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Why Aging Induces Deflation and Secular Stagnation

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Abstract: We provide a quantitative theory of deflation and secular stagnation. In our lifecycle framework, an aging population puts persistent downward pressure on the price level, real interest rates, and output. A novel feature of our theory is that it also recognizes the reactions of government policy. The central bank responds to falling prices by reducing its policy nominal interest rate, and the fiscal authority responds by allowing the public debt–gross domestic product ratio to rise.

JEL classification: E52, E62, G51, D15

Key words: monetary policy, lifecycle, portfolio choice, secular stagnation, nominal government debt, aging, Tobin effect, fiscal policy, deflation

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1 Introduction

In the years prior to the COVID pandemic, many industrialized economies experienced a protracted episode of low and even negative inflation rates and what Rachel and Summers (2019) refer to as secular stagnation: low per capita output growth and low real interest rates. These nominal and real observations are striking because they occurred against a background of significant government stimulus. Central banks reduced their policy nominal interest rates to the effective lower bound (ELB) and pursued a variety of unconventional monetary policies (UMP). Fiscal policy was also loose with debt–output ratios rising to post World War II highs. What are the forces that are driving secular stagnation? Will these forces continue to exert downward pressure on the inflation rate, real interest rates and output growth in future years?

This paper investigates the hypothesis that aging of the population is putting downward pressure on the price level, per capita output and real interest rates. Aging is occurring in many industrialized economies and reflects the combined effects of the post World War II baby boom cohorts moving into retirement, higher life expectancies and lower fertility rates. We model a demographic transition to an older population distribution in a lifecycle model and show how and why aging produces the nominal and real secular stagnation observations that motivate our analysis. A novel feature of our model is that it captures the responses of monetary and fiscal policy to aging. The monetary authority responds to deflationary pressures by lowering the policy rate and the fiscal imbalances created by aging produce large and persistent increases in the debt–output ratio. Aging is ongoing and our results suggest that it will continue to put downward pressure on the inflation rate and per capita output and real interest rates in future years.

We conduct our quantitative analysis using Japanese data. Japan is in the midst of a particularly large demographic transition. Japan experienced a large increase in fertility rates in the period following World War II and these cohorts are now transitioning into retirement. Japan has also experienced large declines in the total fertility rate which fell from 2.14 in 1970 to 1.26 in 2015. Finally, Japan has experienced a modern health miracle. Life expectancies have increased from 60 years for Japanese born in 1950 to 84 years for individuals born in 2014. Japan's population is already falling and we project that the level of the Japanese 21+ population will decline from 102 million in the year 2015 to 80 million in 2055 and that the 70+ share of the population will increase from 23 percent to

38 percent.[1]

Japan is not unique. Other industrialized countries are in the midst of demographic transitions that are of a similar scale. Japan stands out though because its transition started earlier than other industrialized economies and is very rapid. For instance, as recently as the year of 1985 Japan had the lowest old-age dependency ratio in the Group of Seven, but by the year 2005 Japan had the largest old-age dependency ratio in this peer group.[2]

Aging in Japan has been accompanied by nominal and real secular stagnation and government policy has sought to counteract these macroeconomic trends. Real interest rates and the per capita growth rate of GDP have been low since the late 1990s. The policy nominal interest rate (overnight call rate) has been close to zero since the year 1997 and Japan has been the architect of a range of unconventional monetary policies starting with quantitative easing in the year 2001. Finally, fiscal stimulus has also been significant in Japan as measured by the government net debt–output ratio which increased from 0.2 in the year 1990 to 1.5 in 2019.[3]

How does our model account for the secular stagnation facts? One contributing factor is asset demand. Aging increases the size and composition of asset demand. Households hold portfolios of liquid and illiquid assets and a household's demand for each type of asset varies with its age. Young households borrow liquid assets and use them to purchase illiquid assets like homes and durable goods. Older households, in contrast, hold positive amounts of both assets because mortality risk increases with age and because their pension income does not fully replace their previous labor earnings. We calibrate the age–profile of household asset allocations in the model to Japanese data. Then we show that Japan's demographic transition produces persistent increases in aggregate asset demand (in partial equilibrium) for both assets, but that the increase in the demand for liquid assets is particularly large.

Next, we allow prices to adjust to clear markets and government policy to react to the demographic transition. The higher asset demand induced by aging depresses real interest rates. Government debt is a liquid asset and if the supply of nominal government debt doesn't change, then the price level has to fall to clear the market for liquid assets. Aging also reduces the size and average efficiency of the working–aged population which depresses per capita output and the marginal product of capital.

[1] Our projections are based on information provided by the National Institute of Population and Social Security Research. See Table 1 for more details.

[2] Specific details are provided in Section 2.

[3] See Section 2 for more details.

The size and timing of these responses depend on how government policies respond to aging because both monetary and fiscal policy influence the price level and real economic activity in our model. The monetary authority pursues a nominal interest rate targeting rule and the deflationary pressure induced by aging produces persistent declines in the nominal interest rate. We find that this policy reaction smooths out the price level declines and propagates them over time. A lower nominal interest rate transmits to the real sector in two ways. First, we model New Keynesian (NK) nominal price rigidities. This channel turns out to be largely irrelevant for our results. Second, our model features an asset substitution transmission channel of monetary policy as in Tobin (1969), Hagedorn (2018) and Hu et al. (2021). We find that asset substitution is the main transmission channel of monetary policy during Japan's demographic transition.

Changes in the supply of nominally denominated government debt also influence prices and real economic activity. Households require a higher real return on government debt to induce them to hold more of it and the real interest rate on liquid securities increases when government debt is increased. Households save more but also substitute their savings away from physical assets and private investment is crowded out. Still, the increase in household liquid asset demand induced by aging is so large that the model continues to produce deflation (and secular stagnation) during the first 25 years of the transition even when we posit large increases in the supply of (nominal) government debt.

Our claim that demographic change is putting downward pressure on inflation, real interest rates and output is accepted in some policy making circles (see Nishimura, 2011; Fischer, 2016; Shirakawa, 2021), but is controversial among academic economists. For instance, Mian et al. (2021) provide empirical evidence that higher inequality is the main source of the secular decline in the U.S. natural interest rate and argue that aging of the baby boom cohorts is of secondary importance in accounting for it. We show in Section 2 that aging is less of an issue for the U.S. compared to other industrialized economies. However, this result is premised on immigration flows into the U.S. remaining high in future years.

Another reason why the aging hypothesis is controversial is because it is at odds with standard theory about the relationship between population growth rates, real interest rates and inflation. According to the modified golden rule, a slower population growth rate is associated with a lower real return on capital. Then the Fisher equation implies that given the nominal interest rate the inflation rate must increase to equate the real return on nominal government liabilities with the lower real return on capital.

Bullard et al. (2012) provide a political economy explanation for how aging could induce a *decline* in the inflation rate. They observe that older households have no labor income and consume instead out of their savings. Older households prefer a lower inflation rate because it increases the real return on their holdings of nominally denominated assets. If the fraction of the old increases and one assumes that their political influence also increases, then policy makers may take actions to push the price level down. Their hypothesis explains why aging might produce deflationary pressure, but it also implies that periods of deflation should also be periods with high real interest rates. This implication is at odds with the fact that both the inflation rate and real interest rates have been falling at the same time in Japan and other high income countries.

Another important property of our framework is that the price level is jointly determined by monetary and fiscal policy. The price level is determined by a no arbitrage condition in the asset market between capital and nominally denominated government debt as in Hu et al. (2021). For the reasons described in Hagedorn (2021), we refer to this model of price level determination as the demand theory of the price level (DTPL). We explain in Section 3, using a 2-period flexible price overlapping generations (OLG) model, why we prefer the DTPL to the fiscal theory of the price level (FTPL). Under the FTPL, a decline in the fertility rate is inflationary and changes in the nominal interest rate are neutral. However, under the DTPL a lower fertility rate pushes the price level down and monetary policy influences real economic activity.

Eggertsson et al. (2019) also consider aging as a source of low (natural) real interest rates using a lifecycle model.[4] Their premise is that the economy finds itself in a permanent liquidity trap with an upward sloping aggregate demand schedule for goods. The economic mechanisms that produce low interest rates in our model are different. Our baseline analysis abstracts from liquidity traps. Moreover, the empirical performance of our model deteriorates if we impose a lower bound on the nominal interest rate. We interpret these results as indicating that UMP measures have been effective.[5] A second distinction is the set of aggregate variables that we seek to explain. Eggertsson et al. (2019) focus on the evolution of the natural interest rate. We produce transitions with low real interest

[4]A number of other papers have analyzed the effect of aging on real interest rates in flexible price lifecycle models including Braun et al. (2009), Carvalho et al. (2016), Sudo and Takizuka (2020), Gagnon et al. (2021) and Auclert et al. (2021). Fujiwara and Teranishi (2008) introduce nominal rigidities to a perpetual youth model and show that shocks have asymmetric effects on workers and retirees.

[5]Swanson (2021) and Ikeda et al. (2020) provide empirical evidence that UMP are effective tools for monetary stabilization at the ELB.

rates, below trend growth in per capita output, deflation, low nominal interest rates and high government debt–GDP ratios. Indeed, our quantitative framework provides a unified theory about how the economy responds to monetary policy in the short–run, as discussed in Braun and Ikeda (2022), and in the medium term, which is our focus here, under the maintained hypothesis that the ELB is not a binding constraint on the actions of the central bank.

Goodhart and Pradhan (2017) argue that aging will raise the equilibrium real interest rate and increase the inflation rate in future years and Juselius and Takáts (2018) provide empirical evidence that the combination of a lower share of younger population cohorts and a higher share of older cohorts will create inflationary pressure in future years. Our model is a good laboratory for understanding the quantitative significance of their claims.

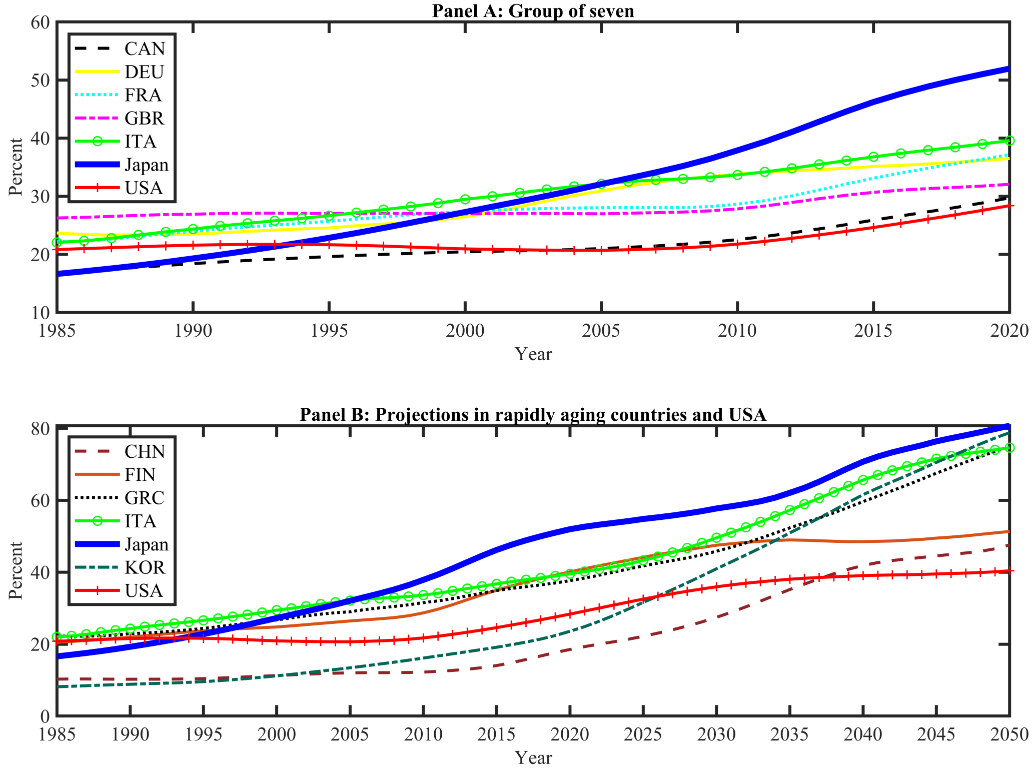
Household demand in our model increases in the first stage of the transition but at horizons of about 50 years, aggregate asset demand for government debt starts to decline. Thus, the second stage of the transition is characterized by rising real interest rates and a rising inflation rate. In fact, both variables eventually overshoot their terminal steady-state values. The intuition for these properties of our model is related to Sargent and Wallace (1981) who analyze how open market operations alter the time–profile of asset demand. It turns out that the increase in the inflation rate in the later stage of our transition is gradual and the peak inflation rate is consequently much smaller than its trough.

The remainder of our paper proceeds as follows. In Section 2 we provide more motivation for our decision to analyze the macroeconomic effects of aging in Japan. In Section 3 we use a simple OLG model to illustrate the workings of the asset substitution channel and show how the reactions of fiscal and monetary policy to a demographic shock influence prices and real economic activity. This section also compares and contrasts the DTPL with the FTPL. Section 4 describes the quantitative model and Section 5 provides an overview of the model calibration. Our main results are reported in Section 6. Section 7 discusses some robustness checks and Section 8 contains our concluding remarks.

