows

$$dy_t = -\kappa_y y_t dt + \sigma_y dB_t^{y,\mathcal{P}}. \tag{9}$$

This corresponds to the special case of the process (1) whereby long run productivity is constant and equal to zero (i.e., $\mu_t = 0$).

The second and main difference is that the economy with risk premia assesses a risk premium when discounting cashflows. We model the risk premium as follows. First, we assume that there is perfect risk sharing between members of the representative household. As a result, workers and firms are symmetric in their assessment of aggregate risks and share a common discount factor. From the Fundamental Theorem of Asset Pricing, the absence of arbitrage in asset markets imply the existence of a "stochastic discount factor" (SDF) Λ_t (see, e.g., Duffie 2001, chapter 6). The SDF Λ_t discounts a stream of cashflows $\{C_s\}_{s\geq t}$ for dates $s \geq t$ to obtain its present value at date t according to the asset pricing equation

$$P_t = \mathbb{E}_t^{\mathcal{P}}\left[\int_t^{\infty} \frac{\Lambda_s}{\Lambda_t} C_s \, ds\right]. \tag{10}$$

That is, the present value of a cashflow C_s at date s is computed using the SDF between date t and date s, Λ_s/Λ_t.

We model the SDF according to

$$\frac{d\Lambda_t}{\Lambda_t} = -r_f dt - \eta_t dB_t^{y,\mathcal{P}}, \tag{11}$$

where r_f is the risk-free rate, and the market price of risk η_t is the risk premium for any asset whose return has a unit exposure to the aggregate productivity shock $dB_t^{y,\mathcal{P}}$. That is, suppose the instantaneous return associated with the asset price (10), $R_{t,t+dt} \equiv (P_{t+dt} + C_t dt)/P_t$, has form $R_{t,t+dt} = \mathbb{E}_t^{\mathcal{P}}[R_{t,t+dt}] + dB_t^{y,\mathcal{P}}$. Then, the asset pricing equation (10) implies that the instantaneous risk premium is $\mathbb{E}_t\left[R_{t,t+dt}\right] - r_f dt = -Cov_t^{\mathcal{P}}\left(d\Lambda_t/\Lambda_t, R_{t,t+dt}\right) = \eta_t dt$.[2]

We specify the market price of risk to vary according to the law of motion

$$d\eta_t = \kappa_\eta\left(\overline{\eta} - \eta_t\right)dt + \sigma_\eta dB_t^{\eta,\mathcal{P}}, \tag{12}$$

where $\overline{\eta} = \mathbb{E}^{\mathcal{P}}[\eta_t]$ is the average market price of risk, $\kappa_\eta > 0$ and $\sigma_\eta > 0$ parameterize the speed of mean reversion and volatility of discount rate shocks, respectively, and $B_t^{\eta,\mathcal{P}}$ is a standard Brownian motion under $\mathcal{P}$ measure. We assume that the discount rate shock $dB_t^{\eta,\mathcal{P}}$ is orthogonal to the productivity shock $dB_t^{y,\mathcal{P}}$; this is in line with the low correlation between productivity growth and asset values in the data (see, e.g., Hall 2017, Section III.A). Specifications (11) and (12) closely correspond to those used in the asset pricing literature for pricing financial assets. For example, Brennan et al. (2004) and Lettau and Wachter (2007) use a closely related expression for valuing stocks; similar specifications are also used to value Treasury bonds in affine term structure models (see, e.g., Singleton 2006). These specifications reflect the consensus from the asset pricing literature that discount rates are time-varying and the risk-free rate is stable (see, e.g., Cochrane 2011 for a summary of the evidence).

For illustrative purposes, we analyze three, progressively more flexible, discount rate processes: a constant $\eta_t = \overline{\eta}$, a time-varying η_t (12), and one in which the discount rate shock $dB_t^{\eta,\mathcal{P}}$ also carries its own market price of risk (see Section 4.3).

Risk-neutral measure. The value functions in the economy with risk premia satisfy

$$J_t = \mathbb{E}_t^{\mathcal{P}}\left[\int_t^{\tau_{sep}} \frac{\Lambda_{t+u}}{\Lambda_t}\left(e^{y_u} - w_u\right)du + \frac{\Lambda_{\tau_{sep}}}{\Lambda_t}V_{\tau_{sep}}\right], \tag{13a}$$

[2]The expression for the instantaneous risk premium can be derived by noting that the gains process $G_t \equiv \int_0^t \Lambda_s C_s\, ds + \Lambda_t P_t$ is a Martingale (this follows from no arbitrage). The expression for the instantaneous risk premium follows from applying Ito's lemma to G_t and equating the drift term to zero.

$$V_t \;=\; \mathbb{E}_t^{\mathcal{P}}\left[-\int_t^{\tau_{match}} \frac{\Lambda_{t+u}}{\Lambda_t} c\, du + \frac{\Lambda_{\tau_{match}}}{\Lambda_t} J_{\tau_{match}}\right], \tag{13b}$$

$$W_t \;=\; \mathbb{E}_t^{\mathcal{P}}\left[\int_t^{\tau_{sep}} \frac{\Lambda_{t+u}}{\Lambda_t} w_u\, du + \frac{\Lambda_{\tau_{sep}}}{\Lambda_t} U_{\tau_{sep}}\right], \tag{13c}$$

$$U_t \;=\; \mathbb{E}_t^{\mathcal{P}}\left[\int_t^{\tau_{match}} \frac{\Lambda_{t+u}}{\Lambda_t} z\, du + \frac{\Lambda_{\tau_{match}}}{\Lambda_t} W_{\tau_{match}}\right]. \tag{13d}$$

These equations are analogous to their counterparts from the economy without risk premia (3). The difference is that cashflows are not discounted at the risk-free rate, but are instead discounted using the SDF according to the asset pricing equation (10). As in the case of the economy without risk premia, we characterize the value functions (13) as the solution to a system of PDEs. To do so, we make use of a tool from the asset pricing literature—risk-neutral valuation (see, e.g., Duffie 2001, chapter 6). This tool computes the present value of a cashflow stream by discounting at the risk free rate but assumes an altered productivity process underlying the cashflow stream.

The risk-neutral measure $\mathcal{Q}$ corresponding to the SDF (11) is defined as

$$\mathcal{Q}(A) = \int_A \exp\left(-\frac{1}{2}\int_0^t \eta_u^2\, du - \int_0^t \eta_u\, dB_u^{y,\mathcal{P}}\right) d\mathcal{P} \tag{14}$$

for any event A in the time t information set, where the integrand is equal to $e^{r_f t}\Lambda_t/\Lambda_0$. Equation (14) shows that $\mathcal{Q}$ places a higher weight on low output realizations (i.e., low realizations for $dB_t^{y,\mathcal{P}}$) when the market price of risk is positive $\eta_t > 0$. The risk-neutral productivity process is

$$dy_t = \kappa_y\left(-\frac{\sigma_y}{\kappa_y}\eta_t - y_t\right) dt + \sigma_y dB_t^{y,\mathcal{Q}} \tag{15}$$

where

$$B_t^{y,\mathcal{Q}} \equiv \int_0^t \eta_u\, du + B_t^{y,\mathcal{P}} \tag{16}$$

is a standard Brownian motion under $\mathcal{Q}$. That is, under the risk-neutral measure, productivity behaves as if it has a time-varying long run productivity equal to $-\sigma_y\eta_t/\kappa_y$.