2 Motivation

This section motivates our formal analysis of Japan. We show that a number of industrialized and large emerging economies are embarking on a transition to a much older population–age distribution. We explain that Japan is interesting because the demographic transition started earlier than other countries and has been rapid and large. Finally, we

Figure 1: Old–age dependency ratios



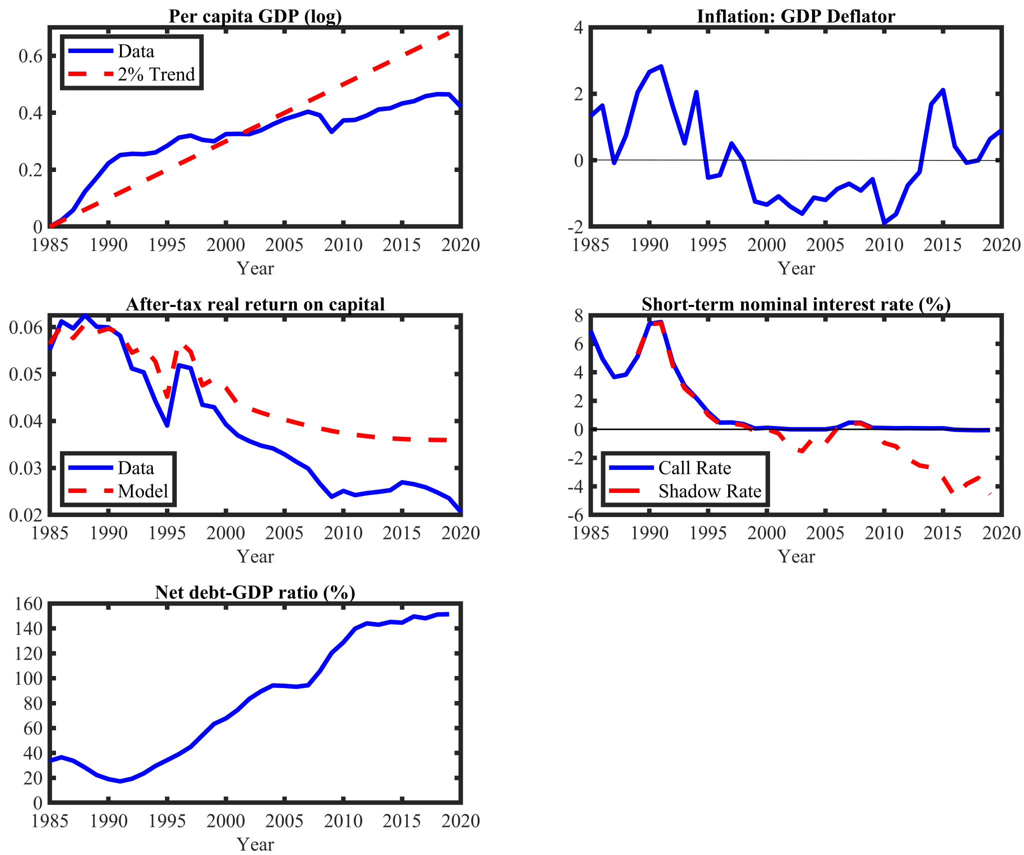
Notes: World Bank Population Division. World Population Prospects: 2019.

document that Japan's demographic transition has been associated with deflation, low real interest rates and GDP growth in spite of significant monetary and fiscal stimulus.

Aging reflects three factors: aging of the baby-boomers, increases in life expectancy and reductions in the fertility rate. The old–age dependency ratio, which we define as the ratio of the 65+ population to the 20–64 population, provides a simple way to summarize the joint impact of these three factors on the age distribution. Panel A of Figure 1 shows that old-age dependency ratios have been steadily rising since 1986 in all members of the Group of Seven. Japan is noteworthy because it is in the midst of a particularly rapid demographic transition. Japan had the lowest old-age dependency ratio in this peer group in the year 1986, but had the highest old-age dependency ratio in the year 2005.

Japan's old-age dependency ratio is projected to continue to increase rapidly until the year 2050 as shown in panel B of Figure 1. The World Bank projects that South Korea, Greece and Italy will also experience large increases in their old-age dependency ratios with peak old-age dependency ratios exceeding 70 percent by 2050. Japan, however, stands out

Figure 2: Aggregate indicators of secular stagnation and government policy in Japan



Notes: GDP and GDP Deflator – Cabinet Office; After-tax real return on capital – authors' calculations using Hayashi and Prescott (2002) methodology; Call rate – Bank of Japan; Shadow rate – Ueno (2017); Net debt–GDP ratio – IMF.

even in this group of rapidly aging societies. Finland shows Northern European countries are also projected to experiencing rapid aging. China is starting from a much lower base, but will also experience rapid aging in future years. The old–age dependency ratio in China is projected to increase by nearly 40 percentage points between 2005 and 2050. Aging is also occurring in the U.S. albeit at a more moderate pace. The United States is benefiting from large immigration flows and, if these flows stop, the projected U.S. old–age dependency ratio rises to the level of Finland in 2050.[6]

Figure 2 shows that aging in Japan has been associated with a protracted episode of deflation, low real interest rates and low growth in per capita GDP. These aggregate out-

[6]The U.S. Census Bureau projects that the U.S. old–age dependency ratio will rise to 49 percent by 2060, if immigration flows drop to zero (Johnson, 2020).

comes emerge in the 1990s, which is when Japan's old-age dependency ratio begins its rapid ascent. The Japanese inflation rate is not only low, but it is also negative for protracted periods of time. The real interest rate, which we measure as the after-tax real return on capital, is also persistently low and declines over time. For purposes of comparison, we report model based projections for this measure of the real interest rate from Braun et al. (2009). Their projections start in the year 2000 and are derived from a computable OLG model that is presented with demographic projections from the National Institute of Population and Social Security Research (IPSS). Interestingly, the model projections are consistent with the subsequent declines in the real interest rate in Japanese data and reflect the fact that aging induces capital deepening in the first stage of the demographic transition.[7] Alternative measures of the real interest rate have a similar pattern (see Han, 2019). Finally, per capita GDP growth exhibits a persistent decline after 1995.

Figure 2 also reports summary statistics about the evolution of monetary and fiscal policy between 1985 and 2020. The call rate, which is the Bank of Japan's policy nominal interest rate, falls steadily to zero in the 1990s and has been close to zero since 1997. The nominal interest rate panel also reports the shadow interest rate of Ueno (2017). The shadow rate uses (imperfect) arbitrage relationships to measure the implied impact of quantitative easing and other UMP on the call rate. According to this estimate from 2001, the year that quantitative easing was introduced, UMP measures have exerted steady and, in recent years, large downward pressure on the shadow call rate. The net debt–GDP ratio, which is based on data from the IMF, rises from about 20 percent of GDP in 1990 to about 150 percent in 2020.

3 2-period OLG Model

Three of the economic mechanisms underlying our main results can be illustrated using a 2-period flexible price OLG model. First, the price level is determinant in our model and it is influenced by both monetary and fiscal policy. The mechanism that determines the price level is different from the FTPL and, as we explain below, it is more difficult to account for our nominal and real secular stagnation observations under the FTPL. A second and related point is that monetary policy affects real economic activity in this simple flexible price model due to an asset substitution or Tobin effect. Third, an increase in the nominal

[7] The scenario reported in Figure 2 assumes that total factor productivity (TFP) growth is constant at its average growth rate for the 1990s.

stock of government debt is also non-neutral when monetary policy follows a Taylor rule.

3.1 The model

The 2-period model analyzed here is essentially the model of Hu et al. (2021). We extend their model by assuming that monetary policy is endogenous and allowing population and the stock of nominal debt to vary over time.

Environment Time is discrete and continues forever: $t = 0, 1, ..$ There is a single good that can be consumed or used to produce new capital. The economy is populated by households who are either young or old. In period t, N_t of young households are born. In the next period, the young generation becomes old and exits the economy in the end of the period. Hence, in period t, there are N_t young households and N_{t-1} old households. The growth rate of the population of young households or "fertility rate" is $n_t = N_t/N_{t-1}$. In the initial period $t = 0$, there are old households with population N_{-1} and each old household is endowed with capital a_{-1} and nominal government bonds $d^n_{-1} \equiv P_{-1}d_{-1}$, where P_{-1} is the price level and d_{-1} is government bonds in real units. In aggregate, the initial capital stock is $K_0 = a_{-1}N_{-1}$ and the initial nominal government debt is $D^n_{-1} = d^n_{-1}N_{-1}$.

Households Households born in period t supply one unit of labor inelastically and earn real wage w_t. They have two ways to save: they can invest in capital which fully depreciates after being used in production in period $t+1$ or they can acquire nominal government bonds. Only old households consume. This assumption isolates the asset substitution channel of monetary policy because asset demand doesn't depend on the interest rate and allows us to show in a transparent way how monetary and fiscal policy influence the price level. The problem of a household born in period t is to maximize consumption c_{t+1} subject to

$$a_t + d_t = w_t \tag{1}$$

$$c_{t+1} = R^k_{t+1}a_t + R_t\frac{P_t}{P_{t+1}}d_t + \xi_{t+1} \tag{2}$$

When young, the household earns w_t and allocates its earnings to capital a_t and government bonds d_t. When old, the household receives the return on capital, $R^k_{t+1}a_t$, the return on government bonds, $R_t(P_t/P_{t+1})d_t$, and a lump-sum transfer ξ_{t+1}, where R_t denotes the nominal interest rate.[8] An interior solution for a_t and d_t satisfies the arbitrage condition between investing in capital and government bonds

$$R^k_{t+1} = R_t \frac{P_t}{P_{t+1}}. \tag{3}$$

Firms Perfectly competitive firms produce the single good according to $Y_t = K_t^\alpha N_t^{1-\alpha}$. Capital depreciates fully after production, and the returns on capital and wages are given, respectively, by

$$R^k_t = \alpha K_t^{\alpha-1} N_t^{1-\alpha} = \alpha k_t^{\alpha-1} \tag{4}$$

$$w_t = (1-\alpha) K_t^\alpha N_t^{-\alpha} = (1-\alpha) k_t^\alpha \tag{5}$$

where $k_t \equiv K_t/N_t$ is capital per young household.[9] Substituting equation (4) into equation (3) yields the Fisher equation

$$\alpha k_{t+1}^{\alpha-1} = R_t \frac{P_t}{P_{t+1}}. \tag{6}$$

Government The government consists of a central bank and a fiscal authority. The central bank sets the nominal interest rate R_t on government debt and the fiscal authority issues one-period nominal debt D^n_t and makes transfers ξ_t to the old.[10] In period t, the government faces the following budget constraint

$$\xi_t = \frac{d^n_t n_t}{P_t} - R_{t-1} \frac{d^n_{t-1}}{P_t} \tag{7}$$

expressed in per capita terms where $d^n_t = D^n_t/N_t$.

Law of motion for capital The aggregate capital stock evolves according to $K_{t+1} = a_t N_t$. Using the budget constraint (1), the law of motion for capital can be expressed in per capita terms as

$$n_{t+1} k_{t+1} = w_t - d_t. \tag{8}$$

[8] The initial old households consume what they earn as $c_0 = R^k_0 a_{-1} + R_{-1} d^n_{-1}/P_0 + \xi_0$.

[9] In the ensuing analysis it is convenient to express period t per capita variables in terms of the population of young households in period t.

[10] Transfers are negative here because they finance interest payments on government debt. But, we use this notation to be consistent with the notation in the quantitative model presented in Section 4.

Market clearing The market clearing condition for period t government debt is

$$\frac{d_t^n}{P_t} = d_t \tag{9}$$

Clearing in the goods market implies $c_t N_{t-1} + K_{t+1} = Y_t$ or

$$\frac{c_t}{n_t} + n_{t+1} k_{t+1} = k_t^\alpha. \tag{10}$$

Combining equations (5), (8), and (9) yields

$$\frac{d_t^n}{P_t} + n_{t+1} k_{t+1} = (1-\alpha) k_t^\alpha \tag{11}$$

which is the aggregate asset market clearing condition. The right hand side is asset demand, which by design depends on wages, but *not* on the real interest rate between period t and $t+1$. The left hand side of the expression is asset supply.

Equilibrium Given $\{d_t^n, R_t\}$, observe that the equilibrium sequence $\{k_t, P_t\}$ is determined by the Fisher equation, (6) and the asset market clearing condition, (11). This strategy for deriving the equilibrium price level is similar to Sargent and Wallace (1981) and Hagedorn (2021) and identical to Hu et al. (2021). Following Hagedorn (2021) we refer to this scheme as the demand theory of the price level (DTPL).

Definition 1 (DTPL equilibrium) *Given the initial capital k_0, the initial nominal obligation $R_{-1} D_{-1}^n$, a sequence of fertility rates $\{n_t\}$, and a sequence of policy variables $\{d_t^n, R_t\}$, a competitive equilibrium for this economy consists of a sequence of prices $\{P_t, R_t^k, w_t\}$, a set of allocations $\{c_t, k_t, d_t\}$ and a sequence of lump-sum transfers $\{\xi_t\}$ that satisfy the firms' optimality conditions (4) and (5), the Fisher equation (6), the government budget constraint (7), and the market clearing conditions (9)-(11).*

3.2 Analytical results

We now examine the effects of monetary policy, fiscal policy and demographic changes on the price level and the real economy.

Proposition 1 (Non-neutrality of money) *In an DTPL equilibrium, monetary policy is not neutral.*

Proof. See Appendix A.1. ■

It is easiest to illustrate the nature of the non-neutralities induced by a change in the nominal interest rate as well as the effects of a lower fertility rate and higher stock of government debt by considering a steady state DTPL equilibrium. Suppose (without loss of generality) that the per capita stock of nominal debt is constant. Then equation (11) implies that the price level is also constant or the gross inflation rate is unity, $\pi = 1$. Since the central bank sets the nominal interest rate, the Fischer equation, (6) implies that the capital stock is given as $k = (\alpha/R)^{1/(1-\alpha)}$. Now consider an increase in R. The previous expression implies that the capital stock falls to equate the real returns on capital and government debt. Then equation (11) implies that the price level falls to clear the asset market. Observe next that changes in the stock of nominal debt and the fertility rate have no real effects in a DTPL steady state equilibrium: $\partial k/\partial d^n = 0$ and $\partial k/\partial n = 0$. To ascertain how the price level responds, note that aggregate demand for real government debt, $d = (1-\alpha)k^\alpha - nk$, is independent of the stock of nominal debt. It then follows from equation (11) that an increase in the steady state stock of nominal debt increases the price level: $\partial P/\partial d^n > 0$. Finally, consider how the price level responds to a change in the steady state fertility rate. Aggregate demand for real government debt is decreasing in the fertility rate. Then, using equation (11) we see that the price level is increasing in the fertility rate: $\partial P/\partial n > 0$. In other words, a drop in the fertility rate decreases the price level. These final two results are summarized in the following proposition.

Proposition 2 (Demographics and fiscal policy) *In the DTPL steady state, a lower fertility rate decreases the price level: $\partial P/\partial n < 0$; a higher issuance of the per capita nominal government debt increases the price level: $\partial P/\partial d^n > 0$.*

Proof. See Appendix A.2. ■

The results reported in Propositions 1 and 2 are premised on the assumption of an DTPL equilibrium and the properties of our model are quite different in a FTPL equilibrium. In Appendix A.3, we analyze an FTPL equilibrium and show that changes in the nominal interest rate are neutral and a lower fertility rate increases the inflation rate in that equilibrium. The main reason for this distinction is that government transfers are held fixed in the FTPL equilibrium and monetary policy doesn't induce redistribution across generations. In the DTPL, in contrast, government transfers are endogenous and change when the central bank alters the nominal interest rate. We will see that the ability of our quantitative model to account for the secular stagnation observations relies heavily on the asset substitution channel of monetary policy. In other words, an important maintained hypothesis in the analysis that follows is that monetary policy induces redistribution.

3.3 Impulse response analysis

Hu et al. (2021) conduct a dynamic theoretical and numerical analysis of shocks to monetary policy and find that an increase in the (exogenous) nominal interest rate crowds out private capital formation and puts downward pressure on prices. Their result suggests that the policy rule pursued by the central bank during a demographic transition will influence the trajectory of the inflation rate and real allocations. We now document that this is the case by computing impulse response functions (IRFs) to a lower fertility shock under two alternative assumptions about the central bank's interest rate targeting rule.

We consider two different monetary policy rules: a fixed interest rate rule and an inflation targeting rule. Specifically the central bank sets the policy interest rate according to

$$R_t = R + \phi_\pi \left(\frac{P_t}{P_{t-1}} - 1 \right). \tag{12}$$

The fixed interest rate rule corresponds to $\phi_\pi = 0$ and the inflation targeting rule corresponds to $\phi_\pi > 1$. Under this policy rule, equation (6) becomes

$$P_{t+1} = \left[\frac{R + \phi_\pi \left(\frac{P_t}{P_{t-1}} - 1 \right)}{\alpha k_{t+1}^{\alpha-1}} \right] P_t. \tag{13}$$

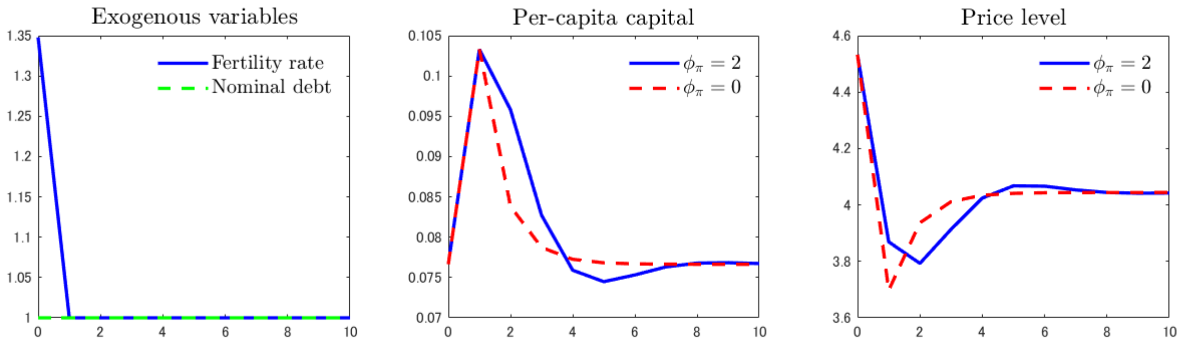
It will be helpful to refer to this equilibrium condition in the discussion that follows.

3.3.1 Reaction of monetary policy to a lower fertility rate

Assume that the economy is in steady state in $t = 0$ and consider a situation in which the fertility rate suddenly decreases to $n < n_0$ in the beginning of period $t = 1$ and stays at the new lower level for all $t = 1, 2...$ For simplicity, assume $\pi_t \equiv P_t/P_{t-1} = 1$ in the initial steady state and consider a baseline fiscal policy such that $d_t^n = d^n$. The amount of the nominal bonds is normalized to unity: $d^n = 1$. We assume that the nominal interest rate is the same between the initial and final steady states: $R_0 = R$.

Figure 3 plots the initial steady state $t = 0$ and then impulse responses when the economy is hit by a sudden decrease in the fertility rate from n_0 to n in the beginning of

Figure 3: Responses to a decline in the fertility rate: alternative monetary policy rules.



period $t = 1$ under the two monetary policy rules: $\phi_\pi = 0$ and $\phi_\pi = 2$.[11]

As shown in Figure 3, the (exogenous) fertility rate drops in $t = 1$. In the case of $\phi_\pi = 0$ (red dashed lines), the price level falls sharply in $t = 0$ and then converges to the terminal steady state from below. The capital stock, in contrast, increases on impact and transitions to the new steady state from above. Computing the transition requires numerical methods (we use a shooting algorithm). However, considerable intuition for the results can be gathered by inspecting the two equilibrium conditions that determine the capital stock and the price level. Consider first the asset market clearing condition, (11) in period 1. The wage rate and nominal debt are predetermined in period 1, but the fertility rate is now lower. It follows that the real supply of assets has to increase. This adjustment occurs in two ways. Households leave the period with more capital, so that k_2 increases. However, capital and government debt are perfect substitutes and have the same real return moving forward. In particular, equation (13) with $\phi_\pi = 0$ is

$$\alpha k_{t+1}^{\alpha-1} = \frac{R}{\pi_{t+1}} \tag{14}$$

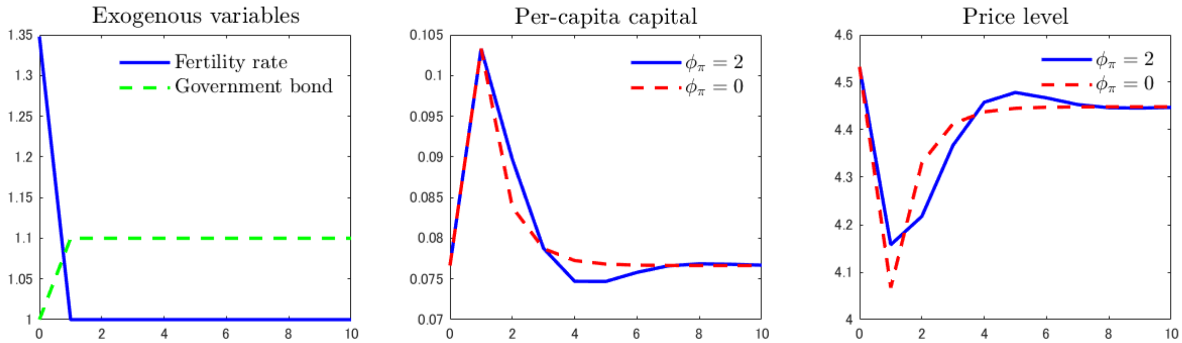
and it follows that the price level falls on impact in period 1 as shown in Figure 3 to compensate households for the lower future real returns on government debt.

Allowing monetary policy to respond to changes in the inflation rate ($\phi_\pi = 2$) affects the asset supply responses in period 1 and asset demand (wages) from period 2 on. The price level now declines less in period 1, but more in periods 2 and 3 compared to the $\phi_\pi = 0$ scenario and the capital stock is now higher in periods 2 and 3.

Thus, the reaction of monetary policy to a lower fertility rate influences the size of the

[11] The rest of the parameterization in this computational example is: $\alpha = 0.3$, $n_0 = (1.01)^{30}$, $n = 1$ and $R = (1.02)^{30}$.

Figure 4: Responses to a decline in the fertility rate and an increase in the stock of nominal government debt: alternative monetary policy rules.



price response in the impact period, and the responses of the inflation rate and capital stock in subsequent periods.

3.3.2 Joint responses of (nominal) government borrowing and monetary policy to lower fertility rates

In our quantitative model we will allow both monetary policy and fiscal policy to vary during the demographic transition. We found that nominal government debt is neutral in our steady–state analysis of this simple 2–period model. We next show that changes in the supply of nominal government debt only have real effects during a transition if they trigger a response by the monetary authority. Figure 4 reports a scenario in which the fertility rate falls and nominal government debt issue is permanently increased by 10 percent in period 1. Consider the $\phi_\pi = 0$ scenario first. Compared to Figure 3, the negative response of the price level in period 1 is much smaller, the terminal steady–state has a higher price level and the trajecory of the capital stock is identical. However, if the monetary policy interest rate rule is endogenous, higher government debt crowds out private capital formation. Comparing Figures 3 and 4, we see that the scenario with high government debt has a lower capital stock in periods 2–5 when $\phi_\pi = 2$.

In our quantitative model we allow for an interest rate smoothing motive for the central bank, model labor supply and introduce two new frictions: nominal price rigidities and household level costs of adjusting illiquid assets. These extensions of the model create new transmission channels for monetary and fiscal policy, but the economic mechanisms documented here will continue to play an important role in understanding how our model accounts for the nominal and real secular stagnation observations.

4 Quantitative Model

We conduct our quantitative analysis using the model of Braun and Ikeda (2022) and readers are referred to that paper for more details on the motivation and discussion of our modeling assumptions. That paper also contains a detailed empirical assessment of the specification and analysis of the short–run household–level and aggregate properties of the model. One of the objectives here is to show that we have a unified quantitative theory of money. Our model has reasonable implications not just at business cycle frequencies, but also at medium–term frequencies. This is why we use essentially the same specification here. In the ensuing discussion we highlight the main features of the model and focus on how household saving decisions change during a demographic transition and how the model accounts for the empirical observations that motivate our analysis.

We consider an OLG economy with representative cohorts. Households can save and/or borrow two assets that differ in terms of their liquidity services. Illiquid assets offer a higher return but are costly to acquire and sell. Liquid assets offer a lower return but are costless to adjust. Depending on where households are in their lifecycle, they choose to borrow liquid assets to purchase illiquid assets or hold positive amounts of both assets.

4.1 Demographic structure

The economy has an OLG structure that evolves in discrete time with a period length of one year. Let j denote the age of the individual as $j = 1, ..., J$. We start keeping track of individuals at age 21 and individuals survive until at most age 120. Thus, model age of $j = 1$ corresponds to age 21, model age $J = 100$ corresponds to age 120 and $J = 100$ cohorts are active in a given year. Let $N_{j,t}$ be the number of individuals of age j in period t, then the population age distribution in period t is given by the $J \times 1$ vector $\mathbf{N}_t \equiv [N_{1,t}, ..., N_{J,t}]'$.

The dynamics of population are governed by

$$\mathbf{N}_{t+1} = \begin{bmatrix} n_{1,t} & 0 & 0 & \dots & 0 & 0 \\ \psi_{1,t} & 0 & 0 & \dots & 0 & 0 \\ 0 & \psi_{2,t} & 0 & \dots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & 0 & 0 \\ 0 & 0 & 0 & \dots & \psi_{J-1,t} & 0 \end{bmatrix} \mathbf{N}_t \equiv \mathbf{\Gamma}_t \mathbf{N}_t,$$

where $n_{1,t}$ is the gross population growth rate of age-1 households between periods t and $t+1$, which we will henceforth refer to as the fertility rate[12], and $\psi_{j,t}$ is the conditional probability that a household of age j in period t survives to period $t+1$. It follows that the aggregate population in period t is given by $N_t = \sum_{j=1}^{J} N_{j,t}$ and that the population growth rate is given by $n_t = N_{t+1}/N_t$. Finally, the unconditional probability of surviving from birth in period $t-j+1$ to age $j = 2, ..., J$ in period t is

$$\Psi_{j,t} = \psi_{j-1,t-1}\Psi_{j-1,t-1}$$

where $\Psi_{1,t} = 1$ for all t.

4.2 Households

Individuals are organized into households. Each household consists of one individual (adult) and children. The number of children varies with the age of the adult and the age of the household is indexed by the age of the adult. Adults face mortality risk and have no bequest motives. At the beginning of each period the adult learns whether she will die at the end of the current period and this rules out accidental bequests. Let $z^i_{j,t} \in \{0,1\}$ index the survival state for the i^{th} household where a value of zero denotes the death state. Death is the only source of idiosyncratic risk faced by households and there are only two types of households in any cohort: surviving households ($z^i_{j,t} = 1$) and non-surviving households ($z^i_{j,t} = 0$).