The asset pricing equation (10) can be conveniently rewritten as

$$P_t = \mathbb{E}_t^{\mathcal{Q}}\left[\int_t^{\infty} e^{-r_f(s-t)} C_s\, ds\right] \tag{17}$$

under the risk-neutral measure. That is, under "risk-neutral pricing" (17), cashflows are discounted at the risk-free rate, but are assumed to be generated by the risk-neutral productivity process (15) (and not by the physical productivity process (9)). The value functions (13) become

$$\begin{aligned}
J_t &= \mathbb{E}_t^{\mathcal{Q}}\left[\int_t^{\tau_{sep}} e^{-r_f u}\left(e^{y_u} - w_u\right) du + e^{-r_f(\tau_{sep}-t)} V_{\tau_{sep}}\right], && \text{(18a)}\\
V_t &= \mathbb{E}_t^{\mathcal{Q}}\left[-\int_t^{\tau_{match}} e^{-r_f u} c\, du + e^{-r_f(\tau_{match}-t)} J_{\tau_{match}}\right], && \text{(18b)}\\
W_t &= \mathbb{E}_t^{\mathcal{Q}}\left[\int_t^{\tau_{sep}} e^{-r_f u} w_u\, du + e^{-r_f(\tau_{sep}-t)} U_{\tau_{sep}}\right], && \text{(18c)}\\
U_t &= \mathbb{E}_t^{\mathcal{Q}}\left[\int_t^{\tau_{match}} e^{-r_f u} z\, du + e^{-r_f(\tau_{match}-t)} W_{\tau_{match}}\right], && \text{(18d)}
\end{aligned}$$

when we use risk-neutral pricing. The difference between (18) and its counterpart from the economy without risk premia (3) is the measure used for computing expectations—the latter uses $\mathcal{P}$ measure while the former uses $\mathcal{Q}$ measure under which low productivity paths are more likely.

Equilibrium. Similar to the economy without risk premia, a Markov equilibrium in the economy with risk premia can be characterized through a single PDE for the match surplus:

$$\begin{aligned}
(r_f + s)S(y,\eta) &= e^y - z - \frac{\beta c}{1-\beta}\theta(y,\eta) + \mathscr{L}^{\mathcal{Q}} S(y,\eta), && \text{(19a)}\\
c &= (1-\beta)q(\theta(y,\eta))S(y,\eta), && \text{(19b)}
\end{aligned}$$

where $\mathscr{L}^{\mathcal{Q}}$ is defined by

$$\begin{aligned}
&\mathscr{L}^{\mathcal{Q}}[\kappa_y, \sigma_y, \kappa_\eta, \overline{\eta}, \sigma_\eta]F(y,\eta)\\
&\equiv -(\sigma_y\eta + \kappa_y y)\partial_1 F(y,\eta) + \frac{1}{2}\sigma_y^2\partial_{11}F(y,\eta) + \kappa_\eta(\overline{\eta}-\eta)\partial_2 F(y,\eta) + \frac{1}{2}\sigma_\eta^2\partial_{22}F(y,\eta)
\end{aligned} \tag{20}$$

for a given function $F(y, \eta)$. Equation (19) is the analog of equation (8) for the characterization of the equilibrium in the economy without risk premia. The difference is that $\mathscr{L}^{\mathcal{Q}}$ in equation (19a) incorporates the laws of motion (12) and (15) under the risk-neutral measure $\mathcal{Q}$ whereas $\mathscr{L}^{\mathcal{P}}$ in equation (8a) incorporates the laws of motion (1) under the physical probability measure $\mathcal{P}$.

3 An equivalence result for unemployment fluctuations

The main result of this section and of this paper is an equivalence result (Proposition 1) which translates between unemployment outcomes in economies with and without risk premia. We subsequently make use of this dictionary in our applications. The equivalence result focuses on the behavior of labor market tightness across the two types of economies—this is sufficient because tightness determines job finding rates and, in turn, unemployment fluctuations through the law of motion (2).

Consider an economy with risk premia, with surplus $S(y, \eta)$ and tightness $\theta(y, \eta)$ characterized by equation (19). We refer to this economy as the "original economy". We demonstrate the link between $S(y, \eta)$ and $\theta(y, \eta)$ and their counterparts from an equivalent artificial economy without risk premia, described below, which we refer to as the "artificial economy" for short. We denote by $S^{\mathcal{Q}}(y^{\mathcal{Q}}, \mu^{\mathcal{Q}})$ and $\theta^{\mathcal{Q}}(y^{\mathcal{Q}}, \eta^{\mathcal{Q}})$ the equilibrium surplus and tightness, respectively, in the artificial economy. To avoid confusion, we used variables *with* $\mathcal{Q}$ superscripts to denote those from the artificial economy, and variables *without* $\mathcal{Q}$ superscripts to denote those from the original economy. This choice of notation reflects the fact that the artificial economy makes use of the risk-neutral measure $\mathcal{Q}$ to determine equilibrium outcomes (we demonstrate this below). Table 1 summarizes the notation for the original economy and the artificial economy.

Description	Original economy Symbol	Value	Artificial economy Symbol	Value
Market price of risk	η_t		n/a	
mean	$\overline{\eta}$	0.625	n/a	
speed of mean reversion	κ_η	0.139	n/a	
volatility	σ_η	0.257	n/a	
Log output per worker	y_t		$y_t^{\mathcal{Q}}$	$= y_t + \sigma_y\overline{\eta}/\kappa_y$
speed of mean reversion	κ_y	0.74	$\kappa_y^{\mathcal{Q}}$	$= \kappa_y$
volatility	σ_y	0.0179	$\sigma_y^{\mathcal{Q}}$	$= \sigma_y$
Long run productivity	n/a		$\mu_t^{\mathcal{Q}}$	$= \sigma_y(\overline{\eta} - \eta_t)/\kappa_y$
speed of mean reversion	n/a		$\kappa_\mu^{\mathcal{Q}}$	$= \kappa_\eta$
volatility	n/a		$\sigma_\mu^{\mathcal{Q}}$	$= \sigma_\eta\sigma_y/\kappa_y$
Value of leisure	z		$z^{\mathcal{Q}}$	$= \exp(\sigma_y\overline{\eta}/\kappa_y)z$
Vacancy maintenance cost	c		$c^{\mathcal{Q}}$	$= \exp(\sigma_y\overline{\eta}/\kappa_y)c$

Table 1: Notation and parameter values. This table summarizes the notation for parameters which are different across the original economy and the equivalent artificial economy. Workers' bargaining power β, the separation rate s, the matching function $m(u, v)$, and the risk-free rate r_f are identical across the two economies. Appendix A explains our choice of parameter values for the original economy. Parameter values for the artificial economy are based on the equivalence result (Proposition 1).

To see the link between the two economies, define

$$y_t^{\mathcal{Q}} \equiv y_t - \mathbb{E}^{\mathcal{Q}}[y_t] \qquad \text{and} \qquad \mu_t^{\mathcal{Q}} \equiv \sigma_y(\overline{\eta} - \eta_t)/\kappa_y \tag{21a, 21b}$$

to be the state variables in the artificial economy, where

$$\mathbb{E}^{\mathcal{Q}}[y_t] = -\sigma_y\overline{\eta}/\kappa_y \tag{22}$$

is the average productivity in the original economy under the risk-neutral measure $\mathcal{Q}$ (i.e., the mean of the risk-neutral productivity process (15) computed under the risk-neutral measure (14)). Note that the mean of productivity (22) is less than zero under the risk-neutral measure while it is zero under the physical measure. Similarly, define

$$z^{\mathcal{Q}} \equiv \exp(-\mathbb{E}^{\mathcal{Q}}[y_t])z \qquad \text{and} \qquad c^{\mathcal{Q}} \equiv \exp(-\mathbb{E}^{\mathcal{Q}}[y_t])c \tag{23a, 23b}$$

to be the value of leisure and the vacancy maintenance cost, respectively, in the artificial economy. Then, the equilibrium quantities in the artificial economy are related to their counterparts from the original economy according to

$$S^{\mathcal{Q}}(y^{\mathcal{Q}}, \mu^{\mathcal{Q}}) = \exp(-\mathbb{E}^{\mathcal{Q}}[y_t])S(y,\eta) \qquad \text{and} \qquad \theta^{\mathcal{Q}}(y^{\mathcal{Q}}, \mu^{\mathcal{Q}}) = \theta(y,\eta). \tag{24a, 24b}$$