Households work, consume, and save for retirement. A household of age j in period t earns an after–tax wage rate of $(1-\tau^w)w_t\epsilon_j$, where τ^w denotes a labor–income tax rate and ϵ_j is the efficiency of labor of an age–j household.[13] All cohorts face the same age–efficiency profile and the efficiency index ϵ_j is assumed to drop to zero for all $j \geq J_r$, where J_r is the mandatory retirement age.

Households provision for retirement by acquiring liquid and illiquid assets. They may save and/or borrow using either asset and the liquid asset is nominally denominated because monetary policy directly controls the nominal interest rate on liquid assets in this economy.

[12] Note that this usage differs from other common definitions of the fertility rate and that the fertility rate, as we have defined it, can be less than unity, indicating a decline in the size of the youngest cohort from one period to the next.

[13] Given that there is only one type of heterogeneity in a cohort, to conserve on notation we do not explicitly index the identity of each household of age j in period t in the ensuing discussion unless it is required to avoid confusion.

The liquid asset earns the nominal interest rate R_{t-1} between period $t-1$ to t and its after-taxed real return is given by $\tilde{R}_{t-1}/\pi_t$, where $\tilde{R}_{t-1} = 1 + (1-\tau^a)(R_{t-1}-1)$. The real return on illiquid assets in period t is R^a_t and its after–taxed return is $\tilde{R}^a_t = 1 + (1-\tau^a)(R^a_t - 1)$.

From the perspective of the household the only distinction between liquid and illiquid assets is that households face costs of adjusting their holdings of illiquid assets as in Aiyagari and Gertler (1991) and Kaplan and Violante (2014). When we parameterize our model, we follow Kaplan et al. (2018) and include physical assets such as homes and durable goods and illiquid financial assets such as equities in our measure of illiquid assets. So the adjustment costs can be interpreted as representing service flows to the financial service sector when, for instance, a household purchases or sells a home. Following Kaplan et al. (2018), we also abstract from the service flow of utility services provided by physical assets. Thus, the benefit from holding illiquid assets is entirely pecuniary in our model.

Adjustment costs on holdings illiquid assets are given by

$$\chi(a_{j,t}, a_{j-1,t-1}, z) = \begin{cases} \frac{\gamma_a(z)}{2}(a_{j,t} - a_{j-1,t-1})^2, & a_{j-1,t-1} > 0 \\ \frac{\gamma_a(z)}{2} a_{j,t}^2, & a_{j-1,t-1} = 0 \end{cases} \tag{15}$$

where $a_{j,t}$ denotes the holdings of illiquid assets in the end of period t and $\gamma_a(z) \geq 0$ is a parameter that governs the size of the adjustment costs for $z = z^i_{j,t} \in \{0, 1\}$. These costs have two main features. First, they vary with the level of the change in assets. Second, they depend on whether the household experiences the death event in the current period. In Braun and Ikeda (2022) we show that this two parameter model of financial frictions allows us to match the main features of the age profiles of liquid asset holdings and illiquid asset holdings in Japanese data.

Given these definitions, the decisions of a surviving household of age–j in period t (i.e., a household with $z^i_{j,t} = 1$) are constrained by

$$\begin{aligned}(1+\tau^c)c_{j,t} + a_{j,t} + \chi(a_{j,t}, a_{j-1,t-1}, 1) + d_{j,t} \\ \leq \tilde{R}^a_t a_{j-1,t-1} + \frac{\tilde{R}_{t-1}}{\pi_t} d_{j-1,t-1} + (1-\tau^w) w_t \epsilon_j h_{j,t} + b_{j,t} + \xi_t,\end{aligned} \tag{16}$$

where $c_{j,t}$ is total household consumption for a household of age j in period t, τ^c is a consumption tax rate, $d_{j,t}$ denotes holdings of the liquid asset, expressed in terms of the final good, at the end of period t, $h_{j,t}$ denotes hours worked, $b_{j,t}$ denotes public pension (social security) benefits, ξ_t is a lump–sum government transfer, and $\chi(\cdot)$ is the transaction cost of adjusting individual holdings of the illiquid asset.[14] We wish to emphasize that there are no ad hoc restrictions on borrowing of surviving households. They are free to borrow against their future earnings and they are also free to take leveraged long positions on illiquid assets, which have a higher return in equilibrium. The only constraint on borrowing of surviving households is the natural borrowing constraint.

If instead the household is in its final period of life ($z^i_{j,t} = 0$), the event is publicly observed by lenders and borrowing is not possible. Thus, the optimal strategy for the household is to consume all of its income and wealth during the current period

$$(1+\tau^c)c_{j,t} = \tilde{R}^a_t a_{j-1,t-1} + \frac{\tilde{R}_{t-1}}{\pi_t} d_{j-1,t-1} + (1-\tau^w) w_t \epsilon_j h_{j,t} + b_{j,t} + \xi_t - \chi(0, a_{j-1,t-1}, 0). \quad (17)$$

The period utility function for a household of age j in period t is given by

$$u(c_{j,t}, h_{j,t}; \eta_j) = \frac{\eta_j \left(c_{j,t}/\eta_j\right)^{1-\sigma}}{1-\sigma} - \frac{\upsilon}{1+1/\nu} h_{j,t}^{1+1/\nu}, \quad (18)$$

where $\sigma > 0$ is the inverse of the elasticity of intertemporal substitution, $\nu > 0$ governs the Frisch elasticity of labor supply, $\upsilon > 0$ is the labor dis–utility parameter, and η_j is the age–family scale profile, which is assumed to be time–invariant. In our model, children are essentially age–specific deterministic demand shocks to household consumption.

We assume that working–age households belong to a labor union.[15] The union respects their marginal utilities and wages are flexible. We analyze the symmetric equilibrium. Thus, hours worked are identical for all workers in period t, $h_{j,t} = \bar{h}_t$ for all $j < J_r$ with $\bar{h}_t$ given by

$$(1-\tau^w)\bar{\epsilon}_t w_t = \upsilon \bar{\lambda}_t^{-1} \bar{h}_t^{\frac{1}{\nu}} \quad (19)$$

where $\bar{\lambda}_t$ is the weighted average of the marginal utilities of working households and $\bar{\epsilon}$ is the weighted average of the efficiency of labor. This specification implies that workers who experience an aggregate or idiosyncratic shock are unable to self–insure by adjusting their hours worked differently from the average worker. Household earnings vary by age because

[14] We are omitting here the dependence of individual choices on the survival event to save on notation. Formally, we have for $z^i_{j,t} \in \{0,1\}$: $c_{j,t}(z^i_{j,t})$, $a_{j,t}(z^i_{j,t})$, $d_{j,t}(z^i_{j,t})$, and $h_{j,t}(z^i_{j,t})$. In what follows this dependence is only made explicit when required.

[15] This is one strategy used in the HANK literature for reducing the impact of wealth on labor supply (see e.g. Hagedorn et al., 2019). If we were to allow households to choose their own labor supply, high wealth households work much less than poor households. The specification we adopt has the properties that households do respond to changes in aggregate wealth, earnings increase with age, but that the length of the workweek is the same for young and older working age households.

the efficiency of a worker's labor depends on the worker's age. The interested reader is referred to Braun and Ikeda (2022) for more details.

The household's optimal choices are given by the solution to

$$
\begin{aligned}
U_j(a_{j-1,t-1}, d_{j-1,t-1}, z_{j,t}) = \max_{\{c_{j,t}, a_{j,t}, d_{j,t}\}} \Big\{ & u\left(c_{j,t}, \bar{h}_t; \eta_j\right) \\
& + \beta z_{j,t}\left[(1-\psi_{j+1})U_{j+1}(a_{j,t}, d_{j,t}, 0) + \psi_{j+1}U_{j+1}(a_{j,t}, d_{j,t}, 1)\right]\Big\},
\end{aligned} \tag{20}
$$

subject to equations (16) and (17) for $z_{j,t} \in \{0, 1\}$ and for $j = 1, ..J - 1$, and $z_{J,t} = 0$, where $\beta > 0$ is the preference discount factor and ψ_{j+1} is the conditional probability that a household of age $j + 1$ survives to the next period.[16] Note that we have imposed no restrictions on the sign or magnitude of asset holdings beyond the natural borrowing constraint. It is thus conceivable, for instance, that households might want to borrow both types of assets. However, in equilibrium, the return on illiquid assets exceeds the return on liquid assets and all private borrowing will be in the form of liquid assets.

4.3 Production of goods and services

The production of goods and services is organized into four sectors.

Final good sector. Firms in this sector are perfectly competitive and combine a continuum of intermediate goods, $\{Y_t(i)\}_{i\in(0,1)}$, to produce a homogeneous final good Y_t, using the production technology: $Y_t = \left[\int_0^1 Y_t(i)^{\frac{1}{\theta}} di\right]^{\theta}$ with $\theta > 1$. Let $P_t(i)$ denote the price of intermediate good i, and P_t denote the price of the final good. Final good firms are price takers in input markets and it follows that demand for intermediate good i is

$$
Y_t(i) = \left(\frac{P_t(i)}{P_t}\right)^{-\frac{\theta}{\theta-1}} Y_t. \tag{21}
$$

The final good is either consumed by households or used as an input in the capital good sector.

Intermediate goods sector. Firms in this sector are monopolistically competitive and each firm produces a unique good indexed by $i \in (0, 1)$. Intermediate goods firm i produces

[16] There is a theoretical possibility that adjustment costs on illiquid assets could exceed the size of beginning of period illiquid assets. Our strategy for parameterizing the adjustment costs on illiquid assets rules this possibility out.

$Y_t(i)$ by combining capital $K_t(i)$ and effective labor $H_t(i)$ with a Cobb-Douglas production function

$$Y_t(i) = K_t(i)^\alpha H_t(i)^{1-\alpha}, \quad 0 < \alpha < 1. \tag{22}$$

Intermediate goods firm i faces demand curve (21), and sets its price $P_t(i)$ to maximize profits subject to a quadratic price adjustment cost function. In a symmetric equilibrium, the condition can be expressed as

$$(\pi_t - 1)\pi_t = \frac{1}{\gamma}\frac{\theta}{\theta - 1}(mc_t - 1) + \Lambda_{t,t+1}\frac{Y_{t+1}}{Y_t}(\pi_{t+1} - 1)\pi_{t+1}, \tag{23}$$

where $\pi_t = P_t/P_{t-1}$ is the gross inflation rate. Equation (23) is the New Keynesian Phillips curve that relates the current inflation rate π_t to the real marginal cost mc_t and the future inflation rate π_{t+1}. In a symmetric equilibrium the aggregate output is

$$Y_t = K_t^\alpha H_t^{1-\alpha}, \tag{24}$$

where K_t denotes the aggregate capital and H_t denotes the aggregate effective labor. The aggregate total profits of intermediate goods firms in period t, $\Omega_t \equiv \int_{i\in(0,1)} \Omega_t(i)di$, are given by

$$\Omega_t = \left[\theta - mc_t - \frac{\gamma}{2}(\pi_t - 1)^2\right] Y_t. \tag{25}$$

Capital good sector. Capital good firms are perfectly competitive and use a linearly homogeneous production technology to produce capital. The representative firm purchases $(1-\delta)K_t$ units of old (depreciated) capital from the mutual fund and I_t units of the final good from the final good firms, and uses the two inputs to produce K_{t+1} units of new capital that is sold back to the mutual fund. Then, the conventional investment identity obtains

$$K_{t+1} = (1-\delta)K_t + I_t. \tag{26}$$

Mutual fund sector. Our economy has two types of illiquid assets – capital and shares in intermediate goods firms – and there is no aggregate uncertainty in the model after time–zero. Thus, a no arbitrage argument implies that the return on the two illiquid assets is the same in all periods except possibly time–zero when their returns will differ if an aggregate time–zero shock occurs. We allocate ownership and the potential time–zero capital gains and losses among households by assuming that households invest in a mutual fund produced by perfectly competitive financial service firms. Each firm holds the market portfolio of the two illiquid assets and pays households the market return on illiquid assets.

To derive the market return on illiquid assets note that the return on capital in period t is given by

$$R_t^k = r_t^k + 1 - \delta. \tag{27}$$

The one period return from investing one unit of the period $t-1$ final good into shares is

$$R_t^v = \frac{\Omega_t + V_t}{V_{t-1}}, \tag{28}$$

where V_t is the share price. We assume that the return on capital and equity is subject to a corporate tax as well as an asset income tax paid by households. Liquid assets, in contrast, will consist primarily of government debt in equilibrium and are taxed once at the household level. To reduce the notational burden, we assume that corporate taxes are paid by the mutual fund. Let τ^k denote the corporate tax rate. Then, perfect competition leads to the arbitrage conditions

$$R_t^a - 1 = (1-\tau^k)(R_t^k - 1) = (1-\tau^k)(R_t^v - 1). \tag{29}$$

for all $t > 0$. From this no-arbitrage restriction the share price is given by

$$V_t = \sum_{i=1}^{\infty} \left(\prod_{j=1}^{i} \frac{1}{R_{t+j}^k} \right) \Omega_{t+i}. \tag{30}$$

Hence, the discount factor $\Lambda_{t,t+1}$ in equation (23) is given by $\Lambda_{t,t+1} = 1/R_{t+1}^k$.

We analyze the evolution of our economy during a demographic transition by solving a two point boundary problem. The terminal condition is determined by a steady state and the initial condition is an initial age–distribution and an initial age–asset distribution.

Equation (29) does not obtain in period $t = 0$ because the response of the price–system generally induces distinct capital gains and losses on shares in intermediate goods firms and capital.

4.4 Government

The government consists of a central bank and a fiscal authority.