To derive the equivalence (24), substitute the change of variables in equations (21), (23), and (24) into equation (19), to get

$$(r_f + s)S^{\mathcal{Q}}(y^{\mathcal{Q}}, \mu^{\mathcal{Q}}) = \exp(y^{\mathcal{Q}}) - z^{\mathcal{Q}} - \frac{\beta c^{\mathcal{Q}}}{1-\beta}\theta^{\mathcal{Q}}(y^{\mathcal{Q}}, \mu^{\mathcal{Q}}) \tag{25a}$$

$$+ \mathscr{L}^{\mathcal{P}}[\kappa_y, \sigma_y, \kappa_\eta, \sigma_\eta\sigma_y/\kappa_y]S^{\mathcal{Q}}(y^{\mathcal{Q}}, \mu^{\mathcal{Q}}),$$

$$c^{\mathcal{Q}} = (1-\beta)q\left(\theta^{\mathcal{Q}}(y^{\mathcal{Q}}, \mu^{\mathcal{Q}})\right)S^{\mathcal{Q}}(y^{\mathcal{Q}}, \mu^{\mathcal{Q}}), \tag{25b}$$

where $\mathscr{L}^{\mathcal{P}}$ is defined in equation (5), and we make use of the facts $S_y = \exp(\mathbb{E}^{\mathcal{Q}}[y_t])S^{\mathcal{Q}}_{y^{\mathcal{Q}}}$, $S_{yy} = \exp(\mathbb{E}^{\mathcal{Q}}[y_t])S^{\mathcal{Q}}_{y^{\mathcal{Q}}y^{\mathcal{Q}}}$, $S_\eta = -\frac{\sigma_y}{\kappa_y}\exp(\mathbb{E}^{\mathcal{Q}}[y_t])S^{\mathcal{Q}}_{\mu^{\mathcal{Q}}}$, and $S_{\eta\eta} = \frac{\sigma_y^2}{\kappa_y^2}\exp(\mathbb{E}^{\mathcal{Q}}[y_t])S^{\mathcal{Q}}_{\mu^{\mathcal{Q}}\mu^{\mathcal{Q}}}$ to derive equation (25a). Comparing equations (8) and (25), we see that $S^{\mathcal{Q}}(y^{\mathcal{Q}}, \mu^{\mathcal{Q}})$ and $\theta^{\mathcal{Q}}(y^{\mathcal{Q}}, \mu^{\mathcal{Q}})$ correspond to the equilibrium surplus and tightness, respectively, of an artificial economy without risk premia.

The artificial economy has current productivity $y_t^{\mathcal{Q}}$, long run productivity $\mu_t^{\mathcal{Q}}$, value of leisure $z^{\mathcal{Q}}$, and vacancy maintenance costs $c^{\mathcal{Q}}$; the relations between these variables and those from the original economy are summarized in equations (21) and (23), respectively. The productivity process in the artificial economy is derived by substituting equations (21a) and (21b) into equations (12) and (15). This yields

$$dy_t^{\mathcal{Q}} = \kappa_y^{\mathcal{Q}}(\mu_t^{\mathcal{Q}} - y_t^{\mathcal{Q}})dt + \sigma_y^{\mathcal{Q}}dB_t^{y,\mathcal{Q}}, \tag{26a}$$

$$d\mu_t^{\mathcal{Q}} = -\kappa_\mu^{\mathcal{Q}}\mu_t^{\mathcal{Q}}dt - \sigma_\mu^{\mathcal{Q}}dB_t^{\eta,\mathcal{P}}, \tag{26b}$$

where the parameters are defined by

$$\kappa_y^{\mathcal{Q}} \equiv \kappa_y, \qquad \sigma_y^{\mathcal{Q}} \equiv \sigma_y, \qquad \kappa_\mu^{\mathcal{Q}} \equiv \kappa_\eta, \qquad \sigma_\mu^{\mathcal{Q}} \equiv \frac{\sigma_\eta\sigma_y}{\kappa_y}. \tag{27}$$

Equations (26) and (27) show effects of risk premia on the dynamics of productivity in the artificial economy. First, positive shocks $dB_t^{y,\mathcal{P}}$ in the productivity process of the original economy (9) translate into positive shocks $dB_t^{y,\mathcal{Q}}$ in the short run productivity process of artificial economy (26a), where the link between the two shocks is given by equation (16). Second, positive discount rate shocks $dB_t^{\eta,\mathcal{P}}$ in the market price of risk process of the original economy (12) translate into negative shocks for the long run productivity process of the artificial economy (26b). Third, the volatility of long run productivity process (26b) $\sigma_\mu^{\mathcal{Q}}$ is scaled by the volatility of the market price of risk σ_η so that more volatile asset markets translate into larger swings in the long run productivity of the artificial economy.

These results indicate that the original economy with risk premia can be viewed as an artificial economy without risk premia, but with an altered productivity process and altered structural parameters. Proposition 1 summarizes this result.

Proposition 1 (Equivalence result)**.** *Let $\theta(y, \eta)$ be the labor market tightness in the original economy with risk premia. Then, $\theta(y, \eta) = \theta^{\mathcal{Q}}(y^{\mathcal{Q}}, \mu^{\mathcal{Q}})$, where $\theta^{\mathcal{Q}}(y^{\mathcal{Q}}, \mu^{\mathcal{Q}})$ is the labor market tightness in the artificial economy without risk premia whose labor productivity evolves according to process* (26) *with the parameters in equation* (27)*. The mappings* (21)*,* (23)*, and* (24) *give the link between the state variables, parameters, and equilibrium objects in the two economies, respectively. All remaining features are identical across the two economies.*

Equilibrium moments. Suppose we are interested in the moment of a labor market statistic $H_t = \mathcal{H}(\{\theta(y_s, \eta_s)\}_{s \leq t})$ in the original economy for a given function $\mathcal{H}$. For example, $\mathcal{H}$ would be determined by the law of motion (2) in the case of the unemployment rate $H_t = u_t$. The moment for this statistic is $\mathbb{E}^{\mathcal{P}}[H_t]$ which computes the moment by drawing the state variables y_t and η_t of the original economy under the physical probability measure $\mathcal{P}$ according to the laws of motion (9) and (12). Proposition 1 establishes an equivalence in labor market policies between the original economy and the artificial economy on a path by path basis. This implies $H_t = H_t^{\mathcal{Q}}$ where $H_t^{\mathcal{Q}} \equiv \mathcal{H}(\{\theta^{\mathcal{Q}}(y_s^{\mathcal{Q}}, \mu_s^{\mathcal{Q}})\}_{s \leq t})$ is the corresponding statistic in the artificial economy. The moment in the original economy can then be computed as $\mathbb{E}^{\mathcal{P}}[H_t^{\mathcal{Q}}]$.

4 High discounts and low fundamental surplus

In this section, we show the link between two leading explanations of unemployment fluctuations—high discounts (Hall, 2017) and low fundamental surplus (Ljungqvist and Sargent, 2017).

The discount rate channel from Hall (2017) works as follows. Risk premia rise following adverse discount rate shocks (e.g., due to events such as financial crises). This leads to a rise in discount rates and a fall in asset valuations; the latter is positively correlated with the value from hiring a new worker. As a result, hiring decreases and unemployment increases.

Ljungqvist and Sargent (2017), henceforth LS, show that, in a wide class of models without risk premia, realistic unemployment fluctuations require a low value for a quantity known as the *fundamental surplus*, defined as "a quantity that deducts from productivity a value that the *invisible hand* cannot allocate to vacancy creation" (p. 2631, emphasis from original text).

In what follows, we use Proposition 1 to first link an economy with risk premia to an equivalent artificial economy without risk premia. The absence of risk premia in the artificial economy allows us to directly apply the LS framework to understand unemployment fluctuations in the artificial economy. This, in turn, allows us to infer unemployment dynamics in the original economy with risk premia. To simplify the exposition, we first consider the special case of a constant risk premium in Section 4.1. We then consider the general case of time-varying risk premia in Section 4.2. Section 4.3 considers the even more general case where discount rate shocks carry its own market price of risk.

4.1 Case I: constant risk premia

Consider a special case of the economy with risk premia described in Section 2.2 in which the market price of risk is constant. That is, $\eta_t = \overline{\eta}$ for all t; this is obtained by setting $\sigma_\eta = 0$ and $\eta_0 = \overline{\eta}$ in the law of motion (12). Next, consider the equivalent artificial economy without risk premia defined in Proposition 1. A constant $\eta = \overline{\eta}$ implies that long run productivity $\mu^{\mathcal{Q}}$ in the artificial economy is a constant and equal to zero (see equation (21b)); that is, $\mu_t^{\mathcal{Q}} = 0$ for all t. This artificial economy corresponds to the canonical matching model with Nash bargaining considered in LS.

Let $\theta(y)$ and $\theta^{\mathcal{Q}}(y^{\mathcal{Q}})$ be the equilibrium tightness in the original economy and the equivalent artificial economy, respectively. Both θ and $\theta^{\mathcal{Q}}$ depend only on productivity because η and $\mu^{\mathcal{Q}}$ are constants. The equality $\theta(y) = \theta^{\mathcal{Q}}(y^{\mathcal{Q}})$ derived in Proposition 1 implies

$$\frac{d\log\theta(y)}{dy} = \frac{d\log\theta^{\mathcal{Q}}(y^{\mathcal{Q}})}{dy^{\mathcal{Q}}}. \tag{28}$$

That is, the elasticity of tightness with respect to productivity is the same across the two economies after taking into account the mapping (21a) between y and $y^{\mathcal{Q}}$. This result allows us to apply the LS results for unemployment fluctuations in economies without risk premia to make inferences regarding unemployment fluctuations in the original economy with risk premia.

LS show that the magnitude of unemployment fluctuations is determined through the elasticity of labor market tightness with respect to productivity shocks. This holds in all models without risk premia in which productivity shocks are the main driving force behind unemployment fluctuations. LS show that for the canonical matching model with Nash bargaining and zero risk premia, the elasticity on the right hand side of equation (28) is proportional to

$$\frac{\exp(y^{\mathcal{Q}})}{\exp(y^{\mathcal{Q}}) - z^{\mathcal{Q}}}. \tag{29}$$

They refer to (29) as the inverse of the "fundamental surplus fraction", with the "fundamental surplus" being the denominator of (29). LS shows that the inverse fundamental surplus fraction (29) evaluated at the mean productivity value $\mathbb{E}^{\mathcal{Q}}[y_t^{\mathcal{Q}}] = 0$, or $1/(1 - z^{\mathcal{Q}})$, is positively related to the level of unemployment volatility in the artificial economy $std^{\mathcal{Q}}(u_t)$. From this, we see that higher values of $z^{\mathcal{Q}}$ imply a higher elasticity of tightness and larger unemployment fluctuations in the artificial economy.

Higher values of $z^{\mathcal{Q}}$ in the artificial economy also leads to larger unemployment fluctuations in the original economy. This follows from first noting the equality of the elasticity of tightness across the two economies (28), and then applying the LS argument above which relates the elasticity of tightness in the artificial economy to the inverse surplus fraction (29).[3] As a result, the effect of risk premia on unemployment fluctuations can be understood in terms of its effect on the value of $z^{\mathcal{Q}}$ in the artificial economy.

A high discount in the original economy is equivalent to a high $z^{\mathcal{Q}}$ in the artificial economy. This follows from equations (22) and (23a) which imply that a high market price of risk $\overline{\eta}$ leads to a high value of $z^{\mathcal{Q}} = z \exp(\sigma_y \overline{\eta}/\kappa_y)$. As a result, high discounts generate a low fundamental surplus and a large elasticity of tightness in the artificial economy $d \log \theta^{\mathcal{Q}}/dy^{\mathcal{Q}}$. Through equation (28), a large $d \log \theta^{\mathcal{Q}}/dy^{\mathcal{Q}}$, in turn, implies a large elasticity of tightness and large unemployment fluctuations in the original economy. These results are illustrated in Figure 1—we see that the value of leisure $z^{\mathcal{Q}}$ in the artificial economy (Panel A), and elasticity of tightness $d \log \theta(y)/dy$ evaluated at the mean value of productivity $y = 0$ (solid

[3]Strictly speaking, the LS result that links the elasticity of tightness $d \log \theta^{\mathcal{Q}}(y^{\mathcal{Q}})/dy$ to the inverse surplus fraction (29) only holds at the mean value of productivity in the artificial economy, $y^{\mathcal{Q}} = \mathbb{E}^{\mathcal{Q}}[y_t^{\mathcal{Q}}] = 0$. This is because their derivations rely on steady-state comparative statics. Equation (28) implies that the value of elasticity in the original economy evaluated at its mean productivity value, or $d \log \theta(y)/dy$ evaluated at $y = \mathbb{E}^{\mathcal{P}}[y_t] = 0$, is instead equal to $d \log \theta^{\mathcal{Q}}(y^{\mathcal{Q}})/dy^{\mathcal{Q}}$ evaluated at $y^{\mathcal{Q}} = -\mathbb{E}^{\mathcal{Q}}[y_t]$. However, our numerical simulations indicate that $d \log \theta(y)/dy$ evaluated at $y = 0$ is close to $d \log \theta^{\mathcal{Q}}(y^{\mathcal{Q}})/dy^{\mathcal{Q}}$ evaluated at $y^{\mathcal{Q}} = 0$ for empirically plausible parameter values. This can be seen in panel B of Figure 1 which compares the elasticity of tightness across the original and the artificial economies. For example, when $\overline{\eta}$ is equal to its data value of 0.625, $d \log \theta(y)/dy$ evaluated at $y = 0$ is 7.6, and $d \log \theta^{\mathcal{Q}}(y^{\mathcal{Q}})/dy^{\mathcal{Q}}$ evaluated at $y^{\mathcal{Q}} = 0$ is 8.2. Similarly, panel B of Figure 2 shows that the two elasticities are close to each other when risk premia varies over time.

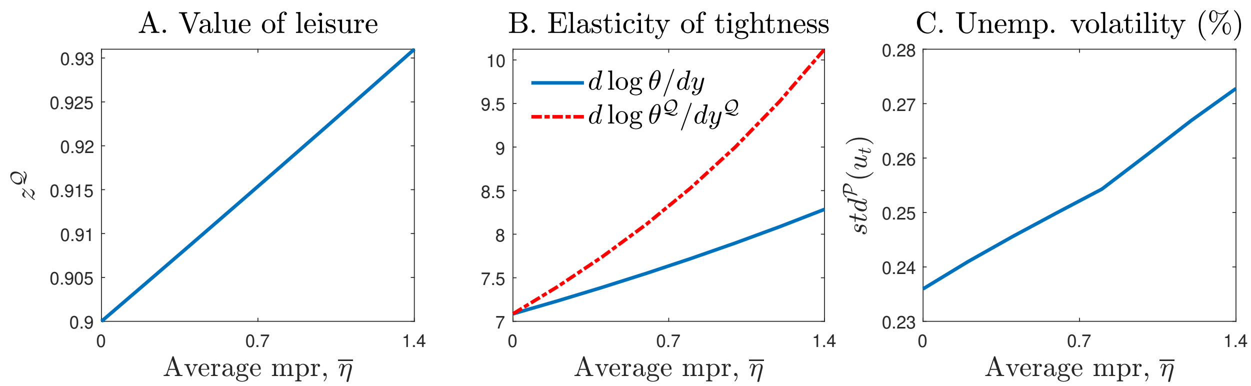

Figure 1: Illustration of equivalence result: constant market price of risk. Panel A plots the value of leisure in the equivalent artificial economy $z^{\mathcal{Q}}$ as we vary the average market price of risk $\overline{\eta}$. The solid line in Panel B plots the elasticity of tightness in the original economy evaluated at $y = \mathbb{E}^{\mathcal{P}}[y_t]$ while the dash-dot line plots the elasticity of tightness in the artificial economy evaluated at $y^{\mathcal{Q}} = \mathbb{E}^{\mathcal{Q}}[y_t^{\mathcal{Q}}]$. Panel C plots the unconditional volatility of unemployment $std^{\mathcal{P}}(u_t)$ in the original economy. We fix $z = 0.9$ and set workers' bargaining power $\beta = 0.06$ so that the model-implied wage to output elasticity (for the $\overline{\eta} = 0$ economy) matches its data counterpart of 0.43. For each value of $\overline{\eta}$, we calibrate vacancy posting costs c so that the model-implied mean unemployment rate is equal that of the data. The remaining parameter values are taken from Table 1 and Appendix A.

line of Panel B) and volatility of unemployment $std^{\mathcal{P}}(u_t)$ (Panel C) in the original economy are all increasing in the market price of risk $\overline{\eta}$.