Central bank. The central bank sets the nominal interest rate R_t following a simple rule that depends on the current inflation rate and the past nominal interest rate

$$\log\left(\frac{R_t}{R}\right) = \rho_r \log\left(\frac{R_{t-1}}{R}\right) + (1-\rho_r)\phi_\pi \log(\pi_t), \tag{31}$$

where R is a constant. The parameter ρ_r governs the inertia of the nominal interest rate, and the parameter $\phi_\pi > 1$ captures the central bank's stance on inflation. A high ϕ_π implies a strong anti–inflation stance and vice versa.

Fiscal authority. The fiscal authority raises revenue by taxing consumption, labor income, capital income, and mutual funds. Total tax revenue is

$$T_t = \sum_{j=1}^{J}\left[\tau^c \bar{c}_{j,t} + \tau^{ka}(R_t^k - 1)a_{j-1,t-1} + \tau^a \frac{(R_{t-1}-1)}{\pi_t} d_{j-1,t-1} + \tau^w w_t \epsilon_j \bar{h}_t\right] N_{j,t}, \tag{32}$$

where $\bar{c}_{j,t} = \psi_{j,t} c_{j,t}(1) + (1-\psi_{j,t}) c_{j,t}(0)$ is the average consumption by surviving and non–surviving households and $\tau^{ka} = \tau^a + \tau^k - \tau^a \tau^k$ is the total tax rate on illiquid assets.

Let D_t^n denote the face value of nominal government debt issued in period t. Then aggregate government expenditures consist of government purchases G_t, nominal interest payments on its debt, net of new issuance, $(R_{t-1}D_{t-1}^n - D_t^n)/P_t$, subsidies to intermediate goods firms, $\tau^f Y_t = (\theta - 1)Y_t$, public pension benefits $B_t \equiv \sum_{j=J_r}^{J} b_{j,t} N_{j,t}$, and lump–sum transfers to households, $\Xi_t \equiv \sum_{j=1}^{J} \xi_t N_{j,t}$. It follows that the government flow budget constraint is given by

$$G_t + \frac{R_{t-1}D_{t-1}^n - D_t^n}{P_t} + \tau^f Y_t + B_t + \Xi_t = T_t \tag{33}$$

and the government bond market clearing condition is given by

$$\frac{D_t^n}{P_t} = D_t \equiv \sum_{j=1}^{J} \bar{d}_{j,t} N_{j,t}, \tag{34}$$

where $\bar{d}_{j,t} = \psi_j d_{j,t}(1) + (1-\psi_j) d_{j,t}(0)$ is the average government bond holdings by surviving and non–surviving households.[17]

[17] Because $d_{j,t}(0) = 0$, the aggregate bond can be arranged as

$$D_t \equiv \sum_{j=1}^{J}\left[\psi_j d_{j,t}(1) + (1-\psi_j) d_{j,t}(0)\right] N_{j,t} = \sum_{j=1}^{J} \psi_j d_{j,t}(1) N_{j,t} = \sum_{j=1}^{J} d_{j,t}(1) N_{j+1,t+1}.$$

We will consider two different scenarios for the time path of government debt. It proves easier to isolate the impact that household asset demand has on real interest rates, the nominal interest rate and the inflation rate if we assume that the supply of per capita nominal government debt is constant. However, we also consider scenarios where the supply of nominal government debt varies over the transition.

For a given time path of nominal government debt, we close the government budget constraint by varying the size of the lump–sum transfer, ξ_t. Benefits from the pubic pension program are modeled in the same way as Braun et al. (2009). A household starts receiving a public pension benefit at the mandatory retirement age of J_r. The real size of the benefit during the household's retirement is constant at a level that is proportional to its average real wage income before retirement

$$b_{j,s+j-1} = \begin{cases} 0 & \text{for } j = 1, ..., J_r - 1 \\ \lambda \left(\frac{1}{J_r - 1} \sum_{j=1}^{J_r - 1} w_{s+j-1} \epsilon_j \bar{h}_{s+t-j} \right) & \text{for } j = J_r, ..., J, \end{cases} \tag{35}$$

where λ is the replacement ratio of the pension benefit and s is the household's birth year. Thus, the public pension system implicitly assumes perfect inflation indexation of pension benefits.

4.5 Competitive equilibrium

Our aim here is to understand the quantitative significance of demographic change for the nominal and real secular stagnation observations that motivate our analysis and that is easiest to ascertain if we conduct an impulse response analysis that holds other aggregate shocks fixed. In the analysis that follows, we assume that at the beginning of time zero households observe the future evolution of the demographic distribution and have perfect foresight about the subsequent evolution of prices and government policy reactions.[18] Consequently, our definition of equilibrium is a perfect foresight competitive equilibrium. More details on the definition of equilibrium can be found in Appendix B.1.

Table 1: Age distribution and 21+ population by decade

	year				
Age/pop 21+	2015	2025	2035	2045	2055
Under 30	0.11	0.11	0.10	0.09	0.09
30–39	0.15	0.12	0.12	0.12	0.11
40–49	0.18	0.16	0.13	0.13	0.13
50–59	0.15	0.17	0.17	0.14	0.14
60–69	0.18	0.14	0.18	0.17	0.15
70+	0.23	0.29	0.31	0.35	0.38
Pop 21+ (millions)	102	100	95	88	80

Notes: Our estimates using data from the IPSS.

5 Model parameterization

5.1 Demographic transition

We assume that new households are formed at age 21 and the size of the household is parameterized in the same way as Braun et al. (2009). In the model individuals face mandatory retirement at age 68 ($J_r = 48$). This is two years older than the age to qualify for full public pension benefits in Japan and is chosen to be consistent with the effective labor–market exit age in 2014 for Japan estimated by the OECD.[19] Finally, the maximum lifespan is set to 120 years ($J = 100$).

Table 1 reports summary statistics for Japan's age distribution and population at 10 year intervals. We limit attention to the 21+ population to make the data consistent with the workings of our model. The combined impact of aging of the baby boom cohorts, lower fertility rates and longer life expectancies are already putting downward pressure on the age 21+ population and this pressure will increase in future years. During this transition the percentage share of the 70+ population in the total 21+ population will increase from 23 percent to 38 percent according to our estimates which are based on data from the IPSS.

[18]Boppart et al. (2017) provide a justification for using this approach in heterogeneous agent economies.
[19]See Pensions at a Glance OECD, 2015.

Table 2: Parameterization of the model

Parameter	Description	Value
Demographics		
J_r	Retirement age	48 (Age 68)
J	Maximum lifespan	100 (Age 120)
$\{\psi_j\}_{j=1}^{J}$	Survival probabilities	Braun and Joines (2015)
Technology		
θ	Gross markup	1.05
γ	Price adjustment cost	41.2
α	Capital share parameter	0.30
δ	Depreciation rate	0.102
$\gamma_a(0)$	Cost of adjusting illiquid assets in death year	0.0723
$\gamma_a(1)$	Cost of adjusting illiquid assets in non-death year	0.203
Preferences		
σ	Inverse elasticity of intertemporal substitution	3
ν	Frisch labor supply elasticity	2
υ	Preference weight on leisure	6.9
β	Preference discount factor	0.996
Monetary Policy		
ρ_r	Interest rule persistence	0.35
ϕ_π	Interest rule inflation elasticity	2
Fiscal Policy		
τ^c	Consumption tax rate	0.05
τ^k	Corporate tax rate	0.35
τ^a	Tax rate on asset income	0.2
τ^w	Tax rate on labor income	0.232
τ^f	Subsidy to intermediate goods firms	$\theta - 1$
G/Y	Government share of output	0.16
λ	Public pension replacement ratio	0.094
D/Y	Net government debt output ratio	1.23

5.2 Other model parameters and model assessment

The remaining model parameters are set as in Braun and Ikeda (2022) and are reported in Table 2. Our previous paper provides an extensive discussion of our calibration strategy and also reports a range of statistics that are designed to assess the fit of our model to Japanese data at business cycle frequencies. Using the same parameterization here adds force to the results that we are about to present because it is clear no adjustments are being made to tailor the parameterization to reproduce a distinct set of macroeconomic observations that are operating at medium term frequencies.

Table 3: Model steady state net worth, liquid and illiquid asset holdings by age.

Age	Net worth	Liquid assets	Illiquid assets
Under 30	0.01	-0.63	0.64
30–39	0.88	-0.85	1.73
40–49	2.85	0.19	2.65
50–59	5.54	2.23	3.31
60–69	7.27	3.63	3.64
70+	4.16	0.94	3.22

Notes: Net worth and asset holdings are reported as ratios relative to income of households aged 50–59. See Braun and Ikeda (2022) for more details.

6 Results

6.1 Household asset demand: partial equilibrium

Our first step in analyzing the macroeconomic effects of aging is to show that asset demand initially increases during a demographic transition and that demand for liquid assets increases by more than demand for illiquid assets.

Table 3 reports net worth and steady state age–asset profiles for households in our model. These age-profiles are calibrated to Japanese survey data from 2014. Liquid asset holdings are negative for younger age groups because these age groups are borrowing liquid assets on net. Net worth increases with age up until retirement which occurs at age 68 in the model and then declines thereafter. The variation in holdings of liquid assets over the lifecycle is particularly large. Younger households borrow liquid assets to purchase illiquid physical and financial assets. Households close to age 68, in contrast, hold large amounts of both liquid and illiquid assets. Then during retirement households draw down both assets.

Next, consider Table 3 and Table 1 together. During the initial stages of the transition to an older age distribution, the fraction of households with high net worth and high liquid asset holdings is stable or increasing. In contrast, the fraction of households with low net worth and negative holdings of liquid assets declines. In particular, the fraction of households aged 50–69 increases from 0.33 in the year 2015 to 0.35 in 2035 while the fraction of households under age 39 declines from 0.26 in the year 2015 to 0.22 in the year 2035. Table 4 shows how these changes in the population distribution affect aggregate demand for liquid and illiquid assets. The changes in aggregate asset demand reported

Table 4: Partial equilibrium aggregate demand for liquid and illiquid assets

Demographic Scenario	Liquid assets	Year	Illiquid assets	Year
Aging of baby boom cohorts	19.83	2038	2.32	2029
Longer life expectancy	0.63	2045	0.07	2044
Lower fertility rates	24.12	2065	6.18	2067
Baseline	27.1	2043	5.24	2053

Notes: The table reports the maximum increase in each type of asset expressed as a percentage of initial assets and the year where the maximum increase occurs. "Aging of baby boom cohorts" reflects changes in the population distribution due to aging of the baby boom cohorts only. "Longer life expectancy" reflects changes in the age distribution due to higher survival rates only. "Lower fertility rates" reflects changes in the birth rate of 21 year olds only and "Baseline" incorporates all three channels.

in the table are partial equilibrium in the sense that they only recognize changes in the population–age distribution and use the steady state age–profile of savings plans.

The dynamics of the population distribution in our model are pinned down by an initial age–distribution, fertility rates in each year, and life–expectancies for each birth cohort. Table 4 helps to understand the contribution of these factors. The "Aging of baby-boom cohorts" scenario holds fertility rates and survival probabilities fixed in all years at their terminal values and uses the initial 2014 age distribution as the initial condition. In the "Longer life expectancy" scenario, survival probabilities vary by birth cohort and gradually increase over time, but the age distribution and fertility rates are fixed in all periods. The "Lower fertility rate" scenario considers the case where fertility rates gradually decline and the initial age distribution and survival probability age–profiles are set to their terminal values. Finally, the "Baseline" demographic scenario incorporates all three factors. The three demographic factors individually and collectively induce persistent increases in liquid and illiquid asset holdings. Aging of the baby–boom cohorts is the strongest factor, but lower fertility rates are also important. A second feature of these results is that aging has a much larger impact on the aggregate demand for liquid assets as compared to the aggregate demand for illiquid assets. As households enter retirement, they prefer to hold a larger share of their portfolios in liquid assets because the replacement rate provided by pension income is less than one. Moreover, they face an elevated mortality risk and can avoid some of the costs of liquidating their holdings of illiquid assets in their death year if they tilt their portfolio towards liquid assets.

6.2 General equilibrium results

The partial equilibrium analysis shows that aging puts steady and persistent upward pressure on asset demand for about 30 years when using the terminal steady state price system. However, it is not clear whether this result survives when prices adjust in each period to clear markets. In addition, price adjustments induce reactions in monetary policy under our assumption that the central bank follows an interest rate targeting rule and price changes affect government revenues, so fiscal policy also adjusts.

We now turn to consider general equilibrium scenarios. Before discussing the results, we wish to reiterate that we are not attempting to make forecasts about the future course of the Japanese economy. Instead we want to understand whether demographic forces are large enough, in isolation, to induce nominal and real secular stagnation which we define as steady and persistent declines in interest rates, the inflation rate and output. Our strategy for answering this question is to conduct a nonlinear impulse response analysis. In the year 2014, households in the model discover that the population distribution is going to evolve over time and they adjust their consumption, savings and labor supply plans accordingly. When making these adjustments, they fully anticipate the future evolution of prices. These assumptions allow us to use global sequence methods to compute an equilibrium.[20, 21]

When solving for the general equilibrium version of the model, we specify an exogenous sequence of nominal government debt. The baseline specification assumes that the stock of per capita nominal debt is held fixed in each period and that lump–sum taxes adjust to satisfy the government budget constraint in each period. This assumption allows us to isolate the important role that the monetary policy response plays in accounting for the secular stagnation observations that motivate this paper. In section 6.2.3 we consider scenarios where the stock of nominal government debt increases during the demographic transition. The monetary authority is assumed to follow a nominal interest rate targeting rule with a serial correlation coefficient ρ_r set to 0.351 and an inflation elasticity, $\phi_\pi = 2$.