How large do discount rates have to be? Equations (22) and (23a) shows that risk aversion multiplies the value of leisure from the original economy z by the scaling factor

$$z^{\mathcal{Q}}/z = \exp(\sigma_y \overline{\eta}/\kappa_y) \tag{30}$$

to arrive at the value of leisure in the equivalent artificial economy without risk premia $z^{\mathcal{Q}}$. From Hagedorn and Manovskii (2008), we know that a high value of $z^{\mathcal{Q}} = 0.955$ is needed to generate a low enough fundamental surplus and realistic levels of unemployment fluctuations in economies without risk premia. We show below that a model with a constant market price of risk generates a small value for the scaling factor (30). Such a model is therefore unable to generate the necessary $z^{\mathcal{Q}} = 0.955$ unless z is high to begin with.

The scaling factor (30) depends on the market price of risk $\overline{\eta}$, and the speed of mean reversion κ_y and volatility σ_y of productivity. In the data, estimates for these values are given by $\overline{\eta} = 0.625$, $\kappa_y = 0.74$, and $\sigma_y = 0.0179$ (see Table 1; Appendix A provides details for these estimates). Plugging these values into equation (30) yields a scaling factor of 1.015. This scaling factor implies that a value of $z = 0.941$ in the original economy is needed to reach $z^{\mathbb{Q}} = 0.955$ in the artificial economy. This value of $z = 0.941$ lies at the high end of estimates for the value of leisure (e.g., Chodorow-Reich and Karabarbounis 2016 estimate z to range between 0.47 and 0.96, depending on preference specifications).

Put differently, our estimate for the scaling factor (30) implies that low to intermediate values of z require counterfactually large values for the average market price of risk $\overline{\eta}$ to generate realistic unemployment fluctuations. To see this, solve equation (23a) to obtain the market price of risk that is required to generate a value of $z^{\mathbb{Q}} = 0.955$ for a given value of z,

$$\overline{\eta}_{required} = \kappa_y(\log 0.955 - \log z)/\sigma_y = -1.90 - 41.34 \log z, \tag{31}$$

where the second equality makes use of the data estimates for κ_y and σ_y. We can then compare the required market price of risk (31) to its data counterpart in order to assess the potential of generating realistic unemployment fluctuations for a given z. The literature has employed a wide range of values for z. At the low end, Shimer (2005) uses a value of $z = 0.4$ which implies a required value of $\overline{\eta}_{required} = 36$. At the high end, Hagedorn and Manovskii (2008) set $z = 0.955$ to match unemployment volatilities; risk premia are no longer necessary in this case and $\overline{\eta}_{required} = 0$. Christiano et al. (2016) use an intermediate value of $z = 0.88$ which implies a required $\overline{\eta}_{required} = 3.4$. Our data estimate for $\overline{\eta}$ is 0.625. These calculations imply that a counterfactually large value of $\overline{\eta}$ is needed to generate realistic unemployment fluctuations for low and intermediate values of z.

4.2 Case II: time-varying risk premia

We now consider the case of time-varying risk premia (TVRP) which is a well-documented feature in the data (see, e.g., Cochrane 2011). We show that increases in risk premia can be viewed as increases in the value of leisure in the equivalent artificial economy without risk premia; our approach therefore provides intuition for how TVRP can lower the fundamental surplus and amplify unemployment fluctuations.

Consider an economy with a time-varying market price of risk η_t that evolves according to equation (12). Proposition 1 implies that labor policies in this economy are equivalent to that of the equivalent artificial economy without risk premia in which long run productivity $\mu_t^{\mathcal{Q}}$ varies according to equation (26b). To see the effect of TVRP on the fundamental surplus in the artificial economy, decompose productivity in the artificial economy $y^{\mathcal{Q}} = \mu^{\mathcal{Q}} + \epsilon^{\mathcal{Q}}$ into the long run component $\mu^{\mathcal{Q}}$ and a transitory component $\epsilon^{\mathcal{Q}} \equiv y^{\mathcal{Q}} - \mu^{\mathcal{Q}}$. The inverse surplus ratio (29) can then be written as $\exp(\epsilon^{\mathcal{Q}})/(\exp(\epsilon^{\mathcal{Q}}) - z^{\mathcal{Q}} \exp(-\mu^{\mathcal{Q}}))$.

Suppose that the market price of risk is initially at its long-run mean $\eta = \overline{\eta}$. A discount rate shock of size $d\eta_t = \Delta\eta$ leads to an inverse fundamental surplus of approximately

$$\frac{\exp(\epsilon^{\mathcal{Q}})}{\exp(\epsilon^{\mathcal{Q}}) - z^{\mathcal{Q}} \exp(-\mu^{\mathcal{Q}})} \approx \frac{1}{1 - z^{\mathcal{Q}}(\Delta\eta)}, \tag{32}$$

where $z^{\mathcal{Q}}(\Delta\eta) \equiv z^{\mathcal{Q}} \exp(\sigma_\eta \Delta\eta/\kappa_y)$ is the effective value of leisure in the artificial economy following the discount rate shock. The approximation (32) assumes that $\epsilon^{\mathcal{Q}}$ mean reverts instantaneously following a discount rate shock, while the effect of the discount rate shock on $\mu^{\mathcal{Q}} = \sigma_y(\overline{\eta} - \eta)/\kappa_y$ (see equation (21b)) is permanent. We make this approximation because the estimates from Table 1 imply that $\mu^{\mathcal{Q}}$ mean reverts slowly following discount rate shocks while $\epsilon^{\mathcal{Q}}$ mean reverts quickly.[4]

[4]The speed of mean reversion for $\mu^{\mathcal{Q}}$ is $\kappa_\mu^{\mathcal{Q}} = \kappa_\eta = 0.139$ which implies a half-life of 5 years. The transitory component has law of motion $d\epsilon_t^{\mathcal{Q}} = \left(-\kappa_y^{\mathcal{Q}} \epsilon_t^{\mathcal{Q}} + \kappa_\mu^{\mathcal{Q}} \mu_t^{\mathcal{Q}}\right) dt + \sigma_y^{\mathcal{Q}} dB_t^{y,\mathcal{Q}} + \sigma_\mu^{\mathcal{Q}} dB_t^{\eta,\mathcal{P}}$ which is obtained by subtracting equation (26b) from equation (26a). To first order, we can ignore the $\kappa_\mu^{\mathcal{Q}} \mu_t^{\mathcal{Q}}$ term in the law of motion because $\kappa_\mu^{\mathcal{Q}} = \kappa_\eta = 0.139$ is small relative to $\kappa_y^{\mathcal{Q}} = \kappa_y = 0.74$. The speed of mean reversion for $\epsilon^{\mathcal{Q}}$

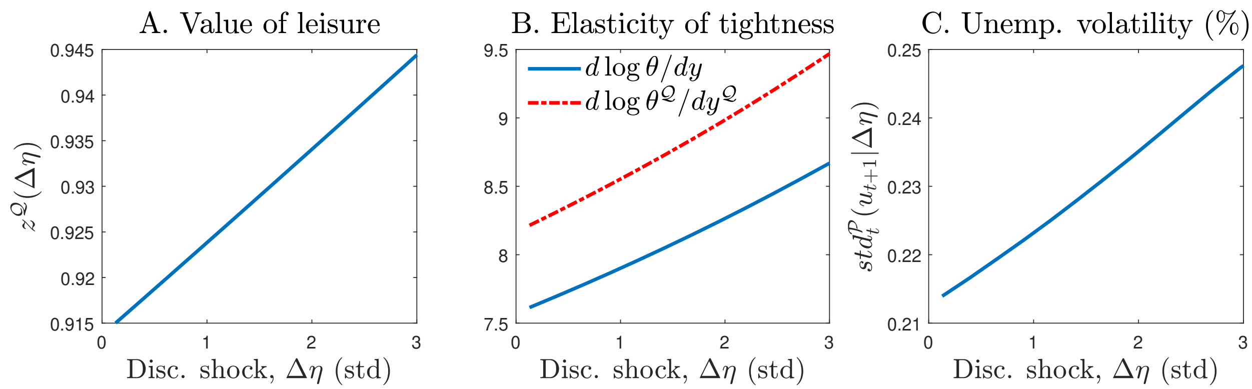

Figure 2: Illustration of equivalence result: time-varying market price of risk. Panel A plots the value of leisure in the artificial economy $z^{\mathcal{Q}}(\Delta\eta)$ following a discount rate shock of size $\Delta\eta$, where the plot displays $\Delta\eta$ in units of unconditional standard deviations. Panel B plots the elasticity of tightness conditional on the discount rate shock. The solid line is for the original economy (when productivity is $y = \mathbb{E}^{\mathcal{P}}[y_t]$), and the dash-dot line is for the artificial economy (when productivity is $y^{\mathcal{Q}} = \mathbb{E}^{\mathcal{Q}}[y_t^{\mathcal{Q}}]$). Panel C plots the conditional volatility of unemployment one year after the shock $std^{\mathcal{P}}(u_{t+1}|\Delta\eta)$. The values in all three panels are computed under the assumption that u_t, y_t, and η_t are at their mean values just prior to the arrival of the discount rate shock. We fix $z = 0.9$ and set workers' bargaining power $\beta = 0.06$ so that the model-implied wage to output elasticity matches its data counterpart of 0.43; we calibrate vacancy posting costs c so that model-implied mean unemployment rate matches the data. The remaining parameter values are taken from Table 1 and Appendix A.

TVRP and unemployment fluctuations. The effect of TVRP on unemployment fluctuations can be seen from equation (32) and is illustrated in Figure 2. A discount rate shock of size $\Delta\eta$ increases the effective value of leisure in the artificial economy from $z^{\mathcal{Q}}$ to $z^{\mathcal{Q}}(\Delta\eta)$ (see Panel A of Figure 2). This lowers the fundamental surplus and increases the inverse surplus fraction (32). Subsequently, the elasticity of tightness becomes larger (Panel B) which leads to a higher volatility in unemployment (Panel C) in the original economy.

To gauge the size of the effect of discount rate shocks on $z^{\mathcal{Q}}(\Delta\eta)$, consider the scaling factor

$$z^{\mathcal{Q}}(\Delta\eta)/z = \exp(\sigma_y(\overline{\eta} + \Delta\eta)/\kappa_y) \tag{33}$$

following a discount rate shock of size $\Delta\eta$. Compared to the scaling factor (30) from an

is therefore approximately $\kappa_y^{\mathcal{Q}} = 0.74$ and the corresponding half-life is 0.9 years.

economy with a constant market price of risk, the scaling factor (33) increases with the size of the discount rate shock $\Delta\eta$. The value of the scaling factor (33) can be estimated using the parameters from Table 1; it is 1.027 for a one unconditional standard deviation discount rate shock (i.e., $\Delta\eta = 0.49$),[5] and 1.04 for a two unconditional standard deviation discount rate shock.

4.3 Case III: priced discount rate shocks

For our third example, consider the economy with risk premia from Section 2.2 in which the SDF (10) is generalized to

$$\frac{d\Lambda_t}{\Lambda_t} = -r_f dt - \eta_t dB_t^{y,\mathcal{P}} - \lambda_\eta(\eta_t) dB_t^{\eta,\mathcal{P}}. \tag{34}$$

This SDF implies that discount rate shocks $dB_t^{\eta,\mathcal{P}}$ carry a market price of risk of $\lambda_\eta(\eta_t)$, and, as before, productivity shocks $dB_t^{y,\mathcal{P}}$ carry a market price of risk of η_t. We call this specification "priced discount rate shocks". We choose this specification because there is recent empirical evidence for this discount rate process (see Kozak and Santosh 2020). More importantly, it illustrates how our framework can be easily adapted to derive the implications of discount rate processes that are even more flexible than the one we considered in Section 4.2. Using our approach, we show that priced discount rate shocks can (1) further lower the fundamental surplus in the equivalent artificial economy, and (2) prolong the propagation of discount rate shocks to unemployment fluctuations.

Our results are for the affine specification

$$\lambda_\eta(\eta_t) = \lambda_{\eta,0} + \lambda_{\eta,1}\eta_t, \tag{35}$$

where $\lambda_{\eta,0}$ and $\lambda_{\eta,1}$ are constants. Appendix B shows that by applying the same procedure used to derive Proposition 1, we can obtain the following generalized equivalence result:

[5] The market price of risk process (12) implies an unconditional standard deviation of $std^{\mathcal{P}}(\eta_t) = \sigma_\eta/\sqrt{2\kappa_\eta}$ for the market price of risk. This value becomes 0.49 when we plug in values from Table 1.

Proposition 2 (Equivalence result, priced discount rate shocks)**.** *Let $\theta(y, \eta)$ be the labor market tightness in the original economy with risk premia and discount rate shocks priced according to equation (35). Then, $S(y, \eta) = \exp(-\sigma_y \mathbb{E}^{\mathcal{Q}}[\eta_t]/\kappa_y) S^{\mathcal{Q}}(y^{\mathcal{Q}}, \mu^{\mathcal{Q}})$ and $\theta(y, \eta) = \theta^{\mathcal{Q}}(y^{\mathcal{Q}}, \mu^{\mathcal{Q}})$, where $S^{\mathcal{Q}}(y^{\mathcal{Q}}, \mu^{\mathcal{Q}})$ and $\theta^{\mathcal{Q}}(y^{\mathcal{Q}}, \mu^{\mathcal{Q}})$ are the surplus and the labor market tightness, respectively, in the artificial economy without risk premia. The mapping between the state variables and parameters in the two economies is*

$$y_t^{\mathcal{Q}} \equiv y_t + \sigma_y \mathbb{E}^{\mathcal{Q}}[\eta_t]/\kappa_y, \qquad \mu_t^{\mathcal{Q}} \equiv \sigma_y(\mathbb{E}^{\mathcal{Q}}[\eta_t] - \eta_t)/\kappa_y, \tag{36a, 36b}$$

$$z^{\mathcal{Q}} \equiv \exp(\sigma_y \mathbb{E}^{\mathcal{Q}}[\eta_t]/\kappa_y) z, \qquad c^{\mathcal{Q}} \equiv \exp(\sigma_y \mathbb{E}^{\mathcal{Q}}[\eta_t]/\kappa_y) c, \tag{36c, 36d}$$

where

$$\mathbb{E}^{\mathcal{Q}}[\eta_t] = (\kappa_\eta \overline{\eta} - \lambda_{\eta,0}\sigma_\eta)/\kappa_\eta^{\mathcal{Q}}, \quad \text{and} \quad \kappa_\eta^{\mathcal{Q}} \equiv \kappa_\eta + \lambda_{\eta,1}\sigma_\eta. \tag{37a, 37b}$$

Labor productivity in the artificial economy evolves according to

$$dy_t^{\mathcal{Q}} = \kappa_y^{\mathcal{Q}}(\mu_t^{\mathcal{Q}} - y_t^{\mathcal{Q}})dt + \sigma_y^{\mathcal{Q}} dB_t^{y,\mathcal{Q}}, \tag{38a}$$

$$d\mu_t^{\mathcal{Q}} = -\kappa_\mu^{\mathcal{Q}} \mu_t^{\mathcal{Q}} dt - \sigma_\mu^{\mathcal{Q}} dB_t^{\eta,\mathcal{Q}}, \tag{38b}$$

where $dB_t^{y,\mathcal{Q}}$ and $dB_t^{\eta,\mathcal{Q}}$ are Brownian shocks under the risk-neutral measure $\mathcal{Q}$, and the parameters are equal to $\kappa_y^{\mathcal{Q}} \equiv \kappa_y$, $\sigma_y^{\mathcal{Q}} \equiv \sigma_y$, $\sigma_\mu^{\mathcal{Q}} \equiv \sigma_\eta \sigma_y/\kappa_y$, and

$$\kappa_\mu^{\mathcal{Q}} \equiv \kappa_\eta^{\mathcal{Q}}. \tag{39}$$

All remaining features are identical across the two economies.