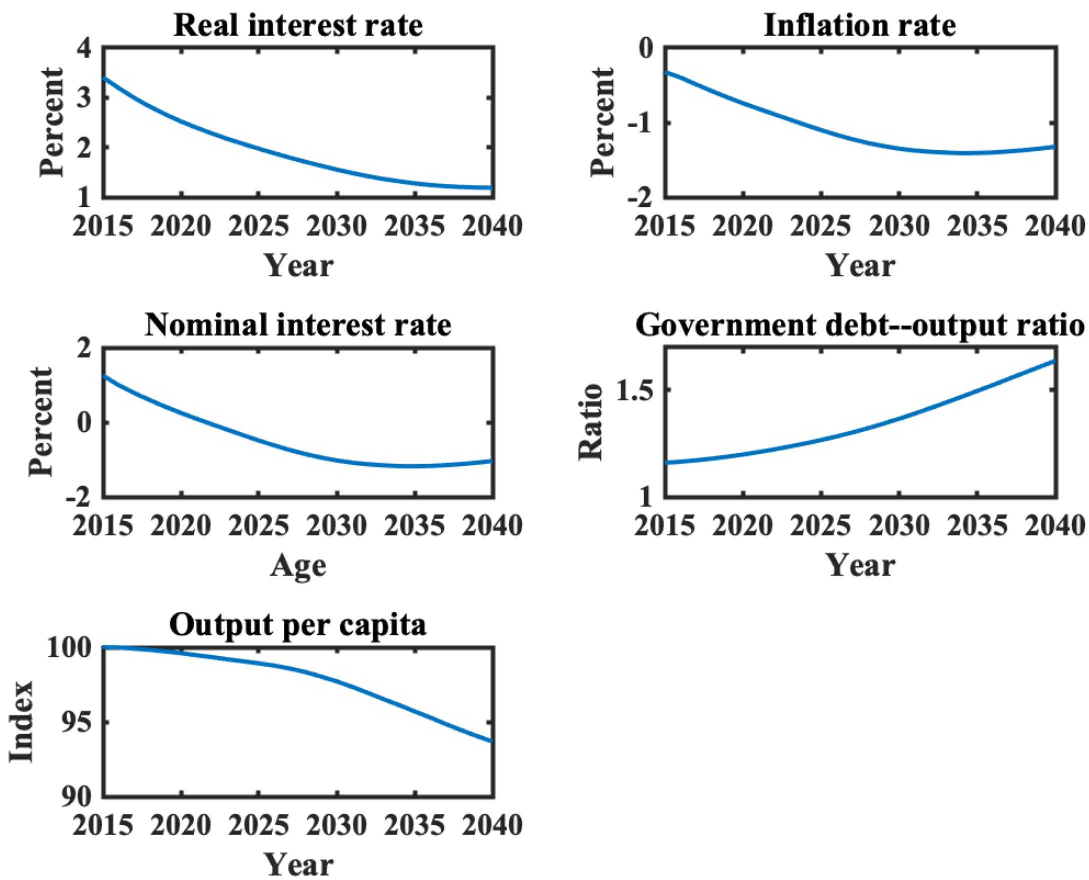
6.2.1 Baseline results

Figure 5 reports results for the baseline scenario. These results indicate that the aging shock induces steady downward pressure on the real interest rate, the nominal interest rate

[20] Chen et al. (2006) find that their results are robust to the model of expectations formation for a demographic shock in a representative agent model.

[21] Computing an equilibrium for a given parameterization of the model can take several days on a 2020 vintage Mac Pro with 16 cores using Matlab's parallel toolbox.

Figure 5: Baseline macroeconomic responses to an aging population distribution



and inflation rate and per capital output. The real interest rate declines from 3.4 percent in 2015 to 1.2 percent in 2040, and the inflation rate falls from -0.32 in 2015 to a low of -1.41 percent in 2035. It is also interesting to see that the model produces a large increase in the government debt–output ratio. It rises from 1.16 in 2015 to 1.84 in 2040. Finally, the model also produces a gradual but steady decline in per capita output at the average rate of -0.23 percent per year between 2015 and 2040.[22]

We now turn to inspect the mechanisms that are inducing these responses. The declines in the real interest rate on illiquid assets and the inflation rate suggest that aging continues to induce persistent increases in asset demand when we assume that markets clear. In the baseline GE simulation, private demand for assets peaks later and at a higher level. The peak increase in private demand for liquid assets is 68.9 percent in the GE simulation as compared to 27.1 percent in the PE scenario and the peak occurs in 2077 in the GE

[22]As a reference point, the output gap – the deviation of output from the potential output, estimated by the Bank of Japan – is -0.21 percent on average between 1990–2020.

scenario as compared to 2043 in the PE scenario. Demand for illiquid assets peak earlier in 2030 in the GE simulation as compared to 2053 in the PE simulation. The peak increase is also a bit higher at 6.1 percent in the GE scenario as compared to 5.2 percent for the PE scenario.

The reason why liquid asset demand increases and the inflation rate falls is particularly easy to understand under our assumption that per capita government nominal debt is constant. The liquid asset market clearing condition (34) in our model can be written as

$$\frac{d_t^n}{P_t} = \frac{\sum_{j=1}^{J} \bar{d}_{j,t} N_{j,t}}{N_t} \tag{36}$$

where d_t^n is per capita nominal government debt which is fixed in this simulation; P_t is the price level; and the right hand side is aggregate household demand for liquid assets. Our analysis of the 2–period OLG model suggests that the reason why the price level is falling is because private demand for liquid assets is increasing.

Higher demand for liquid assets can explain why the inflation rate is falling but, it is still not clear why the model produces concurrent declines in the real interest rate and the inflation rate. To understand why the real interest rate and the inflation rate are moving together, it is helpful to start by pointing out that the Fisher equation doesn't obtain in our quantitative model because liquid and illiquid assets are not perfect substitutes. The analogue of the Fisher equation in our model is approximately given by

$$\frac{\tilde{R}_t}{\pi_{t+1}} = \frac{\tilde{R}^a_{t+1} + \gamma_a \Delta a_{j+1,t+1}}{1 + \gamma_a \Delta a_{j,t}} \tag{37}$$

for a household of age j in period t.[23] Recall that $\tilde{R}_t = 1 + (1 - \tau^a)(R_t - 1)$ is the after–tax nominal interest rate and that the after–tax return on illiquid assets, $\tilde{R}^a_{t+1}$, can be expressed as $\tilde{R}^a_{t+1} \equiv 1 + (1 - \tau^a)(1 - \tau^k)(r^k_{t+1} - \delta)$.

Observe that if the adjustment cost on illiquid assets, γ_a, is set to zero, our model nests the standard Fisher equation. However, in our baseline specification $\gamma_a > 0$ the Fisher equation doesn't obtain because one asset is less liquid than the other. Some households are borrowing liquid assets to acquire illiquid assets while other households are holding positive amounts of both assets, but are net sellers of illiquid assets. No arbitrage restrictions embedded in equation (37) imply that the steady state market price of liquidity,

[23]Equation (37) is an approximation because it assumes that households don't observe their death event at the beginning of their final period of life.

$\tilde{R}_t/(\pi_{t+1}\tilde{R}^a_{t+1})$, is less than one and varies over time during the demographic transition. Intuitively, household level costs of adjusting illiquid assets induce households, to only gradually adjust their holdings of illiquid assets when the aggregate state of the economy changes. The aggregate state of the economy is varying over time due to time variation in the age distribution and the reaction of monetary policy to the resulting changes in the inflation rate.

What is essential to understand for the discussion that follows is that an easing in monetary policy has a level effect and a composition effect. It pushes up the price level and lowers the real return on both assets via the asset substitution effect. However, the two assets are imperfect substitutes and the market price of liquidity also falls persistently. We now show that these reaction effects of monetary policy play a central role in understanding how our baseline specification produces persistent simultaneous declines in the inflation rate and the real interest rate.

6.2.2 The reaction of monetary policy to aging

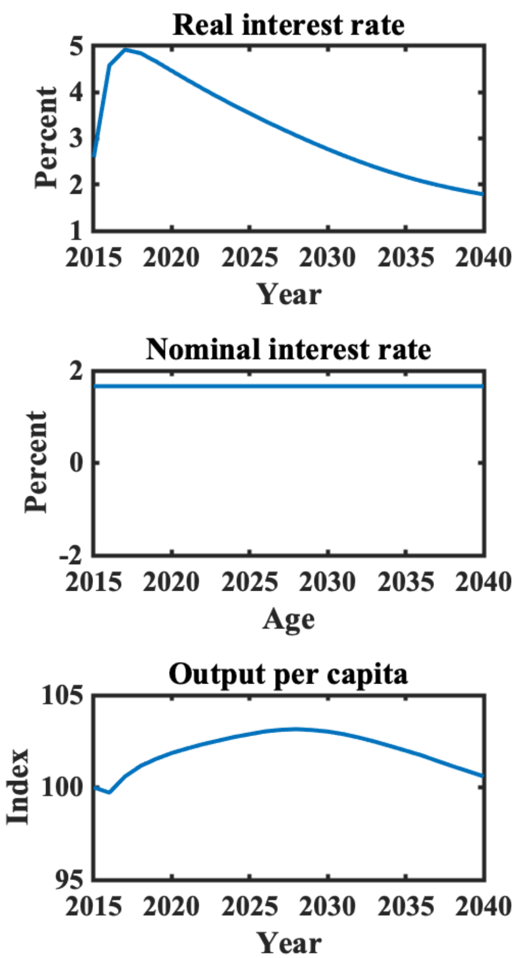
To document the important role of the reaction function of monetary policy, we consider a scenario where the central bank doesn't respond to the deflationary pressure induced by aging by setting ϕ_π to zero. Figure 6 reports the results for the constant R specification. Observe first that monetary policy has an important effect on co–movements between the real interest rate and the inflation rate. When monetary policy is held fixed, the aging shock continues to depress the real interest rate between the years 2018 and 2040, but now the inflation rate plummets on impact, falling by nearly 10 percent, and then recovers in subsequent periods. It follows that the relationship between the inflation rate and the real interest rate is now negative between 2018 and 2040. Thus, this specification fails to produce a persistent period of low real interest rates and low inflation that we have seen in historical data and that the baseline specification reproduces.

A second difference is that output increases in the constant R scenario up until the year 2028, whereas it falls in the baseline scenario. The third difference is that the debt–output ratio increases much more rapidly if monetary policy doesn't react, peaking at 1.6 in 2028 and then gradually declines thereafter.[24]

It follows from the first result that the nominal interest rate targeting rule is playing a central role in the model's ability to account for the persistent concurrent declines in the

[24] The maximum debt–output ratio in the baseline is 2.2 and occurs in the year 2074.

Figure 6: Macroeconomic responses to aging with monetary policy held fixed: $\phi_\pi = 0$



real interest rate and the inflation rate. On the one hand, aging puts persistent downward pressure on the real interest rate even when monetary policy is held fixed. Asset demand increases via the partial equilibrium effects that we discussed in Section 6.1. Aging also induces capital deepening as the size and average labor efficiency of the working population decline. On the other hand, in the baseline specification, monetary policy responds to the deflationary pressure induced by aging by lowering the nominal interest rate and this reaction attenuates the impact response of the aging shock on the price level and propogates the impact of the aging shock on the price level over time. A lower nominal interest rate puts additional downward pressure on the real interest rate. The baseline specification has a lower real interest rate than the constant R specification in each year between 2016 and 2040. Liquid and illiquid assets are imperfect substitutes and lowering the nominal interest rate also shifts the composition of asset demand towards illiquid assets. The collective

Table 5: How prices and aggregate allocations differ in the year 2040 if monetary doesn't respond to aging

Scenario	C	K	H	$\bar{h}$	Y	w	r^a	r^d
Baseline	-0.4	13.9	-5.9	3.1	-0.4	5.78	-1.65	-1.37
Constant R	1.6	10.4	-2.1	7.2	1.5	3.64	-1.05	-0.59

Notes: H is aggregate labor input in efficiency units and $\bar{h}$ is hours per worker. All variables are percentage deviations from steady state except the two interest rate variables which are percentage point differences from steady state.

response of households offsets the Fisher effect and inflation and real interest rates both fall persistently in the baseline specification.

To understand why the output response is different in the two scenarios, consider Table 5 which reports some of the main macroeconomic variables for the year 2040 expressed in terms of deviations from the terminal steady state. Observe that the real return on both illiquid and liquid assets is higher in 2040 in the constant R equilibrium. The capital stock and investment are lower in this equilibrium, but this economy has more wealth. Aggregate illiquid assets and aggregate real holdings of liquid assets are both higher in the constant R equilibrium (the price level is lower). The sign of household consumption varies by age, but aggregate consumption is higher relative to the baseline and aggregate investment is lower. The first effect is larger and this is why output is higher in 2040 in the constant R equilibrium.[25]

Another way to see why output is higher in the constant R equilibrium is to consider the two inputs of production. The capital stock is smaller in this equilibrium, but labor input in efficiency units, H, is much less depressed compared to the baseline specification. Labor input is higher because aggregate labor supply, $\bar{h}$, is higher in the constant R equilibrium. Aggregate labor supply in our model is a marginal–utility based weighted average of labor supply by age (see equation (19)) and a relatively large share of older workers have been negatively impacted by the history of prices they have experienced up to this date. Specifically, the severe deflation that occurred when they were young increased their debt burden and decreased their wealth substantially. They have low consumption in 2040 and prefer to work harder and their preferences are receiving more weight in the labor supply aggregator.