Comparing Propositions 1 and 2, we see that priced discount rate shocks (35) implies two additional features. First, equation (37a) implies that the mean of the market price of risk under $\mathcal{Q}$ measure, $\mathbb{E}^{\mathcal{Q}}[\eta_t]$, is now different from its physical counterpart, $\overline{\eta} = \mathbb{E}^{\mathcal{P}}[\eta_t]$. This leads to a different scaling factor $z^{\mathcal{Q}}/z = \exp(\sigma_y \mathbb{E}^{\mathcal{Q}}[\eta_t]/\kappa_y)$ compared to that from an economy without priced discount rate shocks (30). Second, equations (37b) and (39) imply that priced discount rate shocks alter the speed of mean reversion for the long run productivity process (38b). These additional features have implications when it comes to amplifying and propagating the effect of discount rate shocks on unemployment fluctuations.

Amplification. The effect of priced discount rate shocks on the fundamental surplus in the artificial economy is summarized by the scaling factor $z^{\mathcal{Q}}/z = \exp(\sigma_y \mathbb{E}^{\mathcal{Q}}[\eta_t]/\kappa_y)$. To see the relation between this scaling factor and the one from an economy without priced discount rate shocks (30), consider the special case in which discount rate shocks carry a constant price of risk (i.e., $\lambda_\eta(\eta_t) = \lambda_{\eta,0}$). It then follows from equation (37a) that

$$z^{\mathcal{Q}}/z = \exp(\sigma_y \overline{\eta}/\kappa_y) \exp(-\lambda_{\eta,0}\sigma_y\sigma_\eta/(\kappa_y\kappa_\eta)). \tag{40}$$

The first term, $\exp(\sigma_y \overline{\eta}/\kappa_y)$, is the scaling factor when discount rate shocks are not priced (30). The second term, $\exp(-\lambda_{\eta,0}\sigma_y\sigma_\eta/(\kappa_y\kappa_\eta))$, summarizes the additional effect of priced discount rate shocks. In the data, Kozak and Santosh (2020) find a negative market price of risk for discount rate shocks, $\lambda_{\eta,0} < 0$,[6] which implies that the second term is greater than one. That is, priced discount rate shocks results in a smaller fundamental surplus in the artificial economy and therefore larger unemployment fluctuations.

Propagation. To see how priced discount rate shocks can result in a more persistent effect of discount rate shocks on unemployment fluctuations, write the inverse fundamental surplus (29) in terms of the long run component $\mu^{\mathcal{Q}}$ and the transitory component $\epsilon^{\mathcal{Q}} \equiv y^{\mathcal{Q}} - \mu^{\mathcal{Q}}$,

$$\frac{\exp(\epsilon^{\mathcal{Q}})}{\exp(\epsilon^{\mathcal{Q}}) - z^{\mathcal{Q}}\exp(-\mu^{\mathcal{Q}})} \approx \frac{1}{1 - z^{\mathcal{Q}}\exp(-\mu^{\mathcal{Q}})}. \tag{41}$$

As in Section 4.2, the approximation in equation (41) assumes that $\epsilon^{\mathcal{Q}}$ mean reverts instantaneously following a discount rate shock. Priced discount rate shocks affect the inverse surplus (41) by changing the speed of mean reversion of $\mu^{\mathcal{Q}}$ following a discount rate shock. This

[6]Intuitively, a negative market price of risk results when discount rate shocks covary positively with investor marginal utility. That is, if discount rates are more likely to increase during bad times. In this case, an asset whose return is positively correlated with discount rate shocks insures investors against bad states and therefore carries a negative market price of risk.

is because Proposition 2 (equations (37b) and 39) implies $\mu^{\mathcal{Q}}$ has speed of mean reversion $\kappa_\mu^{\mathcal{Q}} = \kappa_\eta^{\mathcal{Q}} = \kappa_\eta + \lambda_{\eta,1}\sigma_\eta$ which depends on the price of risk of discount rate shocks through $\lambda_{\eta,1}$. A negative $\lambda_{\eta,1}$ decreases the speed of reversion $\kappa_\mu^{\mathcal{Q}}$ so that positive discount rate shocks depress long run productivity in the artificial economy for a longer duration. This depresses the fundamental surplus (in the artificial economy) for a longer duration and therefore prolongs unemployment fluctuations following positive discount rate shocks. It would be interesting to investigate if this mechanism contributes to the slow recovery of unemployment rates following recessions documented in Hall and Kudlyak (2020).

5 Conclusion

We derive an equivalence result which provides a framework to understand unemployment fluctuations in an economy with time-varying discounts from the perspective of an artificial economy without time-varying discounts. In constructing the artificial economy, we build on a tool from the finance literature, namely risk-neutral valuation.

The artificial economy has an altered process for productivity and some of the values of the structural parameters. We show how various properties of the process for time-varying discounts impact the fundamental surplus in the artificial economy. This approach provides a transparent way to understand the effect of these different discount rates processes on the dynamics of the unemployment rate in the original economy. Although we illustrate our approach through three progressively more flexible discount rate processes, our approach is general and can be adapted to understand the effect of other discount rate processes.

Appendix

A Parameter values

The parameter values used in our numerical illustrations are set as follows. We estimate the productivity process (9) using the real output per person series for the nonfarm business sector (i.e., PRS85006163 from FRED) over the period 1951Q1-2019Q4. We log and HP-filter this series using a smoothing parameter of 10000 (as in Shimer 2005). The resulting series has an autocorrelation of 0.831 and a volatility of 0.0082. We convert these values to the parameters for the productivity process (9) using $0.831 = \mathbb{E}^{\mathcal{P}}[z_{t+\Delta}|z_t]/z_t = \exp(-\kappa_y\Delta)$ and $0.0082 = \sqrt{Var^{\mathcal{P}}(z_{t+\Delta}|z_t)} = \sigma_y\sqrt{(1-\exp(-2\kappa_y\Delta))/(2\kappa_y)}$ where $\Delta = 0.25$ for a horizon of a quarter. This results in $\kappa_y = 0.74$ and $\sigma_y = 0.0179$.

We estimate labor market parameters using the sample between 1951 and 2016. We construct labor market tightness using the Barnichon (2010) extended help-wanted index, which is available up to 2016. We obtain job-separation and job-finding rates following Elsby et al. (2009). This results in a mean separation rate of $s = 0.4$ or 3.37% per month. We estimate the curvature of the matching function by regressing the log job-finding rate on log labor market tightness after HP-filtering both series; this yields $\alpha = 0.64$. We set $A = 8.27$ so that the model-implied job-finding rate $f(\theta)$, evaluated at the mean value of tightness in the sample, agrees with its sample mean.

We set the risk-free rate $r_f = 0.013$ to be the difference between the average one year treasury rate and average inflation. The average one year treasury rate is 5.10% over 1962-2019 (based on the DGS1 series from FRED which is available starting from 1962) while average CPI inflation is 3.8% per anum over the same period.

Exercises involving a time-varying market price of risk (e.g., Figure 2) additionally require parameter estimates for the market price of risk process (12). We directly use the estimates from Lettau and Wachter (2007, Table IV) for this process. They fit a discrete time AR(1) process for η_t and find $\eta_{t+\Delta} = (1-0.87)\times 0.625 + 0.87\eta_t + 0.24\varepsilon_{\eta,t+\Delta}$, $\varepsilon_{\eta,t+\Delta} \sim N(0,1)$, is needed to justify stock returns over a horizon of $\Delta = 1$ year. We convert this discrete time process to its continuous time counterpart (12) using $\overline{\eta} = 0.625$, $0.87 = \exp(-\kappa_\eta\Delta)$, and $0.24 = \sigma_\eta\sqrt{(1-\exp(-2\kappa_\eta\Delta))/(2\kappa_\eta)}$, which implies $\kappa_\eta = 0.139$ and $\sigma_\eta = 0.257$.

B Derivation of Proposition 2

Proposition 2 can be derived using the procedure from Section 3 for deriving Proposition 1. The risk-neutral measure implied by the stochastic discount factor (SDF) (34) is equal to

$$\mathcal{Q}(A) = \int_A \exp\left(-\frac{1}{2}\int_0^t \eta_u^2\,du - \int_0^t \eta_u\,dB_u^{y,\mathcal{P}} - \frac{1}{2}\int_0^t \lambda_\eta(\eta_u)^2\,du - \int_0^t \lambda_\eta(\eta_u)\,dB_u^{\eta,\mathcal{P}}\right) d\mathcal{P} \quad \text{(B.1)}$$

for any event A in the time t information set. Compared to equation (14), the additional terms in equation (B.1) account for priced discount rate shocks in the SDF (34). The risk-neutral process for productivity y_t is still given by equation (15) while the risk-neutral process for the market price of risk η_t is

$$d\eta_t = \kappa_\eta^{\mathcal{Q}}\left(\overline{\eta}^{\mathcal{Q}} - \eta_t\right)dt + \sigma_\eta dB_t^{\eta,\mathcal{Q}}, \quad \text{(B.2)}$$

under the risk-neutral meaure (B.1), where $\kappa_\eta^{\mathcal{Q}} \equiv \kappa_\eta + \lambda_{\eta,1}\sigma_\eta$, $\overline{\eta}^{\mathcal{Q}} \equiv (\kappa_\eta\overline{\eta} - \lambda_{\eta,0}\sigma_\eta)/\kappa_\eta^{\mathcal{Q}}$, and $B_t^{\eta,\mathcal{Q}} \equiv \int_0^t \lambda_\eta(\eta_u)\,du + B_t^{\eta,\mathcal{P}}$ is a standard Brownian motion under the risk-neutral measure (B.1).

The equilibrium in the original economy can be characterized following the steps outlined in Section 2.2. In particular, the equilibrium surplus and tightness satisfies equation (19) with risk-neutral parameters for the market price of risk process—that is, with $\mathscr{L}^{\mathcal{Q}} = \mathscr{L}^{\mathcal{Q}}[\kappa_y, \sigma_y, \kappa_\eta^{\mathcal{Q}}, \overline{\eta}^{\mathcal{Q}}, \sigma_\eta]$ in equation (19a). As in Section 3, substituting the change of variables (36), and $S(y,\eta) = \exp(-\sigma_y \mathbb{E}^{\mathcal{Q}}[\eta_t]/\kappa_y) S^{\mathcal{Q}}(y^{\mathcal{Q}}, \mu^{\mathcal{Q}})$ and $\theta(y,\eta) = \theta^{\mathcal{Q}}(y^{\mathcal{Q}}, \mu^{\mathcal{Q}})$, into the equilibrium characterization for the original economy (i.e., equation (19) with $\mathscr{L}^{\mathcal{Q}} = \mathscr{L}^{\mathcal{Q}}[\kappa_y, \sigma_y, \kappa_\eta^{\mathcal{Q}}, \overline{\eta}^{\mathcal{Q}}, \sigma_\eta]$) yields the comparison to the artificial economy without risk premia. The productivity process for the artificial economy (38) follows from substituting the risk-neutral laws of motion (15) and (B.2) for y and η, respectively, into the definitions of $y^{\mathcal{Q}}$ and $\mu^{\mathcal{Q}}$, equations (36a) and (36b), respectively.

References

Regis Barnichon. Building a composite help-wanted index. *Economics Letters*, 109(3):175–178, 2010.

Jaroslav Borovicka and Kararina Borovickova. Risk premia and unemployment fluctuations. Working Paper, New York University, 2018.

Michael J. Brennan, Ashley W. Wang, and Yihong Xia. Estimation and test of a simple model of intertemporal capital asset pricing. *Journal of Finance*, 59(4):1743–1776, 2004.

John Y. Campbell. *Financial Decisions and Markets: A Course in Asset Pricing*. Princeton University Press, first edition, 2018. ISBN 9780691160801.

Gabriel Chodorow-Reich and Loukas Karabarbounis. The cyclicality of the opportunity cost of employment. *Journal of Political Economy*, 124(6):1563–1618, 2016.

Lawrence J. Christiano, Martin S. Eichenbaum, and Mathias Trabandt. Unemployment and business cycles. *Econometrica*, 84(4):1523–1569, 2016.

John H. Cochrane. Presidential address: Discount rates. *Journal of Finance*, 66(4):1047–1108, 2011.

Darrell Duffie. *Dynamic Asset Pricing Theory*. Princeton University Press, Princeton, New Jersey, third edition, 2001. ISBN 0-691-09022-X.

Michael W. L. Elsby, Ryan Michaels, and Gary Solon. The ins and outs of cyclical unemployment. *American Economic Journal: Macroeconomics*, 1(1):84–110, January 2009.

Marcus Hagedorn and Iourii Manovskii. The cyclical behavior of equilibrium unemployment and vacancies revisited. *American Economic Review*, 98(4):1692–1706, September 2008.

Robert E. Hall. High discounts and high unemployment. *American Economic Review*, 107 (2):305–30, February 2017.

Robert E Hall and Marianna Kudlyak. The inexorable recoveries of us unemployment. Working Paper 28111, National Bureau of Economic Research, November 2020.

Patrick J Kehoe, Pierlauro Lopez, Virgiliu Midrigan, and Elena Pastorino. Asset prices and unemployment fluctuations. *Review of Economic Studies*, forthcoming.

Mete Kilic and Jessica A. Wachter. Risk, unemployment, and the stock market: A rare-event-based explanation of labor market volatility. *Review of Financial Studies*, 31(12): 4762–4814, 2018.

Serhiy Kozak and Shrihari Santosh. Why do discount rates vary? *Journal of Financial Economics*, 137(3):740–751, 2020.

Martin Lettau and Jessica A. Wachter. Why is long-horizon equity less risky? a duration-based explanation of the value premium. *Journal of Finance*, 62(1):55–92, 2007.

Lars Ljungqvist and Thomas J. Sargent. The fundamental surplus. *American Economic Review*, 107(9):2630–65, September 2017.

Indrajit Mitra and Yu Xu. Time-Varying Risk Premium and Unemployment Risk across Age Groups. *Review of Financial Studies*, 33(8):3624–3673, 8 2020.

Toshihiko Mukoyama. A note on cyclical discount factors and labor market volatility. Working Paper, 2009.

Christopher A. Pissarides. Short-run equilibrium dynamics of unemployment, vacancies, and real wages. *American Economic Review*, 75(4):676–690, 1985.

Christopher A. Pissarides. *Equilibrium Unemployment Theory*. MIT Press, Cambridge, MA, second edition, 2000.

Robert Shimer. The cyclical behavior of equilibrium unemployment and vacancies. *American Economic Review*, 95(1):25–49, March 2005.

Kenneth J: Singleton. *Empirical Dynamic Asset Pricing: Model Specification and Econometric Assessment*. Princeton University Press, Princeton, New Jersey, 2006. ISBN 0691122970.