[25]Note that in the figures output is reported as an index relative to its 2015 level whereas in Table 5 it is expressed as a percentage deviation from its terminal steady state value.

This is a good point in our analysis to compare our results with previous results in Bullard et al. (2012) and Katagiri et al. (2020). Even though both papers use flexible price OLG frameworks to analyze the effect of demographics on the inflation rate, monetary policy in the settings they consider doesn't affect the inflation rate or real allocations. More importantly, both frameworks have the property that a lower fertility rate increases the real interest rate and then via the Fisher equation, the inflation rate has to fall. Money is a perfect substitute for another interest bearing asset and households only hold both assets if money provides the same real return as the interest bearing asset. One difference between the two papers is the response of monetary policy. Bullard et al. (2012) assume that monetary policy doesn't respond to aging. Katagiri et al. (2020) allow for moentary policy to respond to aging, but they impose the FTPL and monetary policy is passive in the sense of Leeper (1991). As we noted in Section 3, under the FTPL if the real return on the other asset increases, the price level has to fall because money is not an interest bearing asset.

In our model, monetary policy responds to aging by lowering its policy rate and its reaction transmits to the real economy in two ways. First, our model has nominal price rigidities that are the hallmark of NK economics. In particular, intermediate goods firms face quadratic price adjustment costs.[26] Second, in our OLG framework monetary policy affects the real interest rate on liquid and illiquid assets via the asset substitution channel as we explained in Section 3 using a 2–period OLG model.

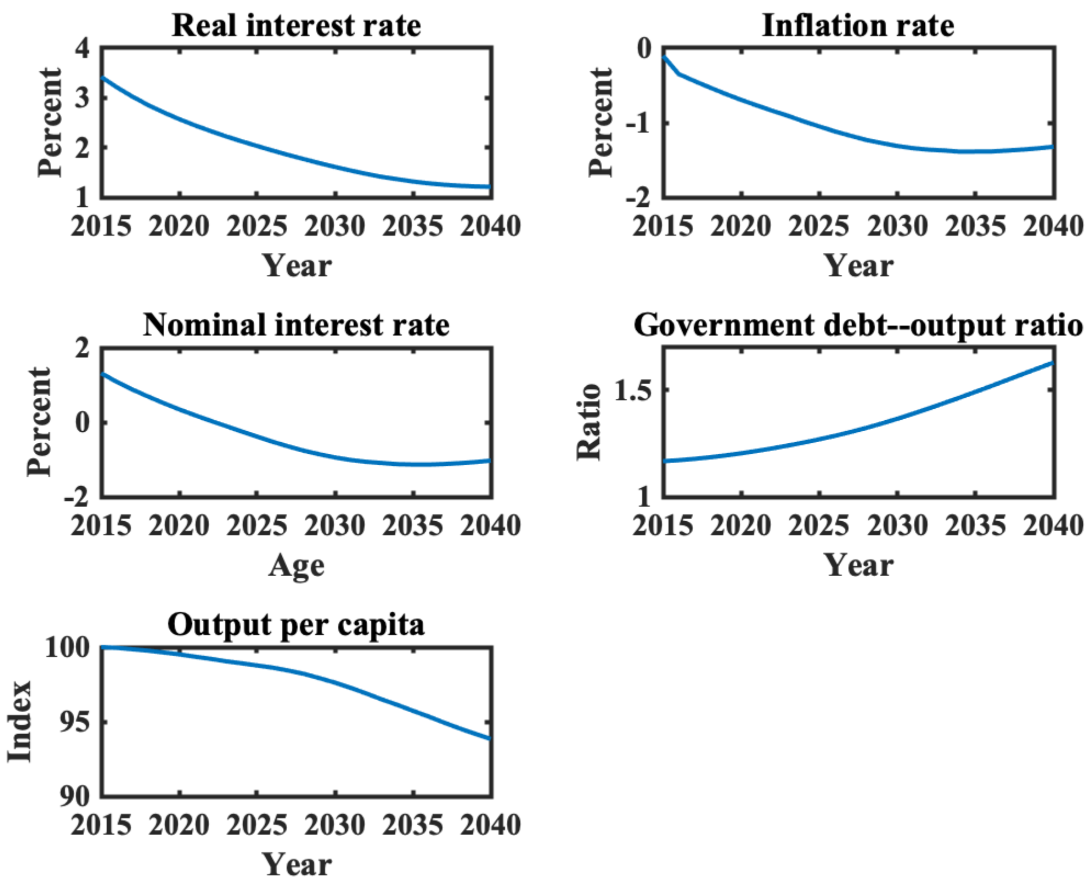
Braun and Ikeda (2022) find that nominal price rigidities help the model to reproduce local projection evidence on the signs and magnitudes of a variety of aggregate variables to monetary policy shocks. However, here we are considering a shock to the age distribution and for this shock the asset substitution channel is more important and nominal rigidities are largely irrelevant. This result can be ascertained by comparing the results in Figure 5 with the results in Figure 7 which reports impulse responses to Japan's aging shock under the assumption that prices are flexible. A comparison of the two figures indicates that the responses are virtually indiscernible.[27]

Thus, the asset substitution channel is the main reason why monetary policy influences inflation and real interest rate co–movements. In our baseline specification, the actions taken by the central bank to offset deflationary pressure depress the real interest rate on

[26] As Rupert and Šustek (2019) document it doesn't automatically follow that an easier monetary policy lowers the real interest rate.

[27] The impact responses are different in the two specifications. But, we have omitted the impact responses to emphasize the secular properties of the two specifications.

Figure 7: Macroeconomic responses to aging with flexible prices

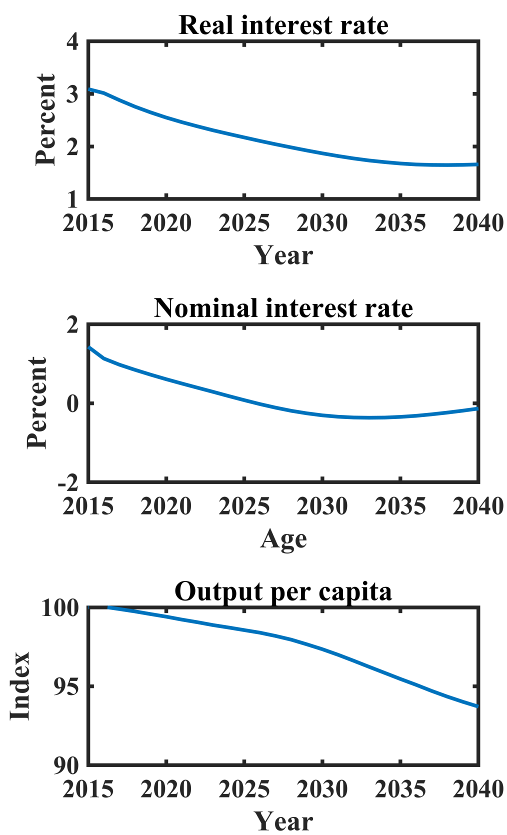


liquid assets. The real return on illiquid assets also falls due to asset substitution. However, the response of the real return on illiquid assets is smaller on impact and more gradual over time because illiquid assets are costly to adjust. Consequently, households allocate a bigger share of their savings to illiquid assets.

6.2.3 Higher government borrowing in an aging society

Up to this point, we have held the supply of per capita nominal government debt fixed. This was a deliberate choice because it made it easier to inspect the mechanisms that allow our model to account for the secular stagnation facts and the baseline specification produces a large increase in the debt–output ratio. Still, the increase in the debt–output ratio in the baseline specification is due entirely to changes in the real value of nominal debt and the nominal stock of government debt has been increasing over time in Japan. In our model a higher supply of government debt is inflationary. This can be seen by inspection of equation (36) and the results from the 2–period model. Thus, it is a quantitative question whether

Figure 8: Macroeconomic responses to aging with higher (nominal) government borrowing



the deflationary sources induced by aging are large enough to overwhelm the inflationary sources induced by higher debt issue.

Before discussing how the results change when the nominal stock of government debt is increased, it is important to point out that our economy is dynamically efficient. The real pretax return on government debt in our baseline specification gets very small, but is positive in all periods including the terminal steady state. During the transition, per capita output declines persistently and then rises to the terminal steady state from below. However, the level of per capita output is constant in the terminal steady state. For the reasons discussed in Blanchard (2023), this property of the model matters when analyzing the macroeconomic effects of higher government borrowing. In our baseline specification, the demographic shock is not large enough to push the real return on government debt below the growth rate of output even temporarily. Thus, a temporary increase in government borrowing has real effects, but higher government debt must eventually be offset by other government policies to prevent the debt–output ratio from exploding.

Figure 8 considers a scenario where the time–profile of government issue is assumed to gradually increase in tandem with the increase in aggregate private demand for liquid assets. The debt–output ratio in 2015 is 1.13, which is close to its baseline value of 1.15. However, the debt–output ratio rises more rapidly over time in the high debt scenario and reaches 2.1 in the year 2040, which is 31 percent higher than its baseline value of 1.6.

Higher government debt attenuates the downward pressure on the price level induced by higher private demand for liquid assets, but the model still predicts that aging is deflationary. The lowest value of the inflation rate in the baseline scenario was -1.4 percent. In this scenario its minimum value is -1 percent and it hits this floor in the year 2032. An inflation rate of -1 percent is not exceptionally low. For instance, the Japanese GDP deflator experienced year over year declines of -0.9 percent on average between 1995–2013. Thus, our main result that aging induces deflationary pressure also obtains even when we posit large increases in the stock of government debt during the transition.

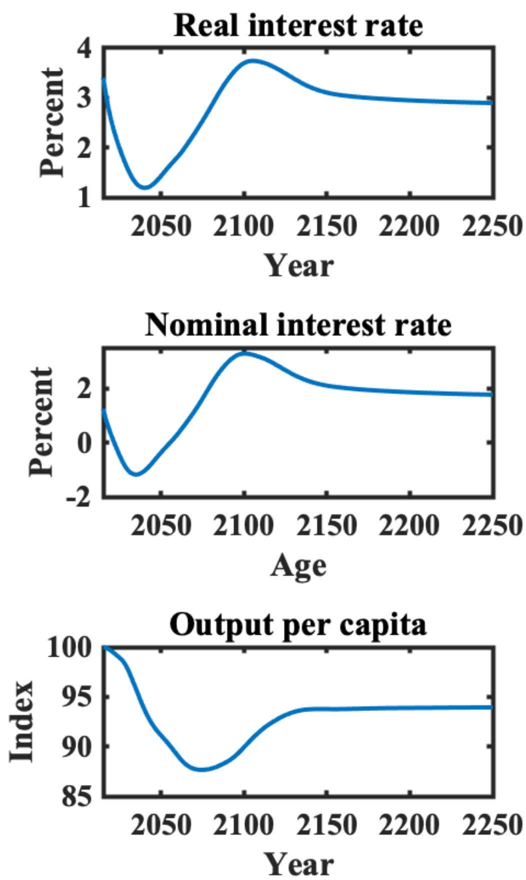
In the baseline scenario 100 percent of the increase in the real stock of government debt between the years 2015 and 2040 was due to revaluation effects induced by deflation. In the current scenario with higher nominal government debt, the majority of the increase in the real stock of government debt over the same time interval is due to higher nominal debt issue and only 41 percent of the increase is due to revaluation effects.

Accommodating households' increased demand for liquid securities, higher government debt issuance attenuates deflationary pressure and this in turn attenuates the decline in the nominal interest rate engineered by the central bank. The nominal interest rate still falls over time but its minimum value is now only -0.36 percent per annum as compared to -1.2 percent in the baseline scenario. This in turn weakens the asset substitution effect. The minimum value of the real interest rate in liquid assets is 0.64 percent in the year of 2034 in the high debt scenario. In the baseline, in contrast, the real return on liquid assets falls to a minimum of 0.23 percent in 2035.

With less downward pressure on the real interest rate on liquid assets, the interest rate on illiquid assets also adjusts to induce households to continue to hold both securities. The real interest rate still declines in this scenario but the magnitude of the peak decline is smaller. It falls to a low 1.65 percent in the year 2039 here as compared to a low of 1.2 percent in the year 2035 in the baseline scenario.

Finally, observe that the output declines are of about the same magnitude in the two scenarios. On the one hand, households have higher returns on their savings in the scenario with higher government debt and aggregate consumption falls less in response to the demographic shock as compared to the baseline. On the other hand, higher government debt

Figure 9: Baseline macroeconomic responses to aging in Japan at longer horizons



issuance crowds out private investment and this offsets most of the consumption gains.

6.2.4 Unpleasant monetary arithmetic

We have seen that aging induces downward pressure on the inflation rate during the first 25 years of the transition. Our model also has interesting implications for how the aggregate economy responds to aging at longer horizons. Private aggregate demand for liquid assets increases persistently during the transition, but at some point it starts to decline. In the baseline specification nominal debt is fixed and it follows that the price level has to increase to clear the market for government debt. This property of our model is similar in spirit to arguments first made in Sargent and Wallace (1981) and indeed, as we show next, our economy has the property that the initial period of deflation that we have documented is followed by a subsequent episode of inflation.

Figure 9 displays the entire transition. Perhaps the most noteworthy feature of it is that the inflation rate experiences overshooting. It rises above its steady state level and then returns to the steady state from above. The reason for this result is that private asset demand peaks during the transition and after this peak it declines. Table 4 shows that the date of the peak for the partial equilibrium baseline scenario is 2043. The period of peak asset demand for the baseline general equilibrium specification occurs in the year 2074, which is the same year that the inflation rate changes sign in Figure 9.

Goodhart and Pradhan (2017) have argued that aging will produce inflation in future years and Juselius and Takáts (2018) provide empirical evidence that the combination of a lower share of younger population cohorts and a higher share of older cohorts will create inflationary pressure in future years. Our model produces a period of overshooting of the inflation rate during the transition. However, the onset of inflation is very gradual and the peak inflation rate is not particularly large. The inflation rate doesn't peak until 2100 and the peak inflation rate is only 0.79 percent. The baseline specification holds government debt fixed. But our conclusion is the same if we consider the higher nominal public debt scenario instead. In this simulation the peak inflation rate is even smaller, 0.53 percent, and the peak occurs in 2097. The reason why overshooting is smaller in the high government debt simulation is because this scenario has less deflation in the first 25 years of the transition.

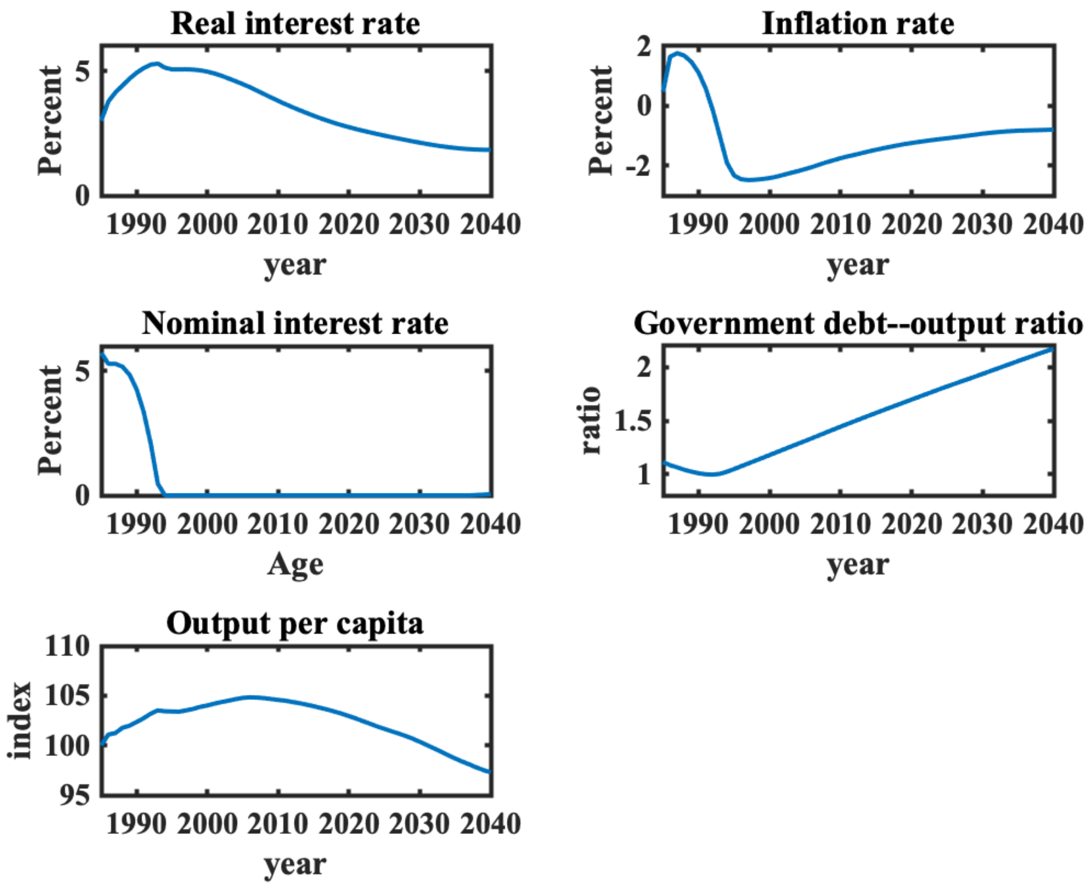
Overshooting in the inflation rate in Figure 9 is accompanied by overshooting in the nominal interest rate and the real interest rate. As household demand for assets falls, both the price level and the real interest rate have to increase to induce households to hold the aggregate stocks of government debt and illiquid assets. The response of the monetary authority magnifies these real interest responses for the reasons we have explained above.

The decline in asset demand has another impact on the fiscal situation of the government. We assume that lump–sum transfers are adjusted each period to clear the government's budget constraint and public transfers decline in the later stage of the transition and approach their terminal steady state from below.

7 Robustness

We have also performed simulations to investigate the robustness of our main result that the transition to an aging population distribution produces a period of secular stagnation. For instance, we have considered scenarios where we impose a zero lower bound on the nominal interest rate. Once this lower bound is hit, the dynamics of the economy resemble those of the ($\phi_\pi = 0$) scenario reported in Figure 6. In particular, the co–movements between the

Figure 10: Macroeconomic responses if demographic shock arrives in 1983



Notes: This simulation imposes the zero-lower bound on the policy nominal interest rate.

inflation rate and the real interest rate turn negative. Our maintained hypothesis that the ELB is not binding in the baseline scenario is also supported by research by Ikeda et al. (2020) who estimate structural non-linear vector autoregressions that embed the ELB for Japan and find that unconventional monetary policy has been, if anything, a more effective stabilization tool than conventional monetary policy at horizons of one year and beyond.

We have also explored how the dating of the demographic shock impacts our conclusions by considering a scenario where households learn about the demographic transition in 1983 instead of 2014. Figure 10 reports the results. This scenario abstracts from other shocks, but still manages to produce a boom period with rising real interest rates, high inflation and above trend output growth. These outcomes are consistent, at a qualitative level, with Japan's experience in the 1980s. It is also interesting that the model then produces a protracted episode of falling real interest rates, deflation and declining output. These latter outcomes are consistent with Japan's experience in the 1990s.

8 Conclusion

In this paper we have shown that a demographic transition to an older age distribution along the lines that Japan is facing now induces strong and persistent downward pressure on real interest rates, the inflation rate and output. Both the response of monetary policy and the transmission channel of monetary policy are important for our results. Our results suggest that Tobin effects are more important than nominal rigidities for understanding the transmission channel of monetary policy for this type of shock and that how monetary policy responds to it matters for aggregate economic activity. In our future work we plan to investigate how welfare of different birth cohorts is impacted by an aging population and analyze the properties of welfare enhancing government policies.

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Appendix (For online publication)

A Analytical Model

A.1 Proof of Proposition 1

In the DTPL equilibrium, the set of two endogenous variables $\{k_{t+1}, P_t\}$ is governed by two equations (6) and (11), which are reproduced here for convenience.

$$\alpha k_{t+1}^{\alpha-1} = R_t \frac{P_t}{P_{t+1}} \tag{A.1}$$

$$\frac{d_t^n}{P_t} + n_{t+1} k_{t+1} = (1-\alpha) k_t^{\alpha} \tag{A.2}$$

Once the equilibrium P_0 is determined, equation (A.2) determines k_{t+1} and equation (A.1) determines P_{t+1} for $t = 0, 1, ..,$. In equation (A.1), the nominal interest rate R_t affects the price level P_{t+1}, which in turn affects k_{t+2} as can be seen from equation (A.2). Hence, the setting of the nominal interest rate can affect the real economy. With $\{k_{t+1}, P_t\}$ on hand, ξ_t is given by equation (7), d_t is given by equation (9), and c_t is given by equation (10).

A.2 Proof of Proposition 2

In the DTPL steady state, the gross inflation rate is unity, $\pi = 1$, because per capita nominal government bonds are assumed to be constant. From equation (6), the capital stock is given by $k = (\alpha\pi/R)^{1/(1-\alpha)}$. The price level in steady state is determined by equation (11) as

$$\frac{d^n}{P} = d = (1-\alpha)k^{\alpha} - nk$$

Since k is independent of n, it follows that $\partial P/\partial n > 0$. It is also obvious that $\partial P/\partial d^n > 0$.

A.3 The FTPL version of the model

We start by defining the FTPL equilibrium of our model.

Definition 2 (FTPL equilibrium) *Given the initial capital k_0, the initial nominal obligation $R_{-1}D_{-1}^n$, a sequence of fertility rates $\{n_t\}$, and a sequence of policy variables $\{\xi_t, R_t\}$, a competitive equilibrium for this economy consists of a sequence of prices $\{P_t, R_t^k, w_t\}$, a set of allocations $\{c_t, k_t, d_t\}$ and a sequence of nominal government bonds $\{d_t^n\}$ that satisfy the firms' optimality conditions (4) and (5), the Fisher equation (6), the government budget constraint (7), and the market clearing conditions (9)-(11).*

Solving the government budget constraint (7) forward while using equation (4) yields the standard equation for the FTPL as

$$\frac{R_{-1}d_{-1}^n}{P_0} = (-\xi_0) + \sum_{t=1}^{\infty}\left(\prod_{\ell=1}^{t}\frac{n_{\ell-1}}{R_\ell^k}\right)(-\xi_t)$$

with the transversality condition $\lim_{t\to\infty}\left[\prod_{\ell=1}^{t}(n_\ell/R_\ell^k)\right]d_t = 0$ imposed. This equation holds in both the DTPL and the FTPL, but a difference is that in the FTPL $\{\xi_t\}$ is set exogenously and $\{d_t^n\}$ is determined endogenously to satisfy the government budget constraint, while in the DTPL $\{d_t^n\}$ is set exogenously and $\{\xi_t\}$ is determined endogenously.

First, we show that monetary policy has no real effect in the FTPL equilibrium. Combining equations (7) and (11) in the initial period yields

$$\frac{R_{-1}d_{-1}^n}{P_0} = (-\xi_0) + n_0\left[(1-\alpha)k_0^\alpha - n_1k_1\right] \tag{A.3}$$

From period $t = 1$ onward, this equation can be written by using equations (6) and (11) as

$$\alpha k_t^{\alpha-1}\left[(1-\alpha)k_{t-1}^\alpha - n_tk_t\right] = (-\xi_t) + n_t\left[(1-\alpha)k_t^\alpha - n_{t+1}k_{t+1}\right] \tag{A.4}$$

Once the equilibrium P_0 is determined, k_1 is given by equation (A.3) and k_{t+1} is given by equation (A.4) for $t = 2, 3, ...$ Since the equilibrium P_0 is determined in such a way that k_{t+1} converges to its steady state value, the real economy is independent of the nominal interest rate. With $\{k_{t+1}, P_t\}$ on hand, d_t^n is given by equation (7), d_t is given by equation (9), and c_t is given by equation (10).

Next, we show that a lower fertility rate increases inflation in the FTPL steady state. In the FTPL steady state, combining equations (6), (7) and (6) yields

$$\left(\alpha k^{\alpha-1} - n\right)\left[(1-\alpha)k^\alpha - nk\right] = (-\xi) \tag{A.5}$$

Assume ξ is set such that $k^* < k < k_{\text{gold}}$, where $k^* \equiv [(1-\alpha)\alpha/n]^{1/(1-\alpha)}$ is the level of capital that maximizes the demand for government bonds, and $k_{\text{gold}} \equiv (\alpha/n)^{1/(1-\alpha)}$ is the golden-rule level of capital. Totally differentiating equation (A.5) with respect to k and n yields

$$\begin{aligned}\frac{\partial k}{\partial n} =& \frac{(1-\alpha)k^\alpha - nk + (\alpha k^{\alpha-1} - n)k}{\alpha(\alpha-1)k^{\alpha-2}((1-\alpha)k^\alpha - nk) + (\alpha k^{\alpha-1} - n)((1-\alpha)\alpha k^{\alpha-1} - n)} \\ =& -\frac{d + (R^k - n)k}{\alpha(1-\alpha)k^{\alpha-2}d + (R^k - n)(n - (1-\alpha)\alpha k^{\alpha-1})}\end{aligned}$$

where $d = (1-\alpha)k^\alpha - nk$. The numerator is positive since we consider the dynamic efficient economy: $R^k - n > 0$. The denominator is also positive since we limit our attention to $k > k^*$. Since $\partial k/\partial n < 0$ and the marginal return of capital is decreasing in k, it follows that $\partial R^k/\partial n > 0$. Since $R^k = R^n/\pi$ and R^n is constant, set by the central bank, the inflation rate is decreasing in n: $\partial \pi/\partial n < 0$.

B Quantitative Model

B.1 Competitive equilibrium

The competitive equilibrium of the quantitative model studied in Section 4 can be defined as follows. Given prices, government policies and demographic variables, all firms maximize their profits, all households maximize their utility, and all markets clear. Here we simply state the two market clearing conditions that have not yet been reported in the main text. First, the aggregate household illiquid assets, denoted by $A_t \equiv \sum_{j=1}^{J} \bar{a}_{j,t} N_{j,t}$ with $\bar{a}_{j,t} = \psi_j a_{j,t}(1) + (1-\psi_j) a_{j,t}(0)$, are equal to the sum of capital and the value of all ownership shares of intermediate goods firms:

$$A_t = K_{t+1} + V_t.$$

Second, Walras' Law implies the market clearing condition for the final good

$$C_t + I_t + G_t = Y_t - \frac{\gamma}{2} (\pi_t - 1)^2 Y_t - X_t,$$

where $X_t = \sum_{j=1}^{J} \bar{\chi}_{j,t} N_{j,t}$, with $\bar{\chi}_{j,t} = \psi_j \chi_{j,t}(1) + (1-\psi_j)\chi_{j,t}(0)$, is the aggregate cost of adjusting illiquid assets. Observe that the aggregate costs of price adjustments and illiquid asset adjustments are modeled as explicit resource costs and consequently subtracted from the aggregate output